UNIT II

SHAFTS AND COUPLINGS

PART - A

1. Classify keys with its applications? (May 2012)

(a) Saddle key- It is applicable where light load is used.

(b) Sunk key – It is used to connect pulleys where is moderate load is applied.

(c) Woodruff key- Used to transmit small amount of torque in automotives.

2. Discuss the forces on key? (Dec 2012, Dec 2014)

(a) Shear force

(b) Bearing force

(c) Tensile force

3. What are the various stresses induced in shafts? (May 2014)

(a) Shear due to torsion

(b) Stress due to bending

(c) Axial stress if an axial load acts.

4. Name any two of the rigid coupling? (May 2014)

(a) Sleeve couplings

(b) Flange couplings

(c) Clamp couplings

5. What is the difference between rigid and flexible coupling? (May 2013, May 2016)

Rigid coupling: It is used in low speed applications where a good axial alignment between connecting shafts can be achieved.

Flexible Coupling: The shafts having longitudinal, lateral and angular misalignment are connected using flexible coupling.

6. How is the strength of a shaft affected by the keyway? (May2014)

The keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced.

7. What is the main use of woodruff key? (Nov 2013)

It is used to transmit less torque in automotive and machine tool industries. The keyway in the shaft is milled in a curved shape whereas the key way in the hub is usually straight.

8. A shaft of 70 mm long is subjected to shear stress of 40 Mpa and has an angle of twist equal to 0.017 radian. Determine the diameter of the shaft. Take G= 80 Mpa? (Nov 2013)

Given data:

Length of the shaft, l= 750mm

Shear stress, τ= 40 N/mm^2

Angle of twist, Θ=0.017 radian

Modulus of rigidity, G=0.8 $\times 10^5$ N/mm^2

To find:

Diameter of the shaft, d

Solutions:

Torsional moment of the shaft, $M_t = (\pi/16) \times \tau \times d^3$

Angle of twist, $\Theta = (M_t \times l)/(GJ)$

Where $J = (\pi/32) \times d^4$

Angle of twist, $0.017 = (2\times40\times750)/(0.8\times10^5\times d)$

d=44.11 mm

Standard diameter, d=45 mm

9. Why a hollow shaft has greater strength and stiffness than solid shaft of equal weight? (Nov 2012)

Stresses are maximum at the outer surface of a shaft. A hollow shaft has almost all the materials concentrated at the outer circumference. So, it has better strength and stiffness for equal weight.

10. Indicate the effects of providing key ways in the shaft? (Nov 2010)

(a) It reduces strengths of the shaft because of material removal.

(b) It increases stress concentration.

11. What do you mean by stiffness and rigidity with reference to shafts? (Dec 2010)

Stiffness is the resistance offered by the shaft for twisting and rigidity is the resistance offered by the shaft for lateral bending.

12. Differentiate between keys and splines? (Nov 2011)

Key: A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.

Splines: Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts or splines. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway. The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.

13. Under what circumstances flexible couplings are used? (Nov 2012)

(a) They are used to join the abutting ends of shafts when they are not in exact alignment.

(b) They are used to permit an axial misalignment of the shafts without under absorption of the power, which the shafts are transmitting.

14. How is flexibility achieved in flexible coupling? (Nov 2010)

(a) Kinematic arrangement such as loosely fit members

(b) Using rubber such as materials

15. Suggest suitable couplings for, shafts with parallel misalignment, shafts with angular misalignment of 100, shafts in perfect alignment?

Flexible coupling such as spring coupling can be used for shafts with parallel misalignment. Universal coupling is suitable for shafts with angular misalignment of 100 . Rigid coupling can be used for shafts in a perfect alignment.

16. Define equivalent torsional moment of a shaft. (April 2017)

The expression $\sqrt{M^2 + T^2}$ is known as equivalent twisting moment and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment.

UNIT – II

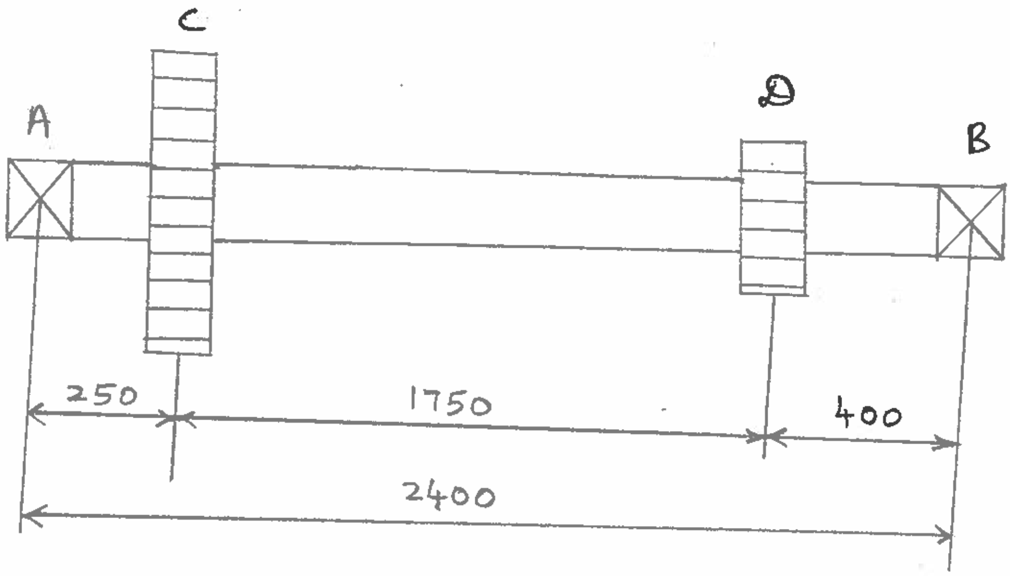
SHAFTS AND COUPLINGS

1. A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250mm and 400mm respectively from the centreline of the left and right bearings. The pitch diameter of the gear C is 600mm and that of gear D is 200mm. The distance between the centre line of the bearings is 2400mm. The shaft transmits 20kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.

Find the diameter of the shaft, if the working stress is 100MPa in tension and 56MPa in shear. The gears C and D weighs 950N and 350N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

[Nov/Dec 2012]

GIVEN DATA:



AC = 250mm ; BD = 400mm

D_C = Pitch Diameter of Gear 'C' = D_c

D_c = 600mm ; ∴ R_c = 300mm

D_D = Pitch Diameter of Gear 'D'

D_D = 200mm ; R_D = 100mm

AB = 2400mm ; Power = P = 20 kW = 20×10^3 W

N = 120 r.p.m. ; $\sigma_t = 100 MPa = 100 N/mm^2$

$\tau = 56 MPa = 56 N/mm^2$

W_c = Weight of Gear 'C' = 950N

W_D = Weight of Gear 'D' = 350N

K_b = Bending Factor = 1.5

K_t = Torsion factor = 1.2

To FIND: d = Diameter of shaft.

SOLUTION:

I Load acting on Shaft at 'C', (P_c):

$$P = \frac{2\pi N T}{60}$$

$$\therefore T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m}$$

$$= 1590 \times 10^3 \text{ N-mm}$$

T = Torque = M_t

$M_t = 1590 \times 10^3$ N-mm

F_{tc} = Tangential Force acting at Gear 'C'.

$$= \frac{M_t}{R_c} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

Load acting on Shaft at C, $P_c = F_{tc} + W_c$

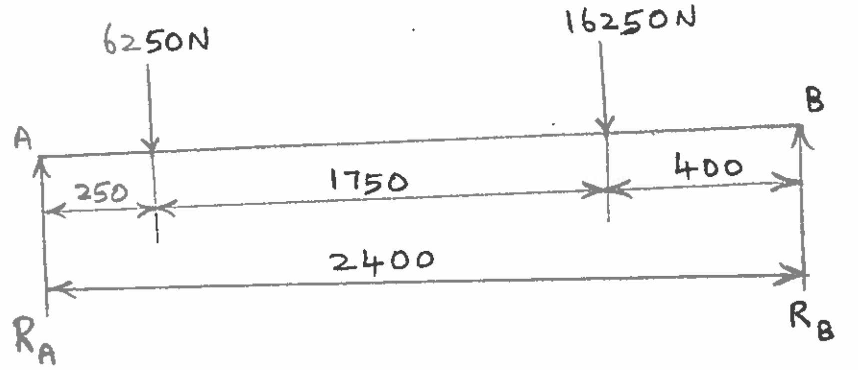
$$P_c = F_{tc} + W_c$$
$$= 5300 + 950 = 6250 \text{ N}$$

II Load acting on Shaft at D, (P_D):

F_{tD} = Tangential Force acting at Gear 'D'

$$= \frac{M_t}{R_D} = \frac{1590 \times 10^3}{100} = 15900 \text{ N}$$

$$P_D = F_{tD} + W_D = 15900 + 350 = 16250 \text{ N}$$



III Reactions at A, R_A and at B, R_B:

$R_A + R_B$ = Total load acting downwards at C & D

$= 6250 + 16250 = 22500\,N$

Taking moments about A

$R_B \times 2400 = (16250 \times 2000) + (6250 \times 250)$

$R_B = \dfrac{34062.5 \times 10^3}{2400}$

$R_B = 14190\,N$

$R_A = 22500 - 14190\,N$

$R_A = 8310\,N.$

IV Bending Moment at C, M_{bC} and at D, M_{bD}:

M_b = Bending Moment

$M_{bC} = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3\,N\text{-}mm$

$M_{bD} = R_B \times 400 = 14190 \times 400 = 5676 \times 10^3\,N\text{-}mm$

$M_{bD} = (R_A \times 2000) - (6250 \times 1750) = 5676 \times 10^3\,N\text{-}mm$

Maximum Bending Moment Transmitted by the Shaft is, $M_b = M_{bD} = 5676 \times 10^3\,N\text{-}mm$

V Equivalent Twisting Moment & Diameter of Shaft.

T_e = Equivalent Twisting Moment

$= \sqrt{(k_b M_b)^2 + (k_t M_t)^2}$

$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$

$= 8725 \times 10^3\,N\text{-}mm$

$$T_e = \frac{\pi}{16} \tau d^3$$

$$8725 \times 10^3 = \frac{\pi}{16} \times 56 \times d^3$$

$$\therefore d^3 = 793 \times 10^3$$

$$d = 92.5 mm$$

VI Equivalent Bending Moment and Diameter of Shaft

$$M_{beq} = \frac{1}{2}\left[(k_b \times M_b) + \sqrt{(k_b M_b)^2 + (k_t M_t)^2}\right]$$

$$= \frac{1}{2}\left[k_b M_b + T_e\right]$$

$$= \frac{1}{2}\left[1.5 \times 5676 \times 10^3 + 8725 \times 10^3\right]$$

$$= 8620 \times 10^3 \text{ N-mm}$$

$$M_{beq} = \frac{\pi}{32} \sigma_b d^3$$

$$8620 \times 10^3 = \frac{\pi}{32} \times 100 \times d^3$$

$$d^3 = 878 \times 10^3$$

$$d = 95.7 mm$$

Taking the larger of two values

$$d = 95.7 mm$$

$$\boxed{d \simeq 100 mm}$$

2. A shaft is supported by two bearings placed 1m apart. A 600mm diameter pulley is mounted at a distance of 300mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25kN. Another pulley 400mm diameter is placed 200mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and $\mu = 0.24$. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley. [APR/MAY 2010]

GIVEN DATA:

$AB = 1m$; $D_C = 600mm$; $R_C = 300mm = 0.3m$

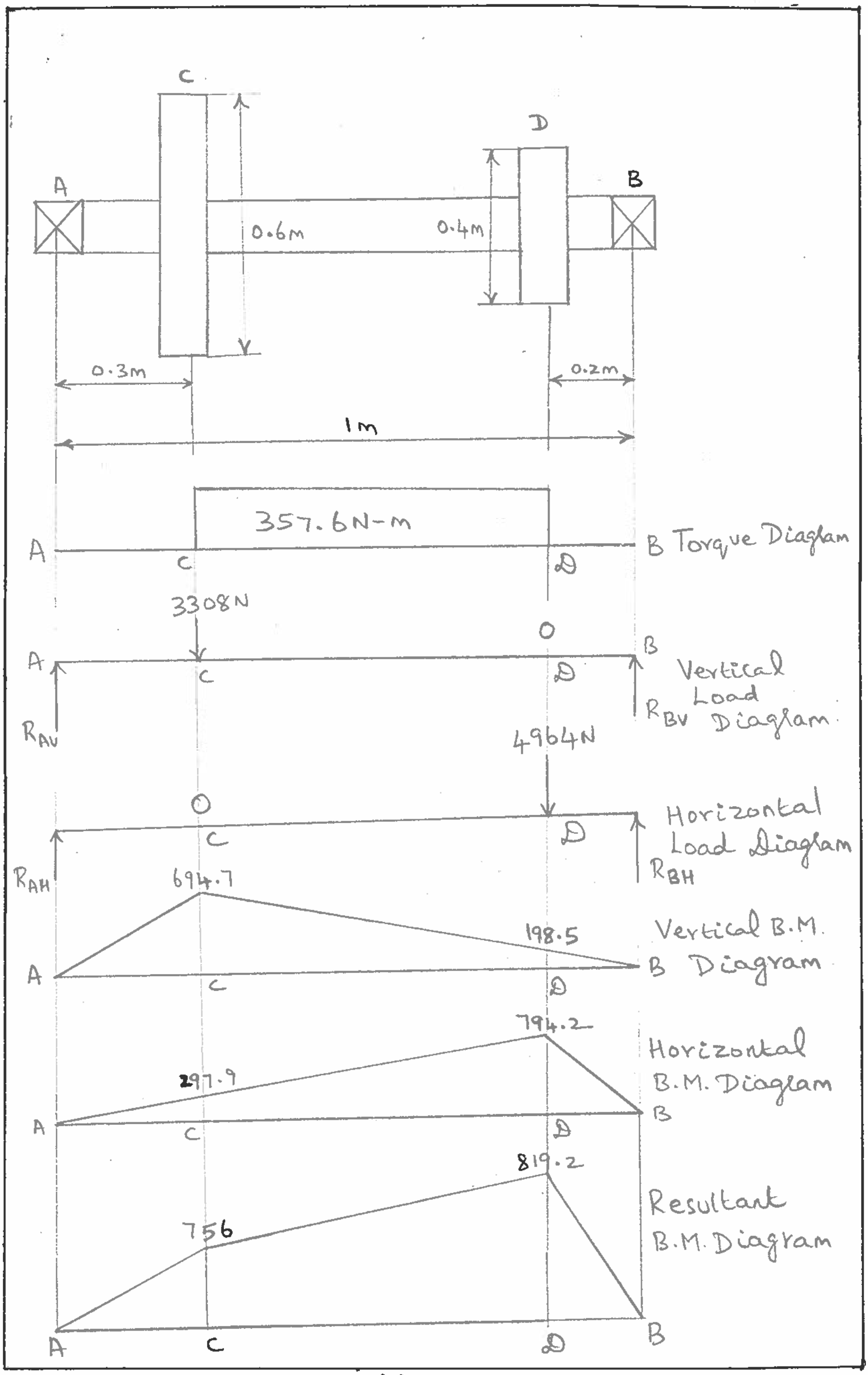
$AC = 300mm = 0.3m$; $T_1 = 2.25kN = 2.25 \times 10^3 N$

$D_D = 400m$; $R_D = 200mm = 0.2m$; $BD = 200mm = 0.2m$

$\theta = 180° = \pi\, rad$; $\mu = 0.24$; $\sigma_b = 63MPa = 63N/mm^2$

$\tau = 42MPa = 42N/mm^2$; $M_{tC} = M_{tD}$

To FIND: d = Diameter of solid shaft



C
D
A
B
0.6m
0.4m
0.3m
0.2m
1m
357.6N-m
A
C
D
B Torque Diaglam
3308N
O
A
C
D
B
Vertical Load Diaglam
R_{AV}
R_{BV}
4964N
O
C
D
Horizontal Load Diaglam
R_{AH}
R_{BH}
694.7
198.5
A
C
D
B
Vertical B.M. Diagram
794.2
297.9
A
C
D
B
Horizontal B.M. Diaglam
819.2
756
Resultant B.M. Diagram
A
C
D
B

I Vertical Loading on Shaft at C and D:

T_1 = Tension in tight side of belt on Pulley C

= 2250 N

T_2 = Tension in slack side of belt on Pulley C

$$\frac{T_1}{T_2} = e^{\mu . \theta} ; \quad \frac{T_1}{T_2} = e^{0.24 \times \pi} ; \quad \frac{T_1}{T_2} = 2.127$$

$$T_2 = \frac{2250}{2.127} = 1058 N$$

Vertical Load acting on Shaft at C.

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 N$$

Vertical Load acting on Shaft at D

$$W_D = 0$$

Torque acting on Pulley C

$$T_C = (T_1 - T_2) R_C = (2250 - 1058) 0.3$$

$$= 357.6 \text{ N-m}$$

II Horizontal Loading on Shaft at C and D:

$$T_D = (T_3 - T_4) R_D = (T_3 - T_4) 0.2$$

$$\frac{357.6}{0.2} = T_3 - T_4 \quad (\because T_C = T_D)$$

$$T_3 - T_4 = 1788 N$$

$$\frac{T_1}{T_2} = \frac{T_3}{T_4} = 2.127$$

$$\frac{T_3}{T_4} = 2.127 ; \quad T_3 = 2.127 T_4$$

$$T_3 = 3376 N$$

$$T_4 = 1588 N$$

Horizontal load acting on shaft at D

$W_D = T_3 + T_4 = 3376 + 1588 = 4964\,N$

Horizontal load acting on the shaft at C

$W_c = 0$

III Bending Moment for Vertical Loading:

R_{AV} = Reaction at A due to vertical Load

R_{BV} = Reaction at B due to Vertical Load

$R_{AV} + R_{BV} = 3308\,N$

Taking moments about A

$R_{BV} \times 1 = W_c \times 0.3$

$= 3308 \times 0.3$

$R_{BV} = 992.4\,N$

$R_{AV} = 3308 - 992.4$

$= 2315.6\,N$

Bending Moment at A, $M_{bAV} = 0$

Bending Moment at B, $M_{bBV} = 0$

Bending Moment at C, $M_{bCV} = R_{AV} \times 0.3$

$= 2315.6 \times 0.3 = 694.7\,N\text{-}m$

Bending Moment at D, $M_{bDV} = R_{BV} \times 0.2$

$= 992.4 \times 0.2 = 198.5\,N\text{-}m$

IV Bending Moment for Horizontal Loading:

R_{AH} = Reaction at Bearing A due to Horizontal Load

R_{BH} = Reaction at Bearing B due to Horizontal Load.

$R_{AH} + R_{BH} = 4964\,N$

Taking Moment about A

$R_{BH} \times 1 = 4964 \times 0.8$

$\therefore R_{BH} = 3971\,N$

$R_{AH} = 4964 - 3971$

$= 993\,N.$

Bending Moment at A, $M_{bAH} = 0$

Bending Moment at B, $M_{bBH} = 0$

Bending Moment at C, $M_{bCH} = R_{AH} \times 0.3 = 993 \times 0.3$

$= 297.9$ N-m

Bending Moment at D, $M_{bDH} = R_{BH} \times 0.2 = 3971 \times 0.2$

$= 794.2$ N-m

V Resultant and Maximum Bending Moment

Resultant Bending Moment at C,

$$M_{bC} = \sqrt{(M_{bCV})^2 + (M_{bCH})^2}$$

$$= \sqrt{(694.7)^2 + (297.9)^2}$$

$$= 756 \text{ N-m}$$

Resultant Bending Moment at D,

$$M_{bD} = \sqrt{(M_{bDV})^2 + (M_{bDH})^2}$$

$$= \sqrt{(198.5)^2 + (794.2)^2}$$

$$= 819.2 \text{ N-m}$$

Maximum Bending Moment, $M_b = M_{bD}$

$$M_b = 819.2 \text{ N-m}$$

VI Equivalent Twisting Moment and Diameter of Shaft.

$$T_e = \sqrt{M_b^2 + M_t^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$

$$T_e = \frac{\pi}{16} \tau d^3$$

$$10^3 \times 894 = \frac{\pi}{16} \times 42 \times d^3$$

$$d^3 = 108 \times 10^3$$

$$\boxed{d = 47.6 \text{mm}}$$

VII Equivalent Bending Moment and Diameter of Shaft:

$$M_{beq} = \frac{1}{2}\left(M_b + \sqrt{M_b^2 + T^2}\right) = \frac{1}{2}(M_b + T_e)$$

$$= \frac{1}{2}(819.2 + 894) = 856.6 \text{ N-m}$$

$$= 856.6 \times 10^3 \text{ N-mm}$$

$$M_{beq} = \frac{\pi}{32}\,\sigma_b\, d^3$$

$$856.6 \times 10^3 = \frac{\pi}{32} \times 63 \times d^3$$

$$d^3 = 138.2 \times 10^3$$

$$\boxed{d = 51.7\text{mm}}$$

Taking larger of two values

$$d = 51.7\text{mm}$$

$$\boxed{d \simeq 55\text{mm}}$$

3. A Solid Circular shaft is subjected to a bending moment of 3000N-m and a torque of 10,000N-m. The shaft is made of 45C8 Steel having ultimate tensile stress of 700MPa and a Ultimate shear Stress of 500MPa. Assuming a Factor of safety as 6. Determine the diameter of the shaft.

[MAY/JUNE 2009]

GIVEN DATA:

$M_b = 3000\ N\text{-}m = 3000 \times 10^3\ N\text{-}mm$

$M_t = 10{,}000\ N\text{-}m = 10{,}000 \times 10^3\ N\text{-}mm$

$\sigma_U = 700\ MPa = 700\ N/mm^2$

$\tau_U = 500\ MPa = 500\ N/mm^2$

$F.O.S = n = 6$

TO FIND:

d = Diameter of the shaft.

SOLUTION:

I Equivalent Twisting Moment & Diameter of Shaft:

$$T_e = \sqrt{(K_b M_b)^2 + (K_t M_t)^2}$$

Assume, K_b & $K_t = 1$.

$$T_e = \sqrt{(3000 \times 10^3)^2 + (10{,}000 \times 10^3)^2}$$

$$= 10.44 \times 10^6\ N\text{-}mm.$$

$$T_e = \frac{\pi}{16}[\tau]\,d^3$$

Where $[\tau]$ = Design shear Stress

$$= \frac{\tau_u}{n} = \frac{500}{6} = 83.3\ N/mm^2$$

$$10.44\times10^6 = \frac{\pi}{16}\times 83.3\, d^3$$

$$d^3 = \frac{16\times 10.44\times10^6}{\pi\times 83.3}$$

$$\therefore \boxed{d = 86\text{mm}}$$

<u>II Equivalent Bending Moment & Diameter of Shaft:</u>

$$M_{beq} = \frac{1}{2}\left[M_b + T_e\right]$$

$$= \frac{1}{2}\left[3000\times10^3 + 10.44\times10^6\right]$$

$$= 6.72\times10^6 \text{ N-mm}$$

$$M_{beq} = \frac{\pi}{32}\left[\sigma_b\right] d^3$$

where $[\sigma_b]$ = Design Bending Stress

$$= \left[\frac{\sigma_u}{n}\right] = \frac{700}{6} = 116.66\text{N/mm}^2$$

$$6.72\times10^6 = \frac{\pi}{32}\times116.66\times d^3$$

$$d^3 = \frac{32\times6.72\times10^6}{\pi\times116.66}$$

$$d = 83.7\text{mm}$$

Taking the larger of two values

$$d = 86\text{mm}$$

$$\boxed{d \simeq 90\text{mm}}$$

4. Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length. [MAY/JUNE 2012]

GIVEN DATA:

$$d_o = d \; ; \; d_i = \frac{1}{2} d_o \; ; \therefore \frac{d_i}{d_o} = 0.5 = k$$

To Find: Comparison of ① Weight ② Strength ③ Stiffness

SOLUTION:

I Comparison of Weight

W_S = Weight of Solid Shaft.

$$= \frac{\pi}{4} d^2 \times l_s + \rho_s$$

$$= \text{Area} \times \text{length} \times \text{Density}$$

W_{HS} = weight of Hollow Shaft

$$= \text{Area} \times \text{length} \times \text{Density}$$

$$= \frac{\pi}{4}\left(d_o^2 - d_i^2\right) \times l_{HS} \times \rho_{HS}$$

$$\frac{W_{HS}}{W_S} = \frac{d_o^2 - d_i^2}{d^2} = \frac{d_o^2 - d_i^2}{d_o^2} = 1 - \left(\frac{d_i}{d_o}\right)^2$$

$$= 1 - k^2 = 1 - (0.5)^2$$

$$\boxed{\frac{W_{HS}}{W_S} = 0.75}$$

II Comparison of Strength:

T_s = Torque Transmitted by Solid Shaft.

$$= \frac{\pi}{16} \tau d^3$$

T_{HS} = Torque Transmitted by Hollow Shaft.

$$= \frac{\pi}{16} \tau d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{16} \tau d_o^3 \left[1 - k^4\right]$$

$$\frac{T_{HS}}{T_s} = \frac{d_o^3 (1-k^4)}{d^3} = \frac{\cancel{d_o^3}(1-k^4)}{\cancel{d_o^3}}$$

$$= \left[1 - (0.5)^4\right]$$

$$\boxed{\frac{T_{HS}}{T_s} = 0.9375}$$

III Comparison of Stiffness

S_s = Stiffness of Solid shaft

$$= \frac{\pi}{32} d^4$$

S_{HS} = Stiffness of Hollow Shaft

$$= \frac{\pi}{32} \left(d_o^4 - d_i^4\right)$$

$$\frac{S_{HS}}{S_s} = \frac{d_o^4 - d_i^4}{d^4} = \frac{d_o^4 - d_i^4}{d_o^4}$$

$$= 1 - \left(\frac{d_i}{d_o}\right)^4 = 1 - k^4$$

$$= 1 - (0.5)^4$$

$$\boxed{\frac{S_{HS}}{S_s} = 0.9375}$$

5. In an axial flow rotary Compressor, the shaft is subjected to a maximum torque of 1500 Nm and a maximum bending moment of 3000 Nm. Neglecting the axial load on the Compressor shaft, determine the diameter of the shaft. Assume that the load is applied gradually. The shear stress in the shaft is limited to 50 N/mm². Also design a hollow shaft for the above Compressor taking inner diameter as 0.4 times the outer diameter. What is the percentage of material saving in hollow shaft.

[NOV/DEC 2014]

GIVEN DATA:

$M_t = 1500 \, Nm = 1500 \times 10^3 \, N\text{-}mm$

$M_b = 3000 \, Nm = 3000 \times 10^3 \, N\text{-}mm$

$[\tau]_{shaft} = 50 \, N/mm^2$; $d_i = 0.4 d_o$; $\frac{d_i}{d_o} = 0.4 = k$.

TO FIND: ① d = diameter of shaft

② d_o & d_i = outside & Inside diameter of hollow shaft.

③ Percentage saving of Material.

I Equivalent Torque

Assume Fatigue factors: $k_b = 1.5$ & $k_t = 1$

$$T_e = \sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \sqrt{(1.5 \times 3000)^2 + (1 \times 1500)^2}$$

$$= 4743 \, N\text{-}m$$

$$= 4743 \times 10^3 \, N\text{-}mm.$$

II Shaft Diameter, d:

$$T_e = \frac{\pi}{16}[\tau]\, d^3$$

$$d^3 = \frac{16 \times T_e}{\pi \times [\tau]} = \frac{16 \times 4743 \times 10^3}{\pi \times 50} = 483118.0129$$

$$d = 78.46\,mm$$

$$\boxed{d \simeq 80mm}$$

III Design of Hollow Shaft:

$$T_e = \frac{\pi}{16}[\tau]\, d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{16}[\tau]\, d_o^3 \left[1 - k^4\right]$$

$$d_o^3 = \frac{16 \times T_e}{\pi \times [\tau] \times (1-k^4)}$$

$$= \frac{16 \times 4743 \times 10^3}{\pi \times 50 \times (1-0.4^4)}$$

$$d_o = 79.14\,mm$$

$$\boxed{d_o \simeq 80mm}$$

$$d_i = d_o \times 0.4 = 80 \times 0.4$$

$$\boxed{d_i = 32mm}$$

IV Percentage Saving of Material:

$$\%\ \text{Saving} = \frac{W_s - W_{HS}}{W_s} \times 100$$

$$= \frac{A_s \times l_s \times \rho_s - A_{HS} \times l_{HS} \times \rho_{HS}}{A_s \times l_s \times \rho_s}$$

$$l_S = l_{HS}$$
$$P_S = P_{HS}$$

$$\therefore \left.\begin{matrix}\%\text{ Saving of}\\ \text{Material}\end{matrix}\right\} = \frac{A_S - A_{HS}}{A_S}$$

$$= \frac{\frac{\pi}{4}d^2 - \frac{\pi}{4}\left(d_o^2 - d_i^2\right)}{\frac{\pi}{4}d^2}$$

$$= \frac{d^2 - \left(d_o^2 - d_i^2\right)}{d^2}$$

$$= \frac{80^2 - \left(80^2 - 32^2\right)}{80^2} \times 100$$

$$\boxed{\%\text{ Saving of Material} = 16\%}$$

6. A mild steel shaft has to transmit 80KW at 200 r.p.m. The allowable shear stress in the shaft is limited to 45N/mm². Allowable shear stress in the key material is 45N/mm². Crushing stress for bolt and key is 160N/mm². Shear stress for bolt material is 30N/mm². Shear stress for cast iron is 8 N/mm². Design and draw a cast iron flange coupling of protected type. [NOV/DEC 2013]

GIVEN DATA:

$P = 80\text{KW} = 80\times10^3\text{W}$; $N = 200\text{ r.p.m}$

$[\tau]_{shaft} = 45\text{N/mm}^2$; $[\tau]_{key} = 45\text{N/mm}^2$

$[\sigma_c]_{bolt} = [\sigma_c]_{key} = 160\text{N/mm}^2$; $[\sigma_c]$ = Design Crushing Stress.

$[\tau]_{C.I} = [\tau]_{Hub} = 8 N/mm^2$

To FIND: Design Flange Coupling.

SOLUTION:

I Design Torque, Td:

$$T_d = \text{Nominal Torque} \times \text{Service factor}$$

Assume Service factor $= 1.25$

$$\text{Power, } P = \frac{2\pi N T}{60}$$

$$\therefore T = \frac{P \times 60}{2\pi N} = \frac{80 \times 10^3 \times 60}{2\pi \times 200}$$

$$= 3819.7 Nm$$

$$T_d = 3819.7 \times 1.25 = 4774.65 N\text{-}m$$

$$= 4774.65 \times 10^3 N\text{-}mm$$

II Shaft Diameter

$$T = \frac{\pi}{16} \tau d^3$$

$$\therefore \tau = \frac{16 \times T}{\pi \times d^3} \leq [\tau]_{shaft}$$

$$\frac{16 \times 4774.65 \times 10^3}{\pi d^3} \leq 45$$

$$d^3 \geq \frac{16 \times 4774.65 \times 10^3}{\pi \times 45}$$

$$\geq 81.45 mm$$

$$\boxed{d \geq 85 mm}$$

III Other Dimensions of the Coupling:

1. Hub Diameter, $D = 2d = 2 \times 85 = 170\,mm$
2. Hub Length, $l = 1.5 \times d = 1.5 \times 85 = 127.5\,mm$
   $\simeq 130\,mm$
3. Bolt Circle Diameter, $B.C.D = 3d = 3 \times 85$
   $= 255\,mm$
4. From DDB. P.No: 5.19, for 85mm Shaft diameter the key dimensions are width, $b = 22\,mm$ and Height, $h = 14\,mm$.
5. Flange Thickness, $t_f = \frac{d}{2} = 42.5\,mm$

IV Design of Hub as a Hollow Shaft:

$$T_d = \frac{\pi}{16} \tau_{hub} \left[\frac{D^4 - d^4}{D} \right]$$

$$\tau_{Hub} = \frac{T_d \times 16 \times D}{\pi (D^4 - d^4)}$$

$$= \frac{16 \times 4774.6 \times 10^3 \times 170}{\pi (170^4 - 85^4)}$$

$$= 5.3\,N/mm^2$$

$$\leq [\tau]_{C.I.}$$

$$\leq 8\,N/mm^2.$$

$\therefore$ Design is safe and Satisfactory.

Note:

No. of bolts = 3, for shaft diameter upto 40mm

No. of bolts = 4, for shaft diameter 40 to 100mm.

No. of bolts = 6, for shaft diameter 100 to 180mm.

V Design of Bolts:

n = No. of bolts = 4

Tangential force acting on the bolt circle = F_t

$$F_t = \frac{T_d}{\text{Bolt Circle Radius}} = \frac{T_d}{(3d/2)}$$

$$= \frac{4774.65 \times 10^3}{\left(\frac{255}{2}\right)}$$

$$= 37,448.2\,N$$

$$\text{Force / bolt} = F_{tb} = \frac{F_t}{n} = \frac{37,448.2}{4} = 9362\,N$$

Shear failure of Bolts

$$[\tau]_{Bolt} = \frac{F_{tb}}{\frac{\pi}{4} \times d_b^2}$$

$$F_{tb} = \frac{\pi}{4} d_b^2 \times [\tau]_{bolt}$$

$$9362 = \frac{\pi}{4} d_b^2 \times 30$$

$$\therefore d_b = 19.93\,mm \simeq 20mm$$

From DDB. P.NO: 5.49

$\boxed{\text{M20 Bolts Can be used}}$

Crushing failure of Bolts:

$$\sigma = \frac{F}{A} ; \quad \sigma_c = \frac{F_{tb}}{t_f \, d_b} = \frac{9362}{42.5 \times 20}$$

$$= 11 \text{ N/mm}^2$$

$$\leq [\sigma_c]$$

$$\leq 160 \text{ N/mm}^2$$

$\therefore$ Design is Safe and Satisfactory

VI Design of key

Shear failure of key

Tangential force on key $= F_{tK}$

$$F_{tK} = \frac{T_d}{(d/2)} = \frac{4774.65 \times 10^3}{(85/2)} = 112344.7 \text{N}$$

Induced Shear Stress, $\tau = \dfrac{F_{tK}}{b \times l} = \dfrac{112344.7}{22 \times 130}$

$$= 39.3 \text{ N/mm}^2$$

$$\leq [\tau]_{key}$$

$$\leq 45 \text{ N/mm}^2$$

Crushing failure of key:

σ_c = Induced crushing Stress

$$= \frac{F_{tK}}{l \times (h/2)} = \frac{112344.7}{130 \times \left(\frac{14}{2}\right)}$$

$$= 12.5 \text{ N/mm}^2$$

$$\leq [\sigma_c] = 160 \text{ N/mm}^2$$

$\therefore$ Design is Safe and Satisfactory

VII Design of Flange:

$$\left.\begin{array}{r}\text{Tangential force}\\ \text{on Hub}\end{array}\right\} = \text{Shearing Area} \times \text{Induced Shear Stress}$$

$$\frac{T_d}{(D/2)} = \pi D t_f \times \tau$$

$$\frac{4774.65 \times 10^3}{(170/2)} = \pi \times 170 \times 42.5 \times \tau$$

$$\tau = 2.47\,N/mm^2$$

$$\leq [\tau]_{Hub}$$

$$\leq 8\,N/mm^2$$

$\therefore$ Design is Safe and Satisfactory

7. A bushed pin type flange coupling is to be designed to transmit 25 kW at a speed of 1000 r.p.m. The following permissible stresses are used shear stress for the shaft and key are 55 N/mm^2; shear stress for the pin is 28 N/mm^2; Bearing pressure on rubber bush is 0.3 N/mm^2 and Crushing stress for the key is 100 N/mm^2 [Nov/Dec 2012]

GIVEN DATA:

$P = 25\,kW = 25 \times 10^3\,W$; $N = 1000$ r.p.m.

$[\tau]_{shaft} = [\tau]_{key} = 55\,N/mm^2$; $[\tau]_{pin} = 28\,N/mm^2$

$[p_b] = 0.3\,N/mm^2$; $[\sigma_c]_{key} = 100\,N/mm^2$

TO FIND: Design Flexible Coupling

I Diameter of the Shaft:

$$P = \frac{2\pi NT}{60}$$

$$\therefore T = \frac{P \times 60}{2\pi N} = \frac{25\times10^3 \times 60}{2\pi \times 1000} = 238.73\,N\text{-}M$$

$$= 238.73 \times 10^3\,N\text{-}mm$$

$$T = \frac{\pi}{16}\tau d^3$$

$$\tau = \frac{16\times T}{\pi d^3} \le [\tau]_{shaft}$$

$$\frac{16\times 238.7\times10^3}{\pi\times d^3} \le 55$$

$$\therefore d^3 \ge \frac{16\times238.7\times10^3}{\pi\times55}$$

$$d = 28.06\,mm$$

$$\boxed{d \simeq 30\,mm}$$

II Selection of Coupling:

From DDB. P.No: 7.108

Max. Rating at 100 RPM

$$= \frac{\text{kW of Power Application} \times \text{Service Factor} \times 100}{\text{RPM of Application}}$$

From DDB. P.No: 7.109

Service Factor = 1.5

$$\text{Maximum Rating} = \frac{25 \times 10^3 \times 1.5 \times 100}{1000}$$

$$= 3.75 \text{ kW} / 100 \text{ r.p.m}$$

From DDB. P.No: 7.108

choose Coupling No. 5 which has a rating of 4 kW per 100 r.p.m.

Data of Coupling No. 5:

$d_b = 25$ mm ; $n = 4$ bolts ; $G = 35$ mm ; $F = 12$ mm

$D = 120$ mm ; $t = 4$ mm

III Bearing Pressure in Rubber Bush:

DDB. P.No: 7.106

$$p_b = \frac{F_t}{d_b\left(G - \frac{2}{3}F\right) \times n}$$

F_t = Tangential Force on Bolt Circle

$$= \frac{\text{Torque}}{\text{Bolt Circle Radius } (D/2)}$$

$$= \frac{238.73 \times 10^3}{(120/2)} = 3978.8 \text{ N}$$

$$p_b = \frac{3978.8}{25\left(35 - \frac{2}{3} \times 12\right) \times 4}$$

$$= 1.47 \text{ N/mm}^2$$

$$> [p_b]$$

$$> 0.3 \text{ N/mm}^2$$

Trial 2:

Choose Coupling No. 7 which has a rating of 16 kW per 100 r.p.m.

$d_b = 40mm$; $n = 6$ bolts ; $G = 45mm$; $F = 16mm$

$D = 190mm$; $t = 5mm$

$$F_t = \frac{238.73 \times 10^3}{(190/2)} = 2513 N$$

$$p_b = \frac{2513}{40\left(45 - \frac{2}{3} \times 16\right) 6}$$

$$= 0.305 N/mm^2$$

Induced Bearing pressure is slightly greater than $[p_b]$. ∴ It can be accepted.

IV Design of Pin:

DDB. P.No: 7.106

M_b = Bending Moment on Pin

$$= \frac{F_t \left[t + \frac{1}{2}\left(G - \frac{2}{3} F\right)\right]}{n}$$

$$= \frac{2513 \left[5 + \frac{1}{2}\left(45 - \frac{2}{3} \times 16\right)\right]}{6}$$

$$= 9284 \text{ N-mm}$$

$$M_b = \frac{\pi}{32} \sigma_b F^3$$

$$\therefore \sigma_b = \frac{32 \times M_b}{\pi \times F^3} = \frac{32 \times 9284}{\pi \times 16^3} = 23 N/mm^2$$

τ_d = Direct Shear Stress

$$= \frac{\text{Force / Pin}}{\text{Sectional Area of Pin}}$$

$$= \frac{(F_t/n)}{\frac{\pi}{4} F^2} = \frac{(2513/6)}{\frac{\pi}{4} \times (16)^2}$$

$$= 2 \ N/mm^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_d^2}$$

$$= \sqrt{\left(\frac{23}{2}\right)^2 + 2^2}$$

$$= 11.73 \ N/mm^2$$

$$\leq [\tau]_{Bolt}$$

$$\leq 28 N/mm^2$$

$\therefore$ Design is Safe and Satisfactory

V Design of key:

From DDB. P.NO: 5.16

For 30mm Shaft diameter, the dimensions of key are; Width, $b = 8mm$ and Height, $h = 7mm$

DDB. P.NO: 7.108

Length of key = Hub Length = E = 63mm

$$\text{Tangential Force on Shaft} = \frac{\text{Torque}}{\text{Radius of Shaft}}$$

$$F_{ts} = \frac{238.73 \times 10^3}{(30/2)} = 15915.3 N$$

Shearing of key:

$$\tau_{key} = \frac{F_{ts}}{bE} = \frac{15915.3}{8 \times 63} = 31.6\ N/mm^2$$

$$< [\tau]_{key} = 55\ N/mm^2$$

$\therefore$ Design is Safe.

Crushing of key:

$$\sigma_c = \frac{F_{ts}}{E \times (h/2)} = \frac{15,915.3}{63 \times (7/2)}$$

$$= 72.2\ N/mm^2$$

$$\leq [\sigma_c]_{key} = 100\ N/mm^2$$

$\therefore$ Design is safe and Satisfactory

VI Design of Hub:

$$\tau_{Hub} = \frac{T_d \times 16 \times D}{\pi (D^4 - d^4)}$$

Here, $T_d = T$

$$= \frac{238.73 \times 10^3 \times 16 \times 120}{\pi (120^4 - 30^4)}$$

$$= 0.7\ N/mm^2$$

$$\leq [\tau]_{Hub\ (or)\ C.I}$$

$$\leq 15\ N/mm^2$$

$\therefore$ Design is Safe and Satisfactory

VII Design of Flange:

Tangential Force on Hub = Shearing Area × Induced Shear Stress.

$$\frac{T}{(D/2)} = \pi D t_f \times \tau \text{, where } t_f = 0.5 \times d = 0.5 \times 30 = 15 mm$$

$$\frac{238.73 \times 10^3}{(120/2)} = \pi \times 120 \times 15 \times \tau$$

$\therefore \tau = 0.7 N/mm^2$

$\leq [\tau]_{Hub} = 15 N/mm^2$

$\therefore$ Design is safe and Satisfactory