1.) Max. principle (or) normal stress theory (or) Max. stress theory (or) Rankine's theory:-

According to this theory, The failure (or) yielding occurs at a point in a member when the maximum principle (or) normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

σ_1 (or) σ_2 (or) σ_3 (whichever is max) = σ_y

Max. Normal Stress = $\frac{\sigma_{yt}}{F.S}$ (Ductile)

$\sigma_t = \frac{\sigma_u}{F.S}$ [Brittle]

For design

σ_1 (or) σ_2 (or) $\sigma_3 = \frac{\sigma_y}{n}$.

where, n = Factor of safety

σ_1 (or) σ_2 (or) $\sigma_3 = \frac{\sigma_y}{n}$ [for Brittle material].

This theory is best suited for brittle material.

2) Max. Shear stress theory (or) Max. Shear theory (or) Guest's theory:-

Acc. to this theory, the failure (or) yielding occurs at a point in a member when the max. shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test.

$\tau_{max} = \frac{\tau_{yt}}{F.S}$ ← shear for yield point

$(\sigma_1-\sigma_2)$ (or) $(\sigma_2-\sigma_3)$ (or) $(\sigma_3-\sigma_1)$ whichever is max. $= \frac{\sigma_y}{n}$.

Problems:

1) A bolt is subjected to a tensile load of 25kN and Shear load of 10kN. Determine the diameter of bolt acceleration to

i) Maximum principle stress theory.

ii) Maximum principle strain theory.

iii) Maximum shear stress theory.

Assume the factor of safety as 2.5.

Yield point stress = $300\ N/mm^2$.

Possion's ratio = 0.25.

Given data:-

$P = 25\ KN = 25\times10^3 N$.

Shear load (P_s) = $10KN = 10\times10^3 = 10\times10^3 N$.

Yield point stress (σ_y) = $300\ N/mm^2$.

Possion's ratio $\left(\frac{1}{m}\right) = (\gamma) = 0.25$

FOS = 2.5.

To find:

$\sigma_t = ?$

$\tau = ?$

Solution-

Axial tensile stress

$$\sigma_x = \sigma_t = P/A = \frac{25\times10^3}{\pi/4\times d^2} = \frac{31.83\times10^3}{d^2}.$$

transverse shear stress

$$\tau = \frac{P_s}{A} = \frac{10\times10^3}{\pi/4\, d^2} = \frac{12.73\times10^3}{d^2}$$

$\sigma_x = \sigma_t$.

$\tau_{xy} = \tau$. $\sigma_y = 0$.

From D.D.B Pg.no. 7.2

$$\sigma_{1,2} = \frac{1}{2}\left[(\sigma x+\sigma y)\right] \pm \sqrt{(\sigma x-\sigma y)^2+4\tau xy^2}$$

Maximum principle stress:-

$$\sigma_1 = \frac{1}{2}\left[\sigma x+\sigma y\right] + \sqrt{(\sigma x-\sigma y)^2+4\tau xy^2}$$

$$\sigma_1 = \frac{1}{2}\left[\frac{31.83\times10^3}{d^2} + \sqrt{\left(\frac{31.83\times10^3}{d^2}\right)^2 + 4\left[\frac{12.73\times10^3}{d^2}\right]^2}\right]$$

$$\sigma_1 = \frac{1}{2}\left[\frac{31.83\times10^3}{d^2} + \frac{1}{d^2}\sqrt{(31.83\times10^3)^2+4(12.73\times10^3)^2}\right]$$

$$\sigma_1 = \frac{1}{2}\left[\frac{31.83\times10^3}{d^2} + \frac{40.76\times10^3}{d^2}\right]$$

$$\sigma_1 = \frac{1}{2}\left[\frac{72.59\times10^3}{d^2}\right]$$

$$\boxed{\sigma_1 = \frac{36.295\times10^3}{d^2}}$$

Minimum principle stress

$$\sigma_2 = \frac{1}{2}\left[(\sigma x+\sigma y) - \sqrt{(\sigma x-\sigma y)^2+4\tau xy^2}\right]$$

$$\sigma_2 = \frac{1}{2}\left[\frac{31.83\times10^3}{d^2} - \frac{40.76\times10^3}{d^2}\right]$$

$$\boxed{\sigma_2 = -\frac{4.446\times10^3}{d^2}}$$

$\therefore$ σ_1 = larger value.

$$\sigma_1 = \frac{36.295\times10^3}{d^2}$$

$$\sigma_1 = \frac{\sigma y}{Fos} = \frac{36.295\times10^3}{d^2} = \frac{300}{2.5}$$

(2.5)

$$\boxed{d = 17.39\,mm}$$

$$FOS = \frac{\text{Yield stress } (\sigma y) \text{ (or) max. stress}}{\sigma \text{ (working stress (or) permissible stress)}}$$

$$\sigma_1 = \frac{\text{Yield stress (or) max. stress}}{FOS}$$

Maximum principal Strain theory:-

$\sigma_1 - \gamma[\sigma_2+\sigma_3] = \frac{\sigma_y}{n\ (FOS)}$ ← Yield stress (from P.S.G DDB pg.no 7.3)

$$\frac{36.295\times10^3}{d^2} - 0.25\left[\frac{-4.446\times10^3}{d^2}\right] = \frac{300}{2.5}$$

$$\frac{36.295\times10^3}{d^2} + \frac{1.11\times10^3}{d^2} = 120$$

d = 17.65mm.

Maximum shear stress theory:- (DDB pg.no 7.3)

Maximum shear stress. $= \sigma_1 - \sigma_2 = \frac{\sigma_y}{n}$ (or) $\frac{\sigma_y}{Fos}$.

$$= \frac{36.295\times10^3}{d^2} - \left[\frac{-4.446\times10^3}{d^2}\right] = \frac{300}{2.5}$$

$$= \frac{40.771\times10^3}{d^2} = \frac{300}{2.5}$$

$$\frac{40.71\times10^3}{d^2} = \frac{300}{2.5} \Rightarrow d = 18.43\text{ mm}$$

Major dia of bolt is 18.43 mm..