Solution:-

Given

$\sigma_t(max) = 140\,MPa \Rightarrow 140\,N/mm^2$.

$R_i = 25mm$.

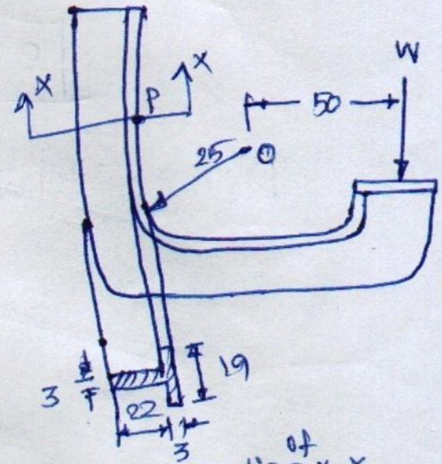
$R_o = 25+25 = 50mm$.

$b_i = 19mm$.

$t_i = 3mm$

$t = 3mm$.

$h = 25mm$.



section of x-x.

All dimensions in mm

Wkt, the area of section at x-x

$A = 3\times22 + 3\times19 = \underline{\underline{123\,mm^2}}$.

Wkt, Radius of curvature of the neutral axis,

$$R_n = \frac{(b_i - t)\,t_i + (t\times h)}{(b_i - t)\log_e\left[\frac{R_i+t_i}{R_i}\right] + t\log_e\left[\frac{R_o}{R_i}\right]}$$

[From PSG DDB Pg-no 6.3].

$$R_n = \frac{(19-3)\,3 + (3\times25)}{(19-3)\log_e\left[\frac{25+3}{25}\right] + 3\log_e\left[\frac{50}{25}\right]}$$

$$R_n = \frac{123}{16\times0.113 + 3\times0.693} = \frac{123}{3.887} = \underline{\underline{31.64mm}}.$$

Radius of curvature of the central axis

$$R = R_i + \frac{\frac{1}{2}h^2\cdot t + \frac{1}{2}t_i^2(b_i - t)}{h\cdot t + t_i(b_i - t)}$$

$$= 25 + \frac{\frac{1}{2}\times25^2\times3 + \frac{1}{2}\times3^2(19-3)}{25\times3 + 3(19-3)} = 25 + \frac{937.5+72}{75+48}$$

$R = 25 + 8.2 \Rightarrow \boxed{R = 33.2mm.}$

Distance b/w central axis to the neutral axis,

$$e = R - R_n = 33.2 - 31.64$$

$$\boxed{e = 1.56\,mm.}$$

Distance b/w the load w ~~to the~~ and the centroidal axis,

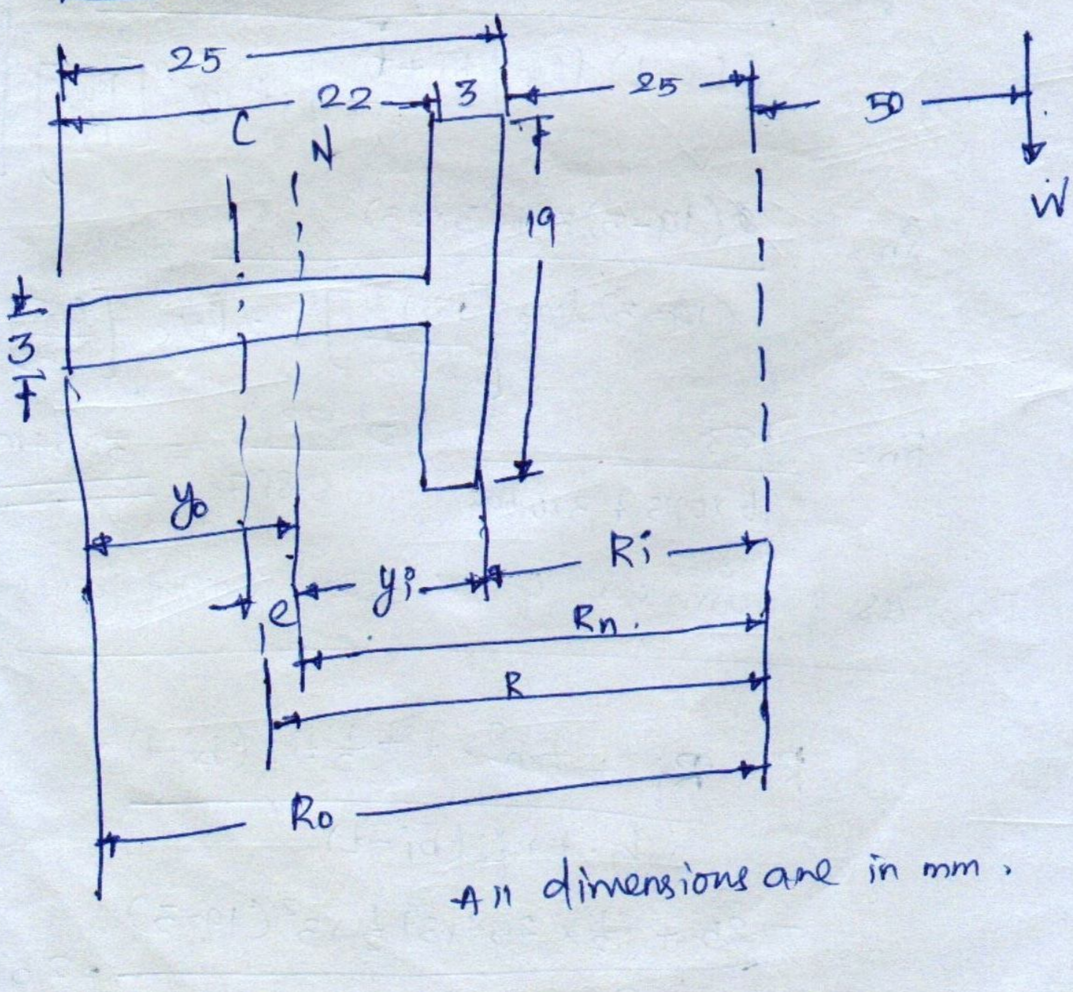
$$x = 50 + R = 50 + 33.2 = 83.2\,mm.$$

$$\boxed{x = 83.2\,mm.}$$

∴ Bending moment about the centroidal axis,

$$M = W.x \Rightarrow W \times 83.2 \Rightarrow 83.2\,W\,N\text{-}mm$$

$$\boxed{M = 83.2\,W\,N\text{-}mm}$$



All dimensions are in mm.

The section at x-x is subjected to a direct tensile load of W and a bending moment of 83.2 W.

The max. tensile stress will occur at point P. (i.e. at the ~~inner~~ inside fibre of the section).

Distance from the neutral axis to the point P,

$$y_i = R_n - R_i = 31.64 - 25 = 6.64\,mm$$

Direct tensile stress at section x-x

$$\sigma_t = W/A = W/123 = 0.008\,W\ N/mm^2$$

Max. bending stress at point P,

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{83.2W \times 6.64}{123 \times 1.56 \times 25}$$

$$\boxed{\sigma_{bi} = 0.115\,W\ N/mm^2}$$

We know that max. tensile stress $\sigma_{t(max)}$

Resultant stress = $\sigma_t + \sigma_{bi}$.

$$140 = \sigma_t + \sigma_{bi} = 0.008\,W + 0.115W = 0.123W$$

$$W = 140/0.123 = 1138\,N$$

$$\boxed{W = 1138\,N}$$

Note:-

wkt, the distance from the neutral axis to the outer fibre,

$$y_o = R_o - R_n = 50 - 31.64 = 18.36\,mm$$

∴ Max. bending stress at the outer fibre,

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{83.2W \times 18.36}{123 \times 1.56 \times 50} = 0.16W$$

Max. stress at outer fibre,

$$= \sigma_t - \sigma_{bo} = 0.008W - 0.16W$$

$$= -0.152\ W\ N/mm^2$$

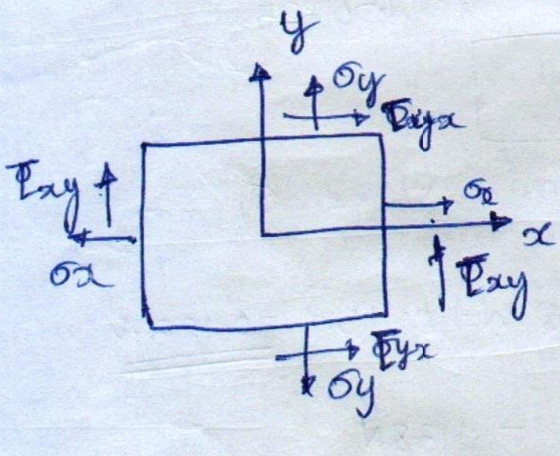
$$= 0.152W\ N/mm^2\ \text{(compressive)}$$

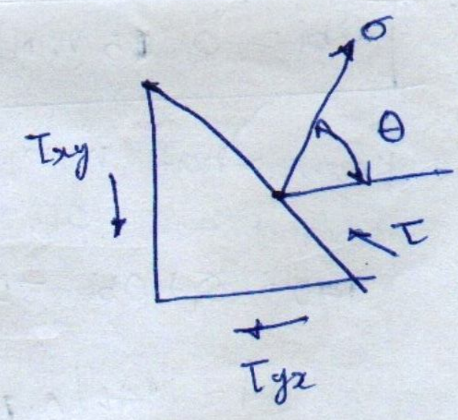
From ~~the~~ above we see that stress at the outer fibre is larger in this case than at the inner fiber, but this stress at outer fibre is compressive.

Principal stresses for variable load combinations

Normal stress, σ

Tangential stress, τ.





$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Max. shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

(or)

Max. Principle Stress

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}$$

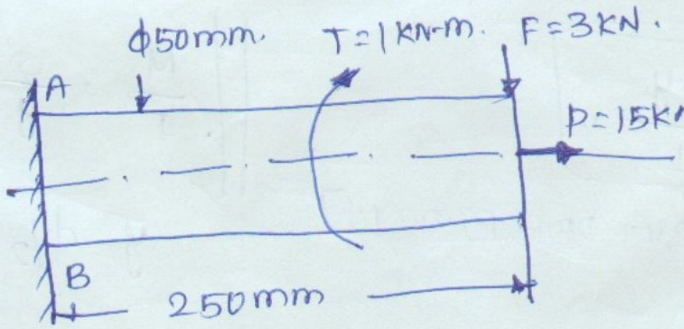
Min principle stress

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \frac{\sigma_x+\sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}$$

Max. shear stress = $\tau_{max} = \frac{\sigma_1-\sigma_2}{2}$.

Problems:-

Calculate ~~the~~ normal stresses at (A) & (B), and also calculate max. shear stresses at (A) and (B).



Solution:-

Given.

$d = 50mm$, $T = 1KN\text{-}m \Rightarrow 1\times10^3 N\text{-}m$. $F = 3KN = 3000N$.

$l = 250mm$.

$P = 15KN = 15\times10^3 N$.

i) consider axial load.

Axial load includes direct stress.

σ_t = Direct stress = $\frac{P}{A} = \frac{15\times10^3}{1963.49}$

$\boxed{\sigma_t = 7.639 \ N/mm^2}$

P = Axial load = Tensile = 15kN.

= $+15\times10^3$ N.

A = $\pi/4 \, d^2 = \pi/4 \times 50^2$ = ~~1963.49~~

= 1963.49 mm2

ii) consider ~~trans~~ transverse (or) bending load.

Bending load = F = 3kN = 3×10^3 N.

Bending moment = Force × length = $3\times10^3\times250$

= 750×10^3 N-mm

Bending load includes bending stress.

$\boxed{\sigma_b = \frac{M\cdot y}{I}}$.

[PSG DDB pg.no 7.1]

$\frac{M}{I} = \frac{\sigma}{y}$ $\quad \sigma_b = \frac{M\cdot y}{I}$

$y = d/2 = 50/2 = 25mm$

$I = \frac{\pi d^4}{64}$

$I = \pi/64 \times (50)^4$

$I = 306.79\times10^3 mm^4$

$\sigma_b = \frac{750\times10^3\times25}{306.79\times10^3}$

$\boxed{\sigma_{b_t} = +61.115 N/mm^2}$ (tensile)

$\boxed{\sigma_{b_c} = -61.115 \ N/mm^2}$ (compressive)

$\boxed{\text{Total stress} = \sigma_x = \sigma_t + \sigma_b}$

at A

$\sigma x = \sigma t + (\sigma b)t = +7.639 + 61.115 = 68.75\,N/mm^2$

at B

$\sigma x = \sigma t + (\sigma b)c = +7.639 + (-61.115) = -53.422\,N/mm^2$.

(iii). Consider torsion

Twisting moment (T) = 1 KN-m = 1000×10^3 N-mm.

$\frac{T}{J} = \frac{\tau}{r}$ [P.S.G DDB Pg.no 7.1]

$\tau xy = \tau = \frac{T \cdot r}{J}$.

$= \frac{1 \times 10^6 \times 25}{613.59 \times 10^3}$

$r = 50/2 = 25$ mm.

$J = \pi/32\, d^4$

$J = \pi/32 \times 50^4$

$J = 613.59 \times 10^3\, mm^4$.

$\boxed{\tau xy = \tau = 40.74\,N/mm^2}$

At (A)

$\sigma x = +68.75\,N/mm^2$.

$\sigma y = 0$.

$\tau xy = +40.74\,N/mm^2$.

At (B)

$\sigma x = -53.476\,N/mm^2$

$\sigma y = 0$

$\tau xy = 40.74\,N/mm^2$.

at (A)

$$\sigma_1 = \text{Max. normal stress} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2_{xy}}.$$

$$= \frac{68.75}{2} + \sqrt{\left(\frac{68.75}{2}\right)^2 + (40.74)^2}$$

$$= 34.375 + 53.30$$

$$\boxed{\sigma_1 = +87.675 \, N/mm^2}$$

$$\sigma_2 = \text{Min. normal stress} = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2_{xy}}.$$

$$= 34.375 - 53.30$$

$$\boxed{\sigma_2 = -18.925 \, N/mm^2}$$

$$\text{Max. shear stress} = (\tau_{max})_{at\,A} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{87.675 - (-18.925)}{2}$$

$$\boxed{(\tau_{max})_{at\,A} = 53.3 \, N/mm^2}$$

At B:-

$$\sigma_1 = \text{Max. normal stress} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2_{xy}}.$$

$$= \frac{-53.476}{2} + \sqrt{\left(\frac{-53.476}{2}\right)^2 + (40.74)^2}$$

$$= -26.738 + 48.730$$

$$\boxed{\sigma_1 = +21.992 \, N/mm^2 \text{ (tensile)}}$$

$\therefore \sigma_2 =$ Min. Normal Stress $= -26.738 - 48.73$

$$\boxed{\sigma_2 = -75.468 \ N/mm^2 \ (\text{compressive})}$$

Max. Shear stress (τ_{max}) at (B) $= \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{21.992 - (-75.468)}{2}$$

$$\boxed{(\tau_{max}) \ \text{at} \ (B) = 48.73 \ N/mm^2}$$

Factor of safety (FOS)

To avoid failure of machine components, the induced maximum stress must be less than the permissible value of stress. (Allowable stress).

$$\text{Factor of Safety} = \frac{\text{Induced Maximum Stress}}{\text{Allowable stress (permissible stress)}}$$

Failure theories:- [From PSG D.D.Book pg no 7.3].

A machine member is subjected to combined loading. i.e, bending and torsion.

It is not possible to decide which combination of normal stress (bending stress due to bending or shear stress due to torsion) causes the failure of the member. In order to predict the failure under combined loads, failure theories are used.