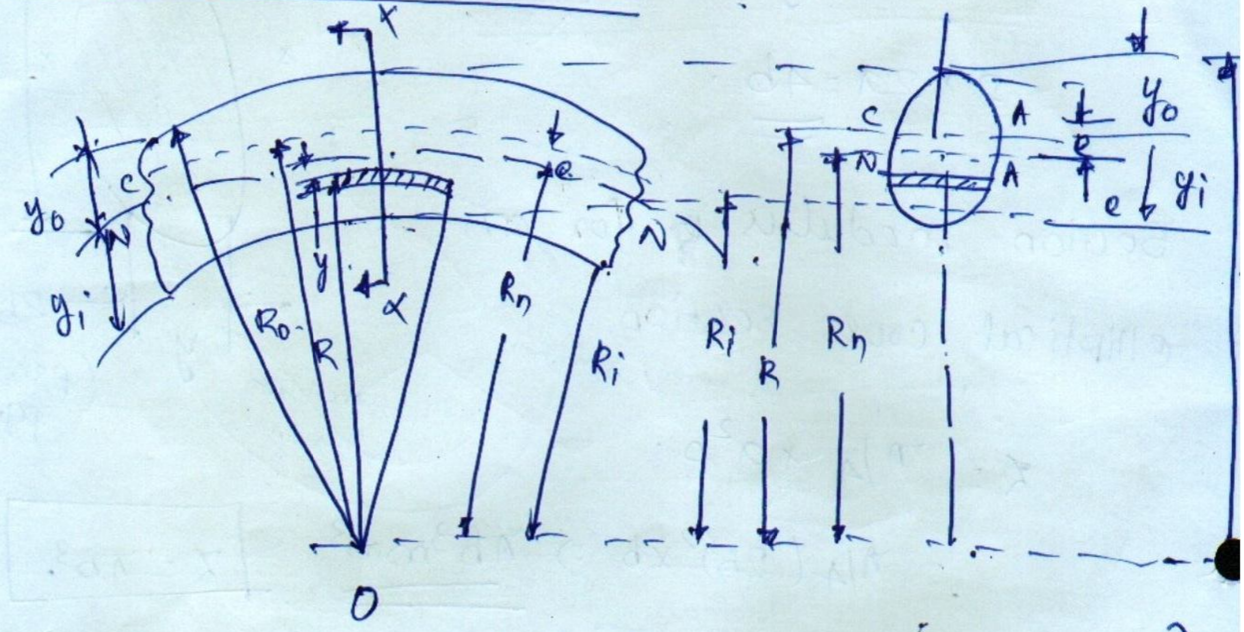
Bending stress in curved beams.



$$\boxed{\sigma_b = \frac{M}{A.e}\left[\frac{y}{R_n-y}\right]}$$ [PSG DDB pg. no 6.2].

M → Bending moment

A → Area of cross section.

e → Distance from centroidal axis to the neutral axis.

$e \to R - R_n'$.

R = Radius of curvature of the centroidal axis.

R_n → Radius of curvature of the neutral axis.

y = Distance from the neutral axis to the fibre under consideration.

Max. bending stress at the inside fibre.

$$\boxed{\sigma_{bi} = \frac{M \cdot \boxed{y_i}\, h_i}{A \cdot e \cdot R_i}}$$

y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$

$$\boxed{y_i = R_n - R_i}$$

$$\boxed{y_i = h_i}$$

R_i = Radius of Curvature of the inside fibre.

Max. bending stress at the outside fibre.

$$\boxed{h_o = y_o}$$

$$\boxed{\sigma_{bo} = \frac{M \cdot \boxed{y_o}\, h_o}{A \cdot e \cdot R_o}}$$

y_o = Distance from neutral axis to the outside fibre, = $R_o - R_n$.

$$\boxed{y_o = R_o - R_n}$$

R_o = Radius of Curvature of the outside fibre.

Note:-

Bending stress at ~~at~~ the inside fibre is tensile.

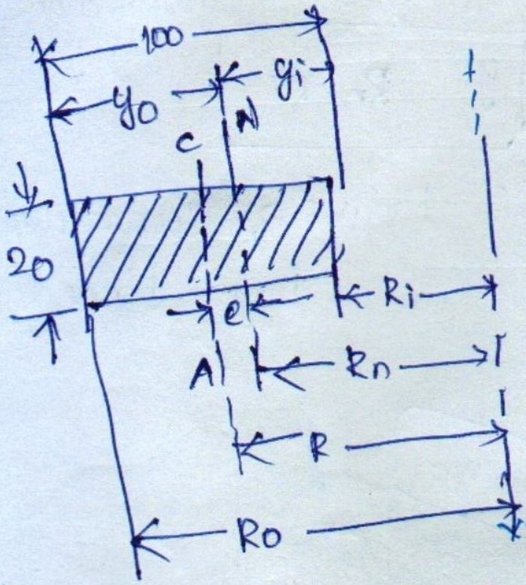
Bending stress at the outside fibre is compressive.

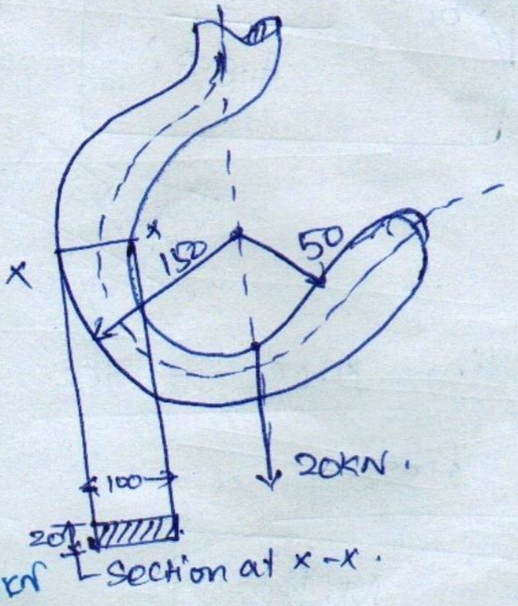
Problems:-

1). The crane hook carries a load of 20kN as shown in figure. The section at x-x is a rectangular whose horizontal side is 100mm. Find the stresses in the inner and outer fibres at the given section.

Solution:

Given = $W = 20kN = 20\times10^3 N$.





All dimensions are in mm.

$R_i = 50mm$; $R_o = 150mm$.

Sh"

Stoff

h=100mm.

b= 20mm.

wkt, area of section at x-x.

$A = b.h = 20\times100 = \underline{\underline{2000mm^2}}$.

Radius of curvature of the neutral axis

$$R_n = \frac{h}{\log_e\left[\frac{R_o}{R_i}\right]} = \frac{100}{\log_e\left[\frac{150}{50}\right]} = \underline{\underline{91.07mm}}$$

Radius of curvature of the centroidal axis

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = \underline{\underline{100mm}}.$$

Distance b/w centroidal axis and neutral axis

$e = R - R_n = 100 - 91.07 \Rightarrow 8.93mm$.

Distance b/w the load and centroidal axis ~~and neutral~~ axis

$$x = R = 100\ mm$$

Bending moment about the centroidal axis

$$M = W \times x = 20\times10^3 \times 100 \Rightarrow \underline{\underline{2\times10^6\ N\text{-}mm}}$$

The section at x-x is subjected to a direct tensile load of $W = 20\times10^3$ N and a bending moment $M = 2\times10^6$ N-mm.

Tensile stress at section x-x.

$$\sigma_t = \overset{\leftarrow load}{W} / A \rightarrow Area \Rightarrow \frac{20\times10^3}{2000} \Rightarrow 10\ N/mm^2 \Rightarrow \underline{\underline{10\ MPa}}$$

Wkt, Distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i$$

$$y_i = 91.07 - 50 = \underline{\underline{41.07\ mm}}$$

Distance from the neutral axis to outside fibre

$$y_o = R_o - R_n = 150 - 91.07 = \underline{\underline{58.93\ mm}}$$