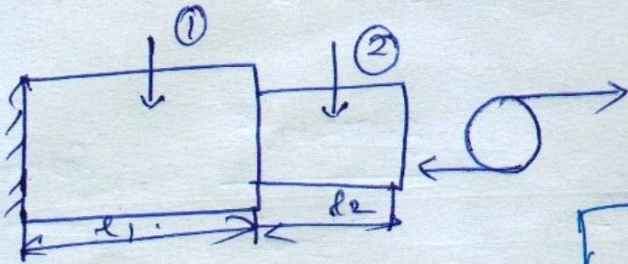
Shafts in parallel and series:-



$1 Pa = 1 N/m^2$

$1 pa = 1 \times 10^5 N/mm^2$

$1 Mpa = 1 \times 10^6 N/m^2$

$$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{r}.$$

$$\theta_1 = \theta_2 = \frac{T l_1}{J_1 \cdot C} + \frac{T l_2}{J_2 \cdot C}$$

Angle of shafts for two shafts are equal means,

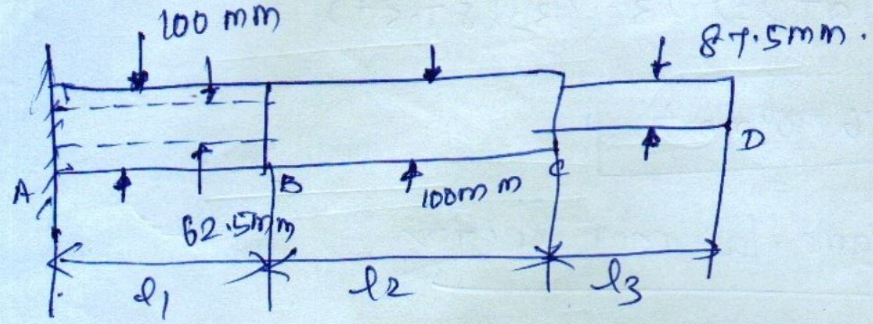
$\theta_1 = \theta_2$

$$\frac{T_1 l_1}{J_1 C} = \frac{T_2 l_2}{J_2 C}.$$

$$\frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{J_1}{J_2}.$$

Problems:-

1). A steel shaft ABCD having total length of 3.5 m cosisting of three lengths. AB is hollow shaft having ~~oust~~ outside and inside diameters of 100mm and 62.5mm respectively, and BC and CD are solid shaft. BC has a diameter of 100mm and CD has a diameter of 87.5 mm. If the angle of twist is same for each section. Find the value of applied torque and total torque of twist if the maximum shear stress in the hollow portion is 77.5mpa and the shear modulus (c) = 82.5Gpa.



overall length = 3.5m = 3500mm.

Solution:-

Polar moment of inertia of section AB (J_1).

$J_1 = \pi/32\ (d_o^4 - d_i^4)$.

$J_1 = \pi/32\ [(100)^4 - (62.5)^4]$.

$J_1 = 8319448.929\ mm^4$ ⇒ $J_1 = 8.32 \times 10^6\ mm^4$

Polar moment of Inertia of section BC (J_2)

$J = \pi/32\ (d^4)$

$J_2 = \pi/32\ (100^4)$

$J_2 = 9.82 \times 10^6\ mm^4$

Polar moment of Inertia of Section (3) (J_3)

$$J_3 = \pi/32 (d^4) \Rightarrow J_3 = \pi/32 (87.5^4)$$

$$\boxed{J_3 = 5.76 \times 10^6 \, mm^4}$$

Angle of twist is same for each section

$$\theta = \theta_1 = \theta_2 = \theta_3 = \frac{Tl_1}{J_1C} = \frac{Tl_2}{J_2C} = \frac{Tl_3}{J_3C}$$

$$\frac{Tl_1}{J_1C} = \frac{Tl_2}{J_2C}$$

$$\frac{l_1}{J_1} = \frac{l_2}{J_2} \Rightarrow \frac{l_1}{J_1} = \frac{l_2}{J_2}$$

$$\frac{l_1}{8.32 \times 10^6} = \frac{l_2}{9.82 \times 10^6} \Rightarrow l_1 = \frac{8.32 \times 10^6}{9.82 \times 10^6} l_2$$

$$\boxed{l_1 = 0.847 l_2} \qquad \boxed{\frac{l_1}{l_2} = 0.847}$$

$$\frac{Tl_1}{J_1C} = \frac{T \cdot l_3}{J_3 \cdot C}$$

$$\frac{l_1}{J_1} = \frac{l_3}{J_3}$$

$$l_1 = \frac{J_1}{J_3} \cdot l_3 \Rightarrow l_1 = \frac{8.32 \times 10^6}{5.76 \times 10^6} l_3$$

$$\boxed{l_1 = 1.444 l_3} \qquad \boxed{\frac{l_1}{l_3} = 1.444}$$

$l = 3500\,mm = l_1 + l_2 + l_3$.

$l_1 + l_2 + l_3 = 3500$.

$(\times \div l_1) \quad = \left(l_1 \times (l_1/l_1)\right) + \left(l_2 \times l_1/l_1\right) + \left(l_3 \times (l_1/l_1)\right) = 3500\,mm$

$$l_1\left[\frac{l_1}{l_1} + \frac{l_2}{l_1} + \frac{l_3}{l_1}\right] = 3500\,mm.$$

$$l_1\left[1 + \frac{l_2}{l_1} + \frac{l_3}{l_1}\right] = 3500.$$

$$l_1\left[1 + \frac{1}{0.847} + \frac{1}{1.444}\right] = 3500.$$

$$l_1[2.8732) = 3500.$$

$$l_1 = \frac{3500}{2.8732}$$

$\boxed{l_1 = 1218.15\,mm.}$

$\boxed{l_1 = 1220\,mm.}$

$l_1/l_2 = 0.847$

$$l_2 = \frac{l_1}{0.847} = \frac{1218.15}{0.847} \Rightarrow \boxed{l_2 = 1438.19\,mm.}$$

$\boxed{l_2 = 1440\,mm.}$

$$\frac{l_1}{l_3} = 1.444.$$

$$l_3 = l_1/1.444 = \frac{1218.15}{1.444} \Rightarrow \boxed{l_3 = 836.67\,mm.}$$

$\boxed{l_3 = 840\,mm.}$

for hollow shaft -

$$T = \pi/16 \times \tau \times \left[\frac{d_o^4 - d_i^4}{d_o}\right]$$

$$T = \pi/16 \times 47.5 \left[\frac{100^4 - 62.5^4}{100}\right]$$

$$\boxed{T = 7.90 \times 10^6 \text{ N-mm}}$$

$$\frac{T}{J_1} = \frac{C\theta_1}{l_1} \Rightarrow \theta_1 = \frac{T l_1}{J_1 C}.$$

$T = \pi/16 \times \tau \times d^3$

$T = \pi/16 \times 47.5 \times 100^3$.

$$\theta_1 = \frac{7.9 \times 10^6 \times 1220}{8.32 \times 10^6 \times 82.5 \times 10^3}.$$

$$\boxed{\theta_1 = 0.014} \Rightarrow \boxed{\theta_1 = 0.014 \text{ rad}}.$$

$$\theta_2 = \frac{T l_2}{J_2 C} \Rightarrow \frac{7.9 \times 10^6 \times 1440}{9.82 \times 10^6 \times 82.5 \times 10^3}$$

$$\boxed{\theta_2 = 0.0140 \text{ rad}}.$$

$$\theta_3 = \frac{T l_3}{J_3 C} = \frac{7.9 \times 10^6 \times 840}{5.76 \times 10^6 \times 82.5 \times 10^3} \Rightarrow \boxed{\theta_3 = 0.01396 \text{ rad}}.$$

$\theta_1 + \theta_2 + \theta_3 = \theta$.

$\theta = 0.014 + 0.014 + 0.01396$.

$$\boxed{\theta = 0.042 \text{ rad}}$$

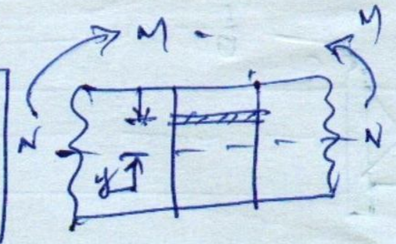
[∴ rad to degree = Ans × $\frac{180}{\pi}$
degree to rad = Ans × $\pi/180$.]

$$\boxed{\theta = 2^\circ 24'}$$

Total angle of twist $(\theta) = 2^\circ 24'$.

Bending Stress in straight beams

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$



M → Bending moment.

σ → Bending stress.

I → Moment of inertia of the cross-section.

y → Distance from the neutral axis.

E → Young's modulus of the material.

R → Radius of curvature of the beam.

$$\boxed{\sigma = \frac{E}{R} \times y}$$

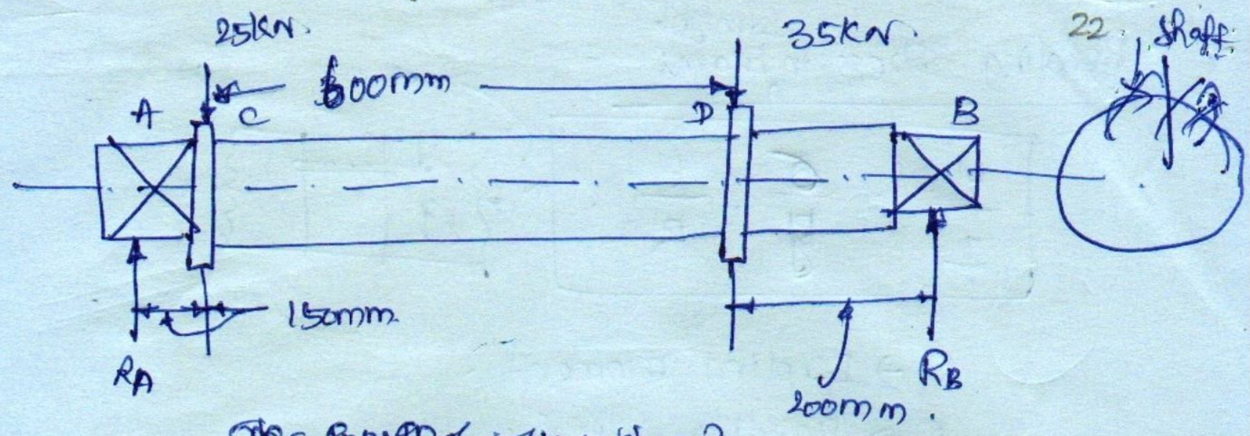
$$I = \frac{\pi}{64} \times d^4$$

$$\boxed{\sigma = \frac{M}{I} \times y} \Rightarrow \frac{M}{I/y} \Rightarrow \frac{M}{Z}$$

$Z = \frac{I}{y}$ = section modulus.

Problems:

1). A pump lever rocking shaft is shown in figure. The pump lever exerts forces of 25KN and 35KN concentrated at 150mm and 200mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100MPa.



Solution:-

Given, $\sigma_b = 100 MPa = 100 N/mm^2$.

Taking moment abt A,

$R_B \times 950 = 35 \times 750 + 25 \times 150$

$R_B \times 950 = 30000 \Rightarrow$ $R_B = 31.58 \times 10^3 N$

$R_A + R_B = 25 + 35$

$R_A = (25+35) - 31.58 \Rightarrow 28.42 \Rightarrow 28.42 \times 10^3 N$

$R_A = 28.42 \times 10^3 N$.

Bending moment at C:-

$R_A \times 150 = 28.42 \times 10^3 \times 150$

$\Rightarrow 4.263 \times 10^6 N\text{-}mm$

Bending moment at D:-

$R_B \times 200 = 31.58 \times 10^3 \times 200$

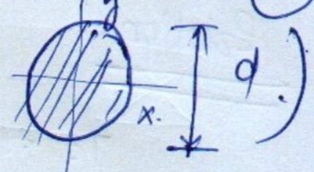
$\Rightarrow 6.316 \times 10^6 N\text{-}mm$

Max. bending at D,

$M = 6.316 \times 10^6 N\text{-}mm$

Section modulus

$Z = \pi/32 \times d^3$. (for shaft,



)

$$Z = 0.0982 d^3$$

$$\sigma_b = \frac{M}{I/y} \Rightarrow \frac{M}{Z}$$

$$100 = \frac{6.316 \times 10^6}{0.0982 d^3} \Rightarrow d^3 = 643.2 \times 10^3$$

$d = 86.3$ (Say) $\Rightarrow$ **90 mm** From DDB. (pg. no 7.25)

~~problem. Not the~~

Q) A cast iron pulley transmitts 10 kW at 400 rpm. The diameter of the pulley is 1.2 metre at it has four straight arms of elliptical cross section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

May/June 2009.

Given data:-

$P = 10\,kW \Rightarrow 10 \times 10^3\,W$; $N = 400\,r.p.m.$

$D = 1.2\,m = 1200\,mm$ (or) $R = 600\,mm$, $\sigma_b = 15\,MPa = 15\,N/mm^2$

Solution:

T= Torque transmitted by the pulley.

$P = \frac{2\pi N T}{60} \Rightarrow 10 \times 10^3 = \frac{2\pi \times 400 \times T}{60}$

$T = \frac{10 \times 10^3}{42} \Rightarrow 238$ N-m.

$T = 238 \times 10^3$ N-mm.

Since, the torque transmitted is the product of the tangential load and the radius of the pully.

Therefore, tangential load acting on the pulley.

$T = W \cdot R$ (W in N, R in mm)

$W = T/R = \frac{238 \times 10^3}{600} = 396.7$ N.

Since, the has four arms, therefore tangential load on each arm,

$W = 396.7/4 \Rightarrow 99.2$ N. $W = 99.2$ N.

Maximum bending moment (M) = W × R (distance) of the arm.

$= W \times R = 99.2 \times 600$

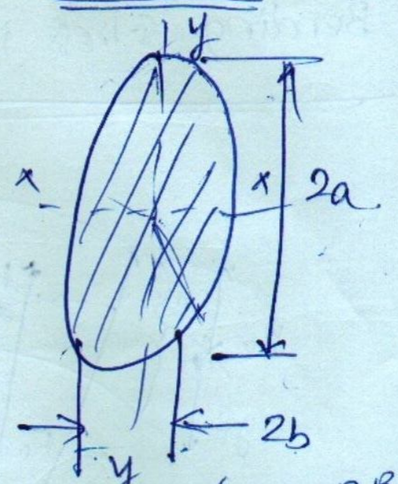
$M = 59520$ N-mm.

2b = Minor axis in mm,

and, 2a = major axis in mm

$= 2 \times 2a = 4b$.

Elliptical



(PSG D.D.B pg.no.6.1)

Section modulus for an elliptical cross section.

a=major b=minor

$Z = \pi/4 \times a^2 b$.

$= \pi/4 (2b)^2 \times b \Rightarrow \pi b^3 mm^3$ $\boxed{Z = \pi b^3}$.

$\sigma b = M/Z$.

$15 = M/Z = \frac{59520}{\pi b^3} \Rightarrow \frac{18943}{b^3}$.

$b^3 = \frac{18943}{15} \Rightarrow 1263 \Rightarrow \boxed{b = 10.8 mm}$

Minor axis ⇒ $2b = 2 \times 10.8 \Rightarrow$ 21.6mm

Major axis ⇒ $2a \Rightarrow 2 \times 2b \Rightarrow 4 \times 10.8 \Rightarrow$ 43.2mm