19ME502: Theory of Machines

Degrees of freedom

Degree-of-freedom (DoF)

- Degree of freedom (also called the mobility *M*) *of a system can be defined as:*

- *the number of inputs* which need to be provided in order to create a predictable output*;*

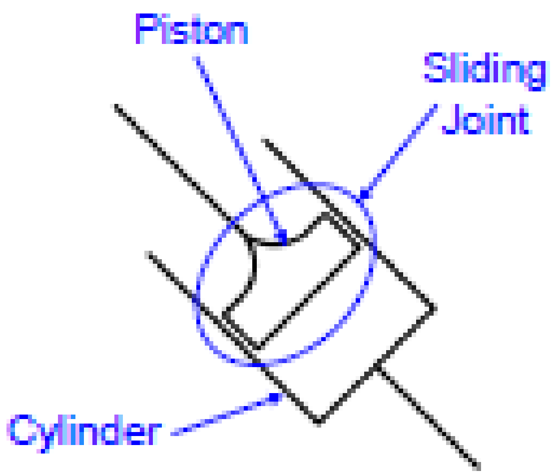
also:

- the number of independent coordinates required to define its position.

Input = Source of motion

The device that introduces/produces motion for a mechanism

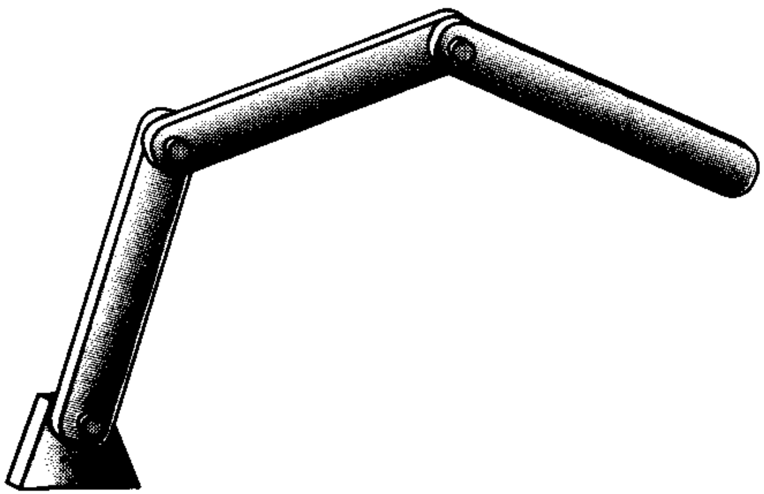
- **Rotary Input**
  - Usually provided by a **motor**
- Linear Input
  - Usually provided by a **linear actuator**
  - Simply a piston in a cylinder moved by pneumatic or hydraulic pressure



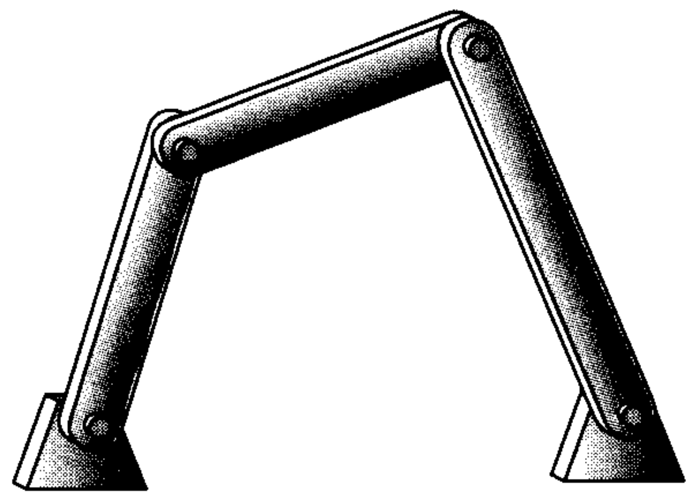
Linear Actuator

Open & Closed Mechanisms

- Kinematic chains or mechanisms may be either open or closed.



(*a*) Open mechanism chain



(*b*) Closed mechanism chain

Open & Closed Mechanisms (contd.)

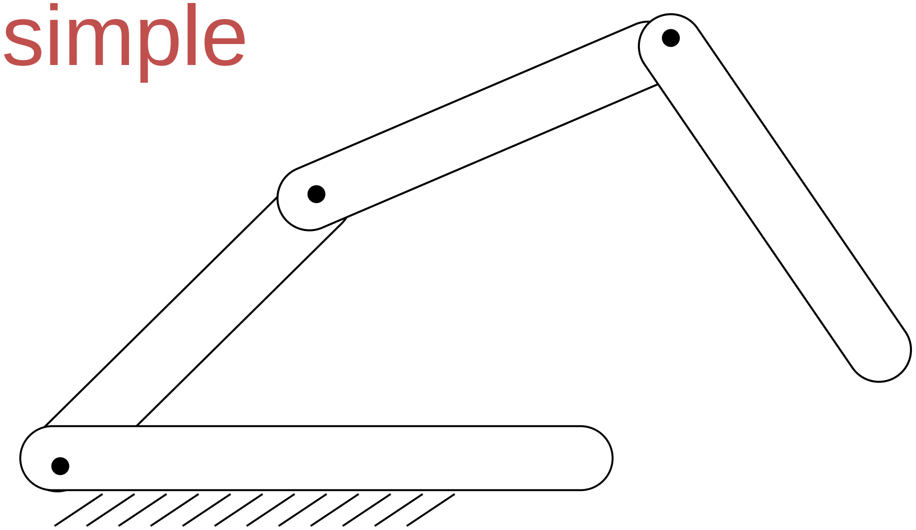
- A ***closed mechanism*** will have no open attachment points or nodes and may have one or more degrees of freedom.

- An ***open mechanism*** of more than one link will always have more than one degree of freedom, thus requiring as many actuators (motors) as it has DOF.

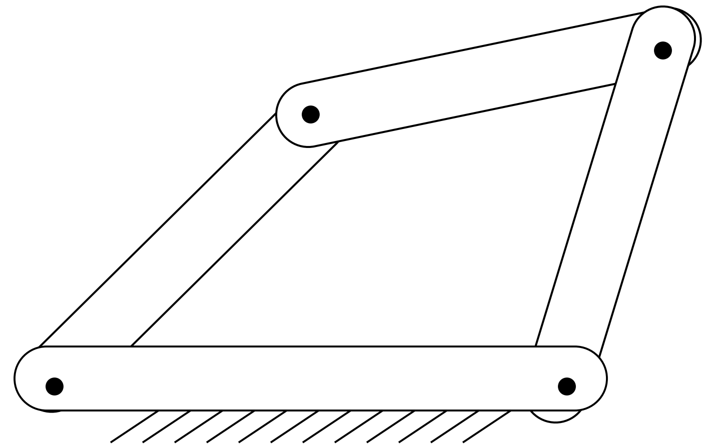
  Ex- Industrial robot

Determining Degrees of Freedom

- For simple mechanisms calculating DOF is simple



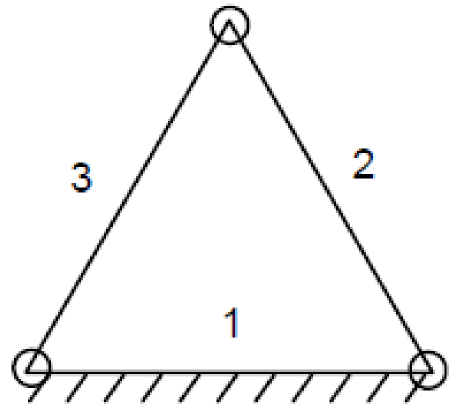
Open Mechanism
DOF=3



Closed Mechanism
DOF=1

Four bar Mechanism

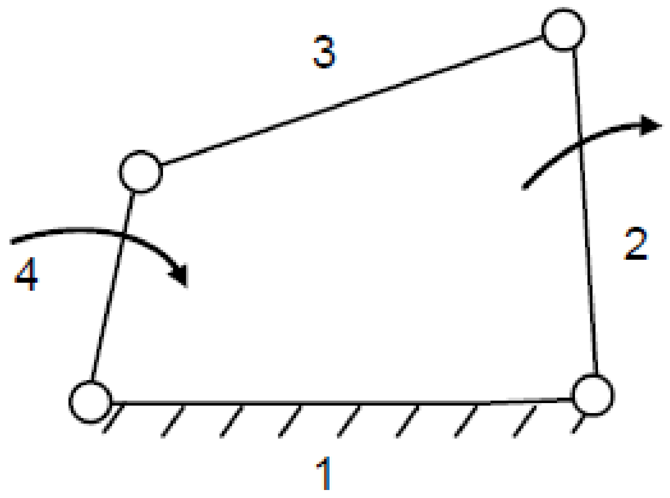
- It may be observed that to form a simple closed chain we need at least three links with three kinematic pairs.



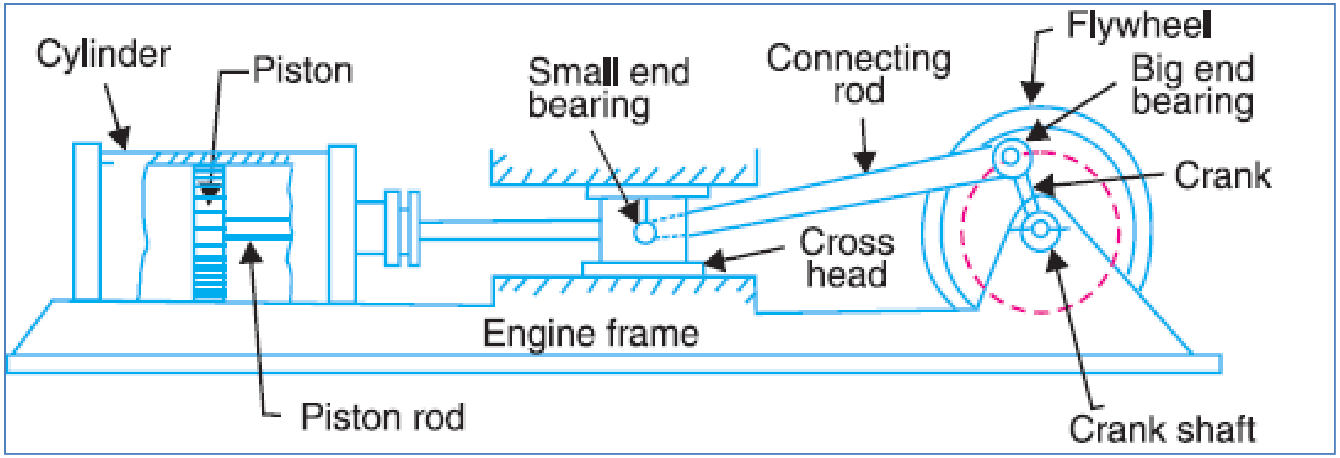
- If any one of these three links is *fixed (ground),* there cannot be relative movement and, therefore, it does not form a mechanism but it becomes a structure which is completely rigid.

Four bar Mechanism (contd.)

- Thus, a simplest mechanism consists of *four links*, each connected by a kinematic lower pair (revolute etc.), and it is known as **four bar mechanism**.
- For example, **reciprocating engine mechanism** is a planner mechanism in which link 1 is fixed, link 2 rotates and link 4 reciprocates.



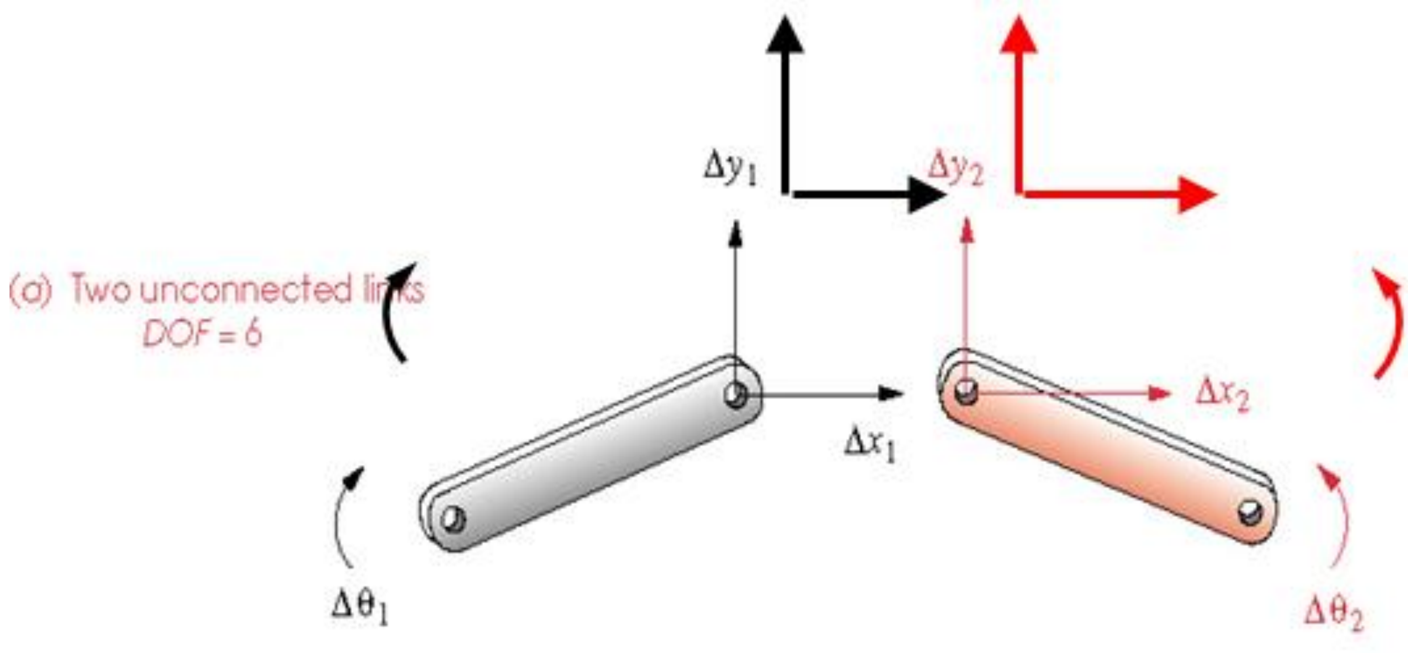
Reciprocating engine mechanism



- The expansion of burning fuel in the cylinders periodically pushes the piston down, which, through the connecting rod, turns the crankshaft.
- The continuing rotation of the crankshaft drives the piston back up, ready for the next cycle.
- The piston moves in a reciprocating motion, which is converted into circular motion of the crankshaft, which ultimately propels the vehicle.

Degree of Freedom in Planar Mechanisms

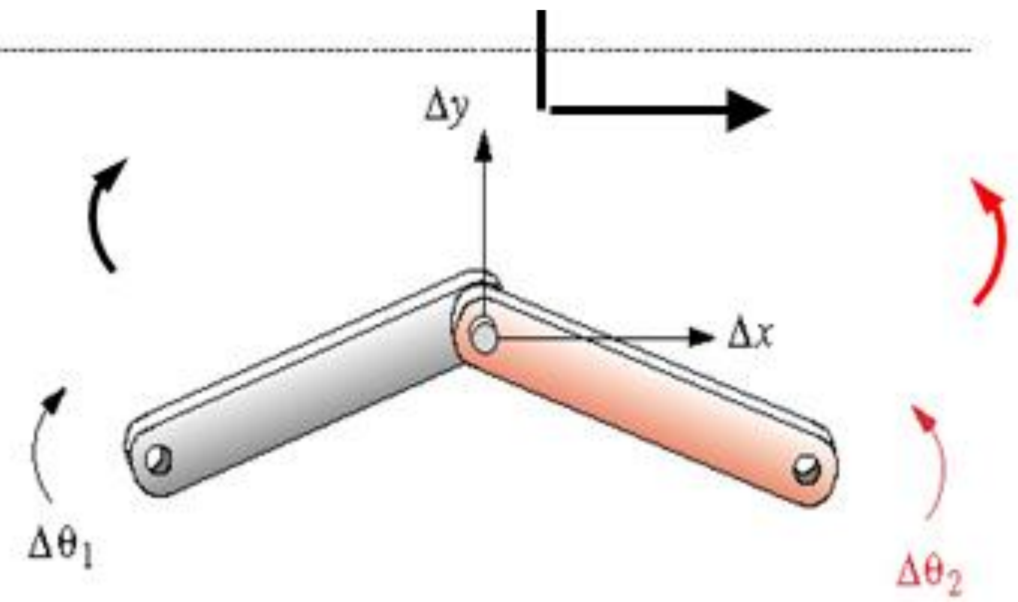
- Any link in a plane has 3 *DOF. Therefore, a system of* ***L*** *unconnected links in the same plane will have* ***3L DOF****, as shown in Figure, where the two unconnected links have a total of six DOF.*



Degree of Freedom in Planar Mechanisms (contd.)

- When these links are connected by **a full joint** in as in Figure, ΔY_1 and ΔY_2 are combined as ΔY, and Δx_1 and Δx_2 are combined as Δx. This removes two DOF, leaving **four DOF**.

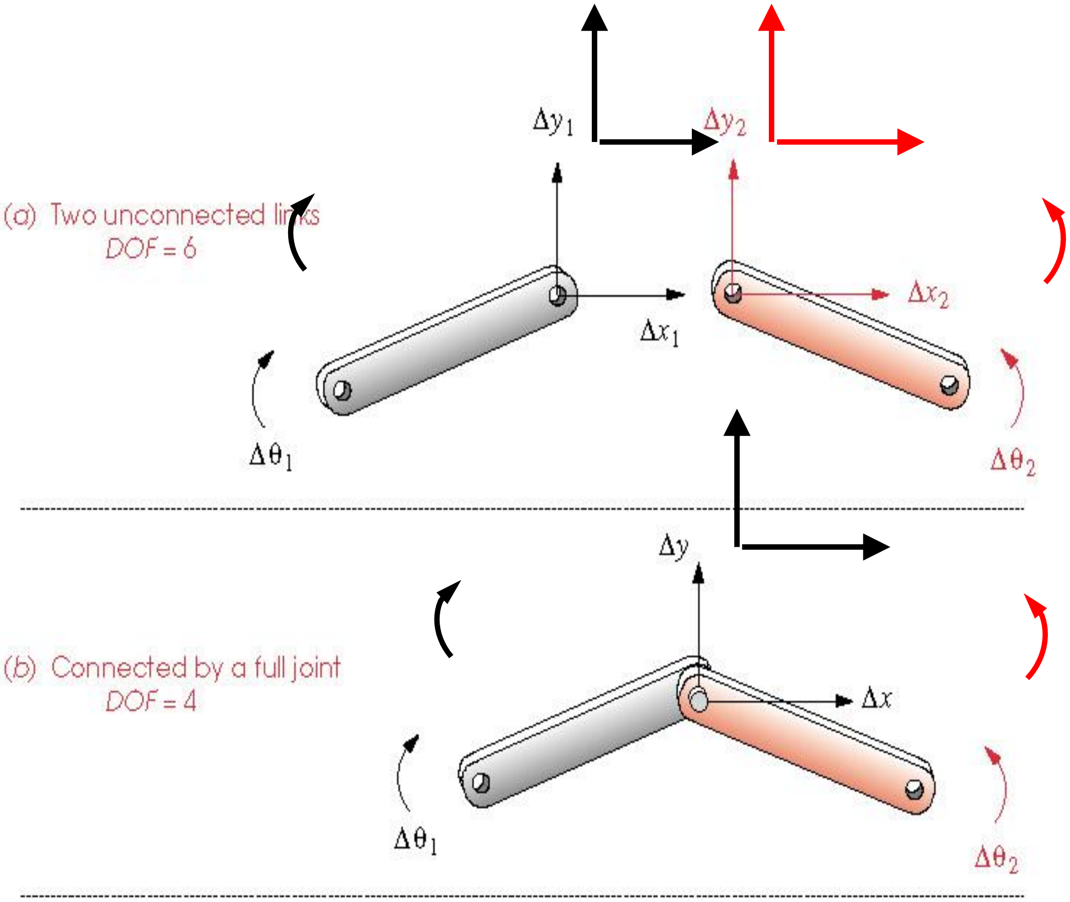
b) Connected by a full joint
DOF = 4



Degree of Freedom in Planar Mechanisms

Two unconnected links: 6 DOF
(each link has 3 DOF)

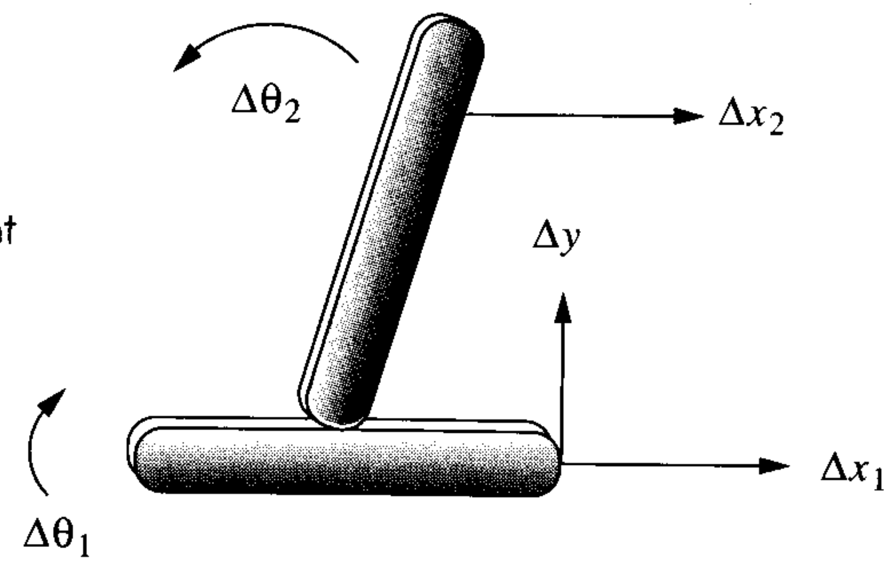
When connected by a full joint: 4 DOF
(each full joint eliminates 2 DOF)



Degree of Freedom in Planar Mechanisms (contd.)

- In Figure the **half joint** removes only one DOF from the system (*because a half joint has two DOF*), leaving the system of two links connected by a half joint with a total of **five DOF**.

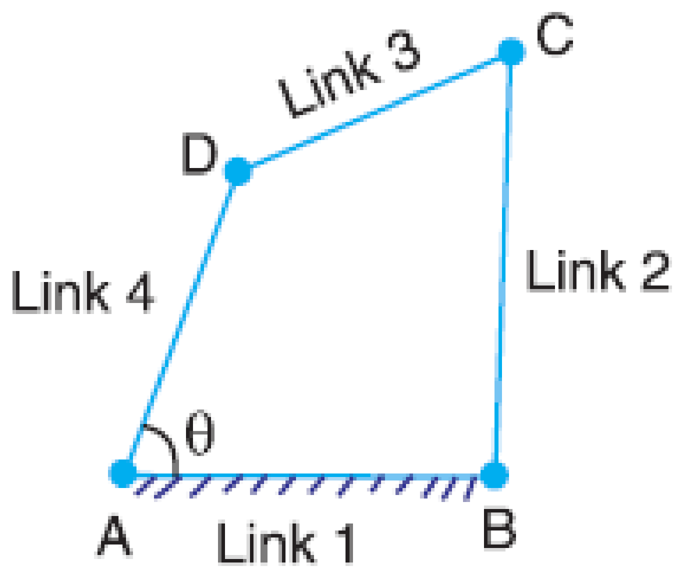
(c) Connected by a roll-slide (half) joint
DOF = 5



Another example

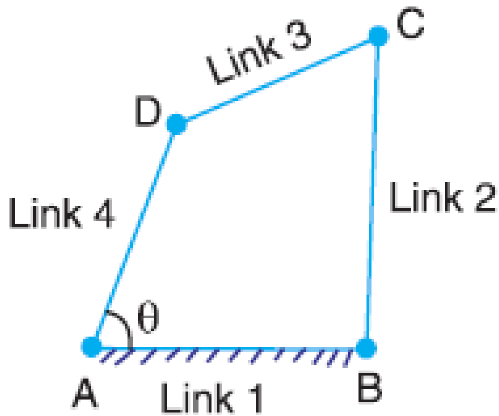
- Consider a **four bar chain**, as shown in figure. A little consideration will show that only one variable such as **θ** is needed to define the *relative positions of all the links*.

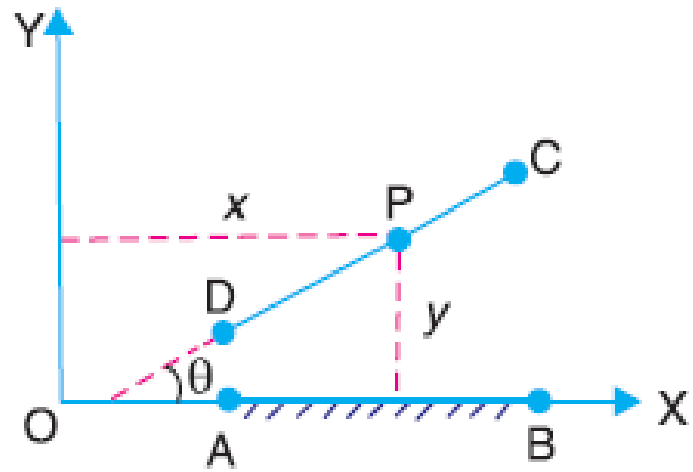
- In other words, we say that the **number of degrees of freedom** of a four bar chain is one.



Another example (contd.)

- Consider two links AB and CD in a plane motion as shown in Figure.





- The link *AB* with coordinate system *OXY* is taken as the reference link (or fixed link).
- The position of point P on the moving link *CD* can be completely specified by the **three variables**. *i.e.* the coordinates of P denoted by *x* and *y*, and inclination θ of link CD *w.r.t.* x-axis or link AB.

Another example (contd.)

- In other words, we can say that each link of a mechanism has **three degrees of freedom** before it is connected to any other link.

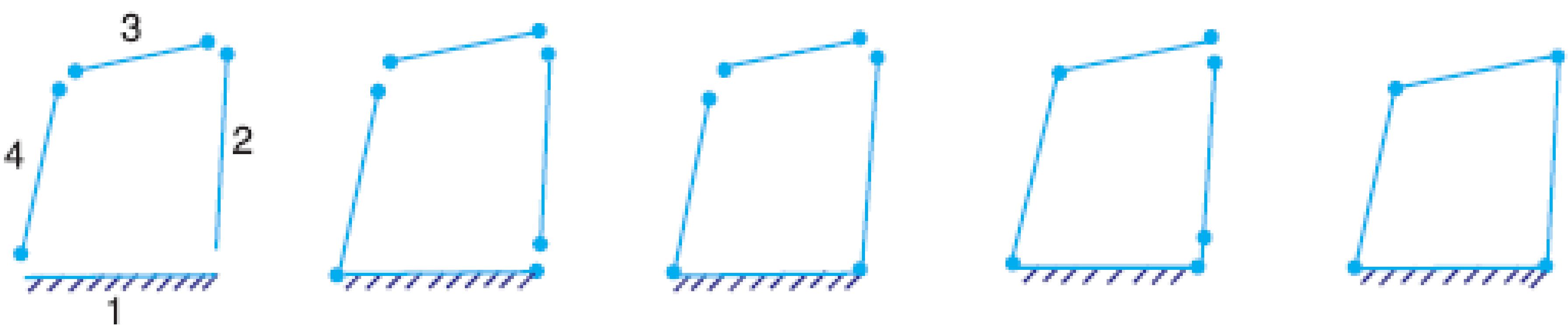
- But when the *link CD* is **connected** to the *link A B* by a turning pair at **A**, the position of link CD is now determined by a single variable θ and thus has one degree of freedom.

- We have seen that when a link is connected to a fixed link by a turning pair *(i.e.* lower pair), two degrees of freedom are destroyed.

Another example (contd.)

- We have seen that when a link is connected to a fixed link by a turning pair *(i.e.* lower pair), two degrees of freedom are destroyed.

- This may be clearly understood from Figure given below, in which the resulting **four bar mechanism** has one degree of freedom.



(*a*) $n = 9$ (*b*) $n = 7$ (*c*) $n = 5$ (*d*) $n = 3$ (*e*) $n = 1$

Determining DoF's

- Now let us consider a plane mechanism with ***l*** number of links.

- Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be *(l - 1)* and thus the total number of degrees of freedom will be ***3 (l - 1)*** before they are connected to any other link.

Determining DoF's

- In general, a mechanism with ***l*** number of links connected by **j** number of **binary joints** or lower pairs (i.e. single degree of freedom pairs) and **h** number of higher pairs (i.e. two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$$M = 3\,(l - 1) - 2j - h$$

- This equation is called **Gruebler's criterion** for the movability of a mechanism having plane motion.
- If there are no two degree of freedom pairs *(i.e.* higher pairs), then $h = 0$. Substituting $h = 0$ in equation, we have

$$M = 3\,(l - 1) - 2j$$

- Gruebler's equation for planar mechanisms

$$M = 3\,(l - 1) - 2j$$

- Note that the value of j must reflect the value of all joints in the mechanism; i.e. *half joints count as 0.5 b/c they only remove 1 DOF*. A modified form of Gruebler's equation for clarity is known as **Kutzbach's modification**, which take into account full and half joints separately;

$$M = 3\,(L - 1) - 2J_1 - J_2$$

Where

J_1= Number of 1 DOF (full) joints

J_2= Number of 2 DOF (half) joints

Important Note !!

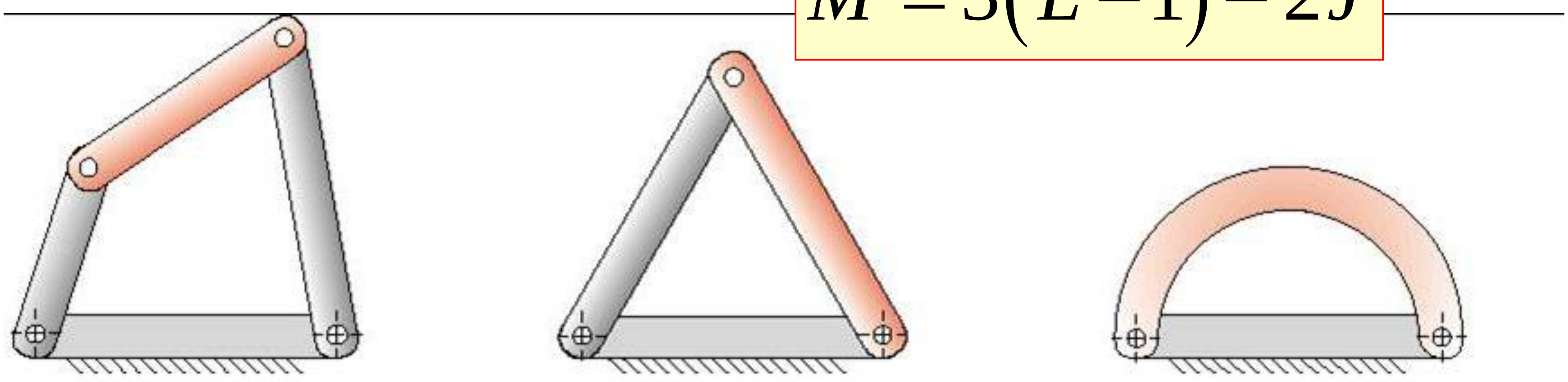
It should be noted that

Gruebler's/Kutzbach's equation has **no information in it about link sizes or shapes, only their quantity.**

Mechanisms and Structures

- If DoF > 0, it's a mechanism
- If DoF = 0, it's a structure
- If DoF < 0. it's a preloaded structure (will have built in stresses with manufacturing error)

$$M = 3(L-1) - 2J$$



(a) Mechanism—DOF = +1 (b) Structure—DOF = 0 (c) Preloaded structure—DOF = −1

Delta Triplet (Truss)