SNS COLLEGE OF ENGINEERING

(Autonomous)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

19EC502 – TRANSMISSION LINES AND ANTENNAS

III YEAR/ V SEMESTER

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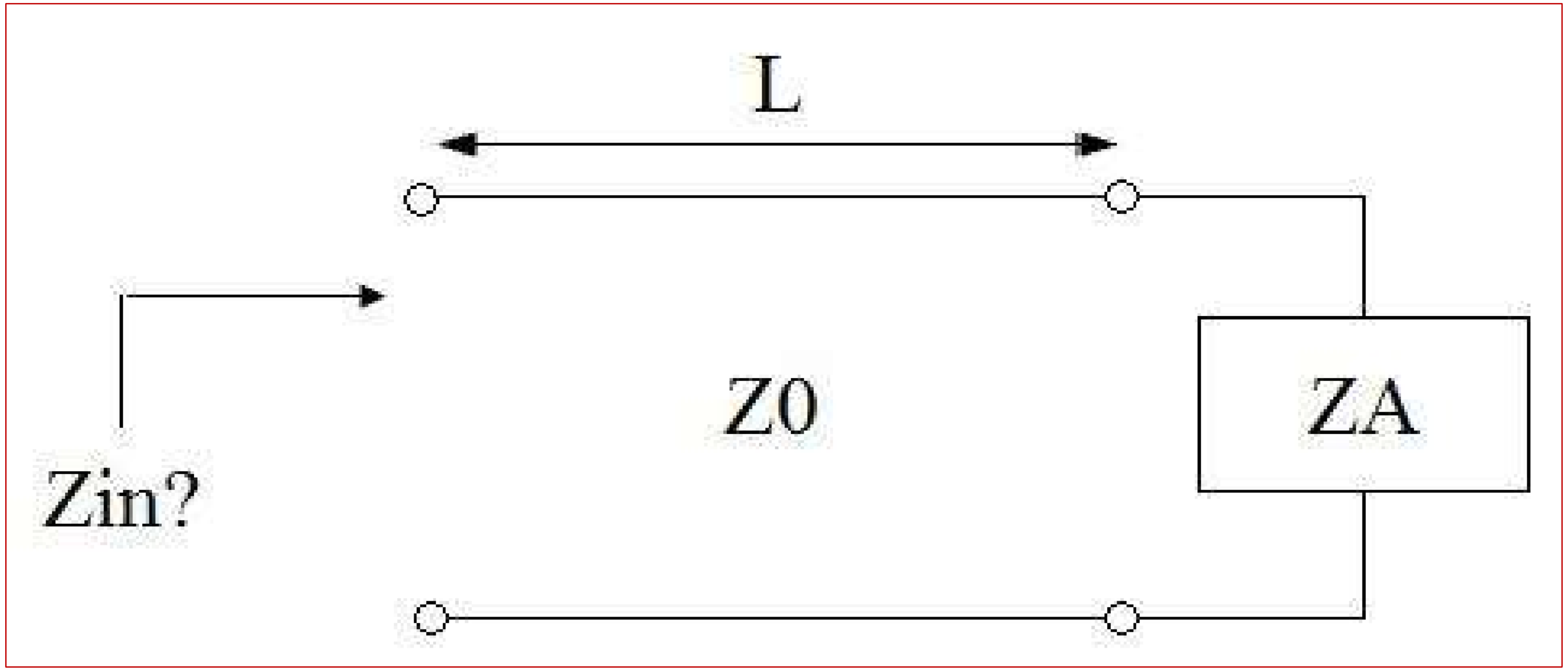
UNIT 1 – TRANSMISSION LINE THEORY
INPUT & TANSFER IMPEDANCE

INPUT IMEDANCE OF A LINE



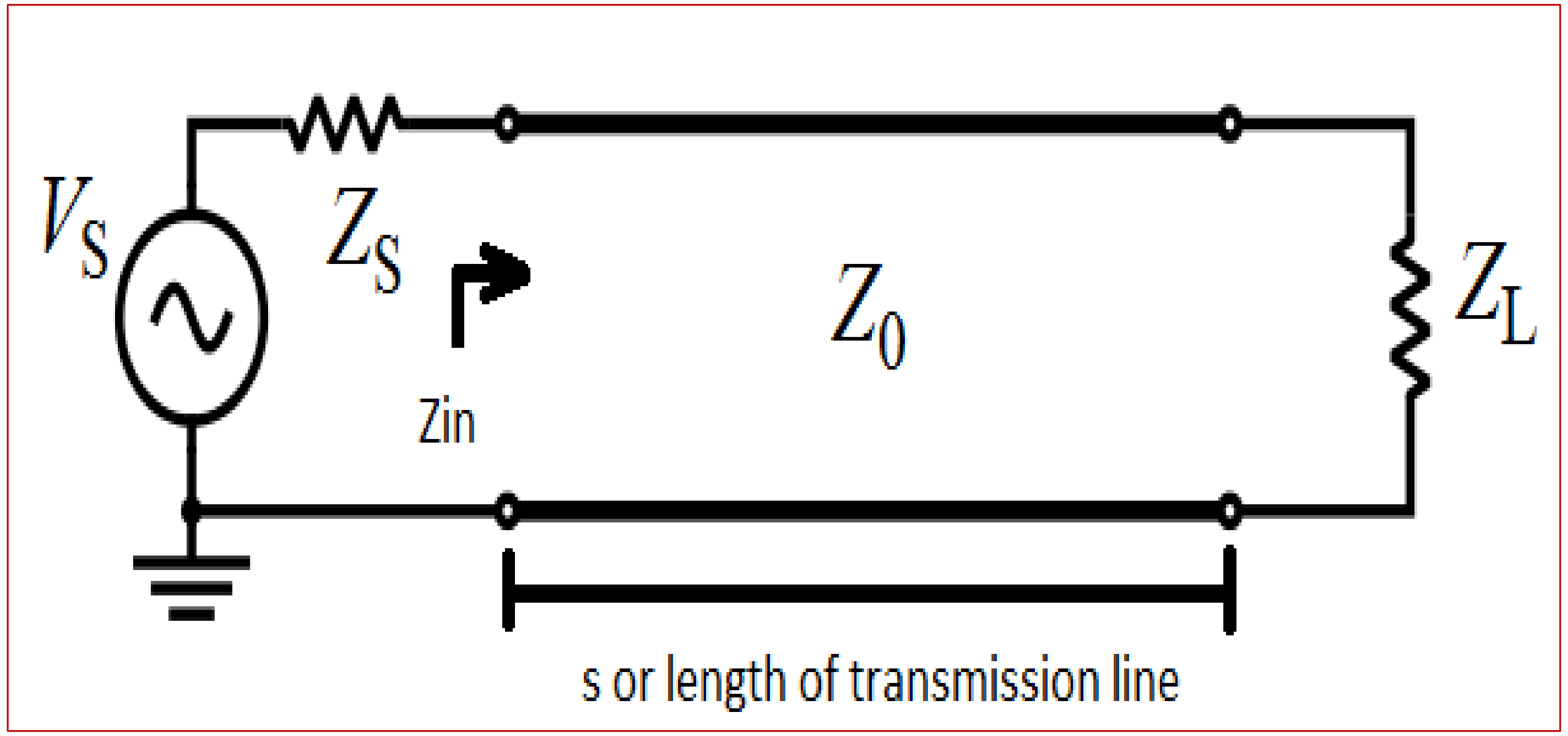
What is Zin or Input impedance of a transmission line?

INPUT IMEDANCE OF A LINE

- Input impedance of a transmission line is defined as the impedance measured across the input terminals of the transmission lines
- It is the impedance seen looking into the sending end or the input terminals
- It is also the impedance at the input into which the source must work when the line is connected

INPUT IMEDANCE OF A LINE

- Also known as driving point impedance
- Denoted by Zin = Vs/Is
- Also known as driving point impedance



INPUT IMEDANCE OF A LINE – STANDARD FORMS

- **FIRST FORM**

The voltage and current expressions – Hyperbolic form

$E = E_R \cosh\sqrt{zy}\, s + I_R Z_0 \sinh\sqrt{zy}\, s$ ---------- (1)

$I = I_R \cosh\sqrt{zy}\, s + \frac{E_R}{Z_0} \sinh\sqrt{zy}\, s$ ---------- (2)

To find input voltage & input current for the transmission line of length l, replace s by l, $\sqrt{zy}$ by γ, E by Es & I by Is in equations (1) & (2),

$E_s = E_R \cosh\gamma l + I_R Z_0 \sinh\gamma l$ ---------- (3)

$I_s = I_R \cosh\gamma l + \frac{E_R}{Z_0} \sinh\gamma l$ ---------- (4)

INPUT IMEDANCE OF A LINE

Input Impedance $Z_s = E_s/I_s$

Therefore, Eqn (3) / Eqn (4) gives,

$$\frac{E_s}{I_s} = \frac{I_R Z_R \cosh\gamma l + I_R Z_0 \sinh\gamma l}{I_R \cosh\gamma l + \frac{I_R Z_R}{Z_0} \sinh\gamma l}$$

$$Z_s = Z_0 \frac{Z_R \cosh\gamma l + Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l + Z_R \sinh\gamma l} \quad \text{-------- (5)}$$

Equation (5) is one of the standard form of input impedance of a transmission line.

INPUT IMEDANCE OF A LINE

- **SECOND FORM**

$$E = \frac{E_R (Z_R + Z_0)}{2Z_R} \left[e^{\sqrt{zy}\, s} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\sqrt{zy}\, s} \right] \quad \text{------------ (6)}$$

$$I = \frac{I_R (Z_R + Z_0)}{2Z_R} \left[e^{\sqrt{zy}\, s} - \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\sqrt{zy}\, s} \right] \quad \text{------------ (7)}$$

To find input voltage & input current for the transmission line of length l, replace s by l, √zy by γ, E by Es & I by Is in equations (1) & (2) & by getting Es/ Is

- **SECOND FORM**

$$E = \frac{I_R Z_R (Z_R + Z_0)}{2Z_R}\left[e^{\gamma l} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\gamma l} \right] \quad \text{------------ (6)}$$

$$I = \frac{I_R (Z_R + Z_0)}{2Z_0}\left[e^{\gamma l} - \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\gamma l} \right] \quad \text{------------ (7)}$$

$$Zs = Z_0 \left[\frac{e^{\gamma l} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\gamma l}}{e^{\gamma l} - \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\gamma l}} \right] \quad \text{------------------ (8)}$$

Eqn (8) is the another form of input impedance of a transmission line

INPUT IMEDANCE OF A LINE

Input impedance is given by

$$Z_{\mathrm{in}}(\ell) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma\ell)}{Z_0 + Z_L \tanh(\gamma\ell)}$$

tanh(jθ) = j tan θ

Subs $\gamma = j\beta$, $Z_0 \tan h \gamma l = j Z_0 \tan \beta l$

INPUT IMEDANCE OF A LOSSLESS LINE

- For a lossless transmission line, Input impedance is purely imaginary and is given by γ = j β
- Therefore the input impedance is given by,

$$Z_{\mathrm{in}}(\ell) = Z_0 \frac{Z_{\mathrm{L}} + j Z_0 \tan(\beta\ell)}{Z_0 + j Z_{\mathrm{L}} \tan(\beta\ell)}$$

where $\beta = \frac{2\pi}{\lambda}$ is the wavenumber.

INPUT IMEDANCE OF SPECIAL CASES OF LINE

- **Matched load**

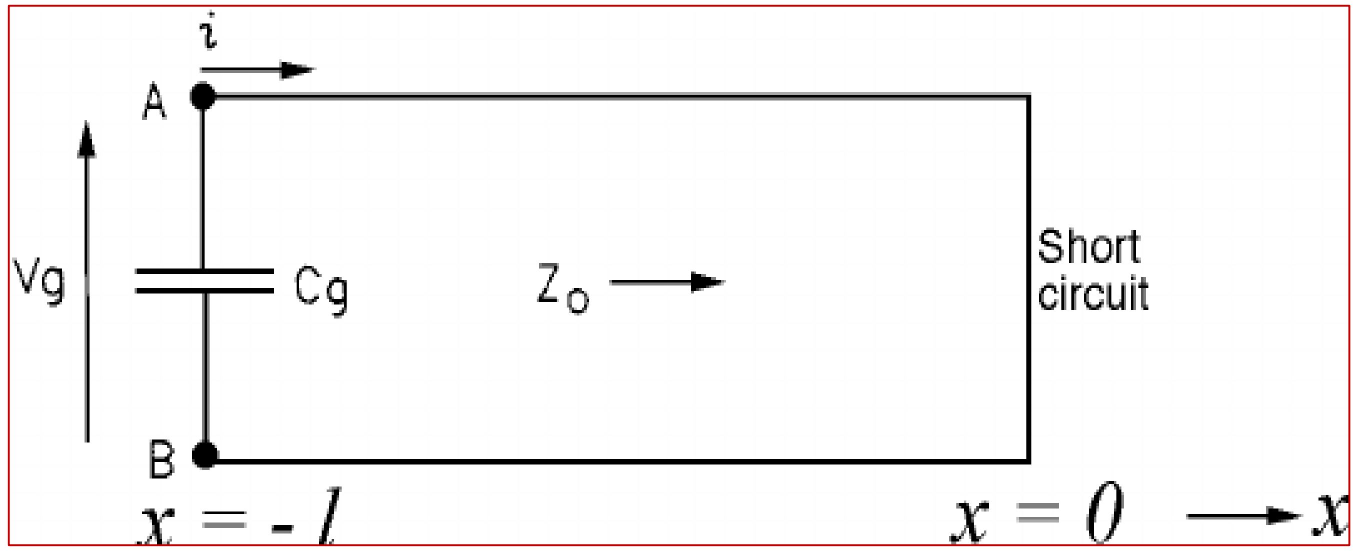
Another special case is when the load impedance is equal to the characteristic impedance of the line (i.e. the line is matched), in which case the impedance reduces to the characteristic impedance of the line so that,

$$Zin = Z_0 = Z_L$$

- **Short line**

For the case of a shorted load (i.e $Z_L = 0$), the input impedance is purely imaginary and a periodic function of position and wavelength (frequency)

$$Zin(l) = j\, Z0 \tan(\beta l)$$

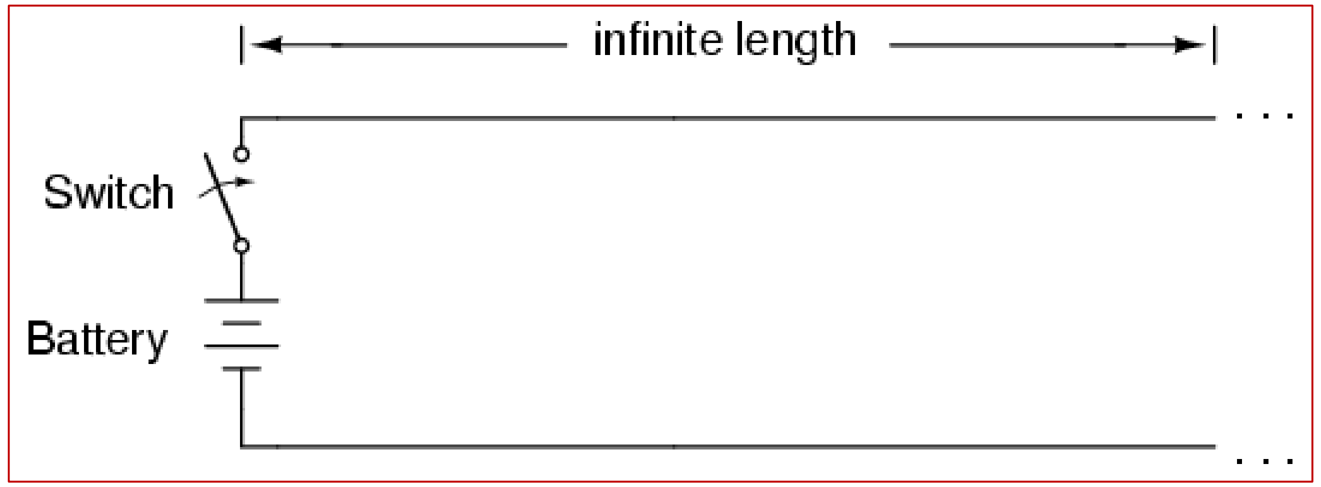


INFINITE LINE

- When the length l of the line is infinite, i.e l approaches to infinity, thus

$$Zin=Z_0$$

- Hence it is concluded that a line of infinite length irrespective of the type of terminating load, has an input impedance Z_0, thus behaving like a line of finite length terminated in its characteristic impedance Z_0



TRANSFER IMPEDANCE

- Input voltage of a transmission line is

$$E_s = \frac{E_R (Z_R + Z_0)}{2Z_R}\left[e^{\sqrt{zy}\,s} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\sqrt{zy}\,s}\right]$$

Subs E_R by $I_R Z_R$ in the above expression, we get

$$E_s = \frac{I_R Z_R (Z_R + Z_0)}{2Z_R}\left[e^{\sqrt{zy}\,s} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\sqrt{zy}\,s}\right]$$

Subs. Reflection co-efficient $k = (Z_R - Z_0) / (Z_R + Z_0)$

TRANSFER IMPEDANCE

- Transfer Impedance $Z_T = Es/ I_R$

$$Z_T = \frac{Es}{I_R} = \frac{(Z_R + Z_0)}{2} e^{\sqrt{zy}\, s} + \frac{(Z_R - Z_0)}{2} e^{-\sqrt{zy}\, s}$$

Rearranging the above expression, we get,

$$Z_T = \frac{Z_R}{2} e^{\sqrt{zy}\, s} + \frac{Z_R}{2} e^{-\sqrt{zy}\, s} + \frac{Z_0}{2} e^{\sqrt{zy}\, s} - \frac{Z_0}{2} e^{-\sqrt{zy}\, s}$$

$$Z_T = Z_R \cosh \sqrt{zy}\, s + Z_0 \sinh \sqrt{zy}\, s$$

THANK YOU