SNS COLLEGE OF ENGINEERING

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

COURSE NAME : 19EC513 – IMAGE PROCESSING AND COMPUTER VISION

III YEAR / V SEMESTER

Unit I- DIGITAL IMAGE FUNDAMENTALS AND TRANSFORMS

Topic : Image transforms - Properties of 2D DFT

Fourier Transform: Fourier transform is the input tool that is used to decompose an image into its sine and cosine components.

Properties of Fourier Transform:

- **Linearity:**

Addition of two functions corresponding to the addition of the two frequency spectrum is called the linearity. If we multiply a function by a constant, the Fourier transform of the resultant function is multiplied by the same constant. The Fourier transform of sum of two or more functions is the sum of the Fourier transforms of the functions.

Case I. If h(x) -> H(f) then ah(x) -> aH(f)

Case II. If h(x) -> H(f) and g(x) -> G(f) then h(x)+g(x) -> H(f)+G(f)

- **Scaling:**

Scaling is the method that is used to the change the range of the independent variables or features of data. If we stretch a function by the factor in the time domain then squeeze the Fourier transform by the same factor in the frequency domain.

If f(t) -> F(w) then f(at) -> (1/|a|)F(w/a)

- **Differentiation:**

Differentiating function with respect to time yields to the constant multiple of the initial function.

- If f(t) -> F(w) then f'(t) -> jwF(w)
- **Convolution:**

It includes the multiplication of two functions. The Fourier transform of a convolution of two functions is the point-wise product of their respective Fourier transforms.

- If f(t) -> F(w) and g(t) -> G(w) then f(t)*g(t) -> F(w)*G(w)
- **Frequency Shift:**

Frequency is shifted according to the co-ordinates. There is a duality between the time and frequency domains and frequency shift affects the time shift.

- If f(t) -> F(w) then f(t)exp[jw't] -> F(w-w')
- **Time Shift:**

The time variable shift also effects the frequency function. The time shifting property concludes that a linear displacement in time corresponds to a linear phase factor in the frequency domain.

- If f(t) -> F(w) then f(t-t') -> F(w)exp[-jwt']

The Discrete Fourier Transform (DFT) transfers an image from the spatial domain to the frequency domain. It is one of the most important transforms in image processing, which enables us to decompose an image into its sine and cosine components.

The output image after applying the Fourier transformation is represented as a linear combination of a collection of sine and cosine waves of different frequencies.

Consider a 1D function, $\{f(x), 0 \le x \le N-1\}$. The general form of a transformation is

$$g(u) = \sum_{x=0}^{N-1} T(u,x) f(x);\ 0 \le u \le N-1 \qquad (1)$$

where $T(u,x)$ is called the **forward kernel** of transformation and $g(u)$ is the transformed image.

If the transformation is the Discrete Fourier Transform.

Then,

$$g(u) = \sum_{x=0}^{N-1} \frac{1}{N} e^{-i2\pi\frac{x}{N}} f(x);\ u = 0,1,2,\ldots N-1 \qquad (2)$$

The inverse 1-D DFT will then be,

$$f(x) = \sum_{x=0}^{N-1} e^{i2\pi\frac{ux}{N}} g(u) \qquad (3)$$

As can be seen the signal is written as a linear combination of an orthogonal set of basis functions. Similarly, an image can be transformed into a set of “basis images”, which can be used for representing the image.

We can extend the transform to 2-D image.

Consider an image $f(x,y)$ of size $M \times N$. The 2-D DFT of $f(x,y)$ is defined as follows:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) e^{-i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \quad (4)$$

And the inverse 2-D DFT is given by:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v) e^{i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \quad (5)$$

Note that while the image values $f(x,y)$ are going to be real, the corresponding frequency domain data is going to be complex. There will be one matrix containing real values $R(u,v)$ and the other matrix $I(u,v)$ will contain the imaginary component of the complex value. The amplitude spectrum or the magnitude for 2D DFT is given by

$$|F(u,v)| = [R^2(u,v) + I^2(u,v)]^{1/2} \quad (6)$$

The power spectrum of the 2-D DFT is defined as

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v) \quad (7)$$

and the phase spectrum of the 2-D DFT is given by

$$\text{Ø}(u,v) = \tan^{-1} \frac{I(u,v)}{R(u,v)} \quad (8)$$

Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x, y)$	$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u, v)$	$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$
3) Polar representation	$F(u, v) = \lvert F(u, v) \rvert e^{j\phi(u,v)}$
4) Spectrum	$\lvert F(u, v) \rvert = \left[R^2(u, v) + I^2(u, v) \right]^{1/2}$ $R = \text{Real}(F); \quad I = \text{Imag}(F)$
5) Phase angle	$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)} \right]$
6) Power spectrum	$P(u, v) = \lvert F(u, v) \rvert^2$
7) Average value	$\bar{f}(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) = \frac{1}{MN} F(0, 0)$

Name	Expression(s)
8) Periodicity (k_1 and k_2 are integers)	$F(u, v) = F(u + k_1M, v) = F(u, v + k_2N)$ $= F(u + k_1M, v + k_2N)$ $f(x, y) = f(x + k_1M, y) = f(x, y + k_2N)$ $= f(x + k_1M, y + k_2N)$
9) Convolution	$f(x, y) \star h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n)h(x - m, y - n)$
10) Correlation	$f(x, y) \Join h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f^*(m, n)h(x + m, y + n)$
11) Separability	The 2-D DFT can be computed by computing 1-D DFT transforms along the rows (columns) of the image, followed by 1-D transforms along the columns (rows) of the result. See Section 4.11.1.
12) Obtaining the inverse Fourier transform using a forward transform algorithm.	$MNf^*(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F^*(u, v)e^{-j2\pi(ux/M + vy/N)}$ This equation indicates that inputting $F^*(u, v)$ into an algorithm that computes the forward transform (right side of above equation) yields $MNf^*(x, y)$. Taking the complex conjugate and dividing by MN gives the desired inverse. See Section 4.11.2.

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$
3) Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u, v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$
6) Convolution theorem[†]	$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$ $f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$

Name	DFT Pairs
7) Correlation theorem†	$f(x, y) \star h(x, y) \Leftrightarrow F^*(u, v) H(u, v)$ $f^*(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$
8) Discrete unit impulse	$\delta(x, y) \Leftrightarrow 1$
9) Rectangle	$\text{rect}[a, b] \Leftrightarrow ab\frac{\sin(\pi ua)}{(\pi ua)}\frac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $j\frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\right]$
11) Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $\frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) *Differentiation* (The expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.)	$\left(\frac{\partial}{\partial t}\right)^m\left(\frac{\partial}{\partial z}\right)^n f(t, z) \Leftrightarrow (j2\pi\mu)^m(j2\pi\nu)^n F(\mu, \nu)$ $\frac{\partial^m f(t, z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu, \nu); \frac{\partial^n f(t, z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu, \nu)$
13) *Gaussian*	$A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

Any Query????

Thank you……