UNIT – I PROPERTIES OF MATTER

TOPIC – IV BENDING MOMENT

2.13 BENDING OF BEAMS

Beam

A beam is a rod or bar of uniform cross-section (either circular or rectangular) of a homogeneous, isotropic elastic material whose length is large compared to its thickness.

When such a beam is fixed at one end and loaded at the other, within the limit of elasticity a bending is produced due to the moment of the load. The deformation produced by the load brings about restoring forces due to elasticity tending to bring the strip back to its original position. In equilibrium position,

Restoring couple = Bending couple

These two couples act in the opposite directions. The plane in which these couples act is called the plane of bending. The moment of the couple due to the elastic reactions (restoring couple) which balances the external couple due to the applied load is called the bending moment.

Assumptions

While studying about the bending of beams, the following assumptions have to be made.

i. The length of the beam should be large compared to other dimensions.
ii. The load(forces) applied should be large compared to the weight of the beam.
iii. The cross section of the beam remains constant and hence the geometrical moment of inertia Ig also remains constant.
iv. The shearing stresses are negligible.
v. The curvature of the beam is very small.

Bending of a beam and neutral axis:

Let us consider a beam of uniform rectangular cross section Fig.2.11. A beam may be assumed to consist of a number of parallel longitudinal metallic fibres placed one over the other and are called as filaments as shown in Fig 2.12.

Let the beam be subjected to deforming forces at its ends as shown in Fig 2.13. Due to the deforming forces the beam bends. We know the beam consists of many filaments. Let us consider a filament AB at the centre of the beam.

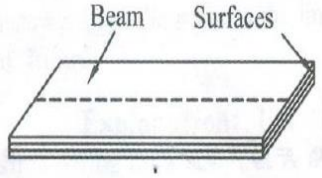

Fig 2.11

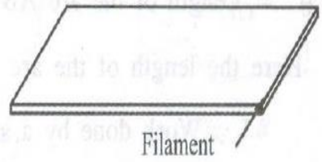

Fig 2.12

It is found that the filaments (layers) lying above AB gets elongated, while the filaments (layers) lying below AB gets compressed. Therefore the filament (i.e) layer AB which remains unaltered is taken as the reference axis called as NEUTRAL AXIS and the plane is called as neutral plane. Further, the deformation of any filament can be is measured with reference to the neutral axis.

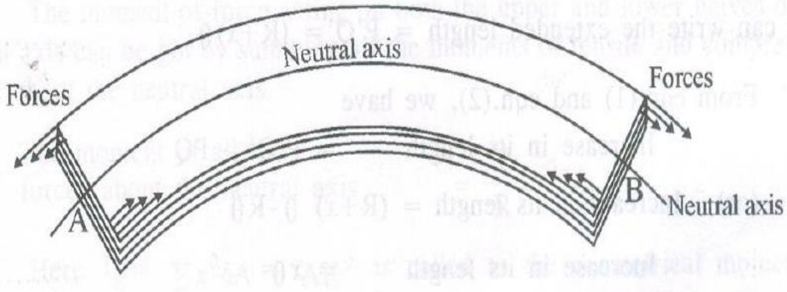

Fig. 2.13

2.14 EXPRESSION FOR THE BENDING MOMENT:

Let us consider a beam under the action of deforming forces. The beam bends into a circular arc as shown in Fig 2.14. Let AB be the neutral axis of the beam. Here the filaments above AB are elongated and the filaments below AB are compressed. The filament AB remains unchanged.

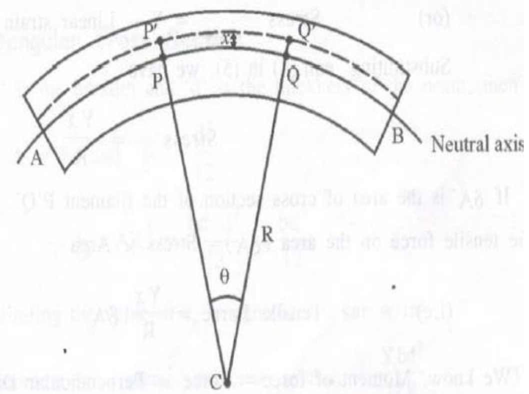

Fig. 2.14

Let PQ be the arc chosen from the neutral axis. If R is the radius of curvature of the neutral axis and θ is the angle subtended by it at its centre of curvature 'C'.

Then we can write original length PQ = Rθ --------------------(1)

Let us consider a filament P'Q' at a distance 'x' from the neutral axis.

We can write the extended length = P'Q'= (R+x)θ --------------------(2)

From eqn.(1) and eqn.(2), we have

Increase in its length = P'Q' – PQ

Increase in its length = (R+x)θ – Rθ

Increase in its length = xθ --------------------(3)

We know Linear Strain = $\frac{Increase\ \ in\ length}{Original\ \ length}$

(or) Linear Strain = $\frac{x\theta}{R\theta}$

Linear Strain = $\frac{x}{R}$ --------------------(4)

We know,

The Young's modulus of the material Y = $\frac{Stress}{Linear\ \ Strain}$

(or) Stress = Y x Linear strain --------------------(5)

Substituting eqn.(4) in (5), we have

Stress = $\frac{Yx}{R}$

If δA is the area of cross section of the filament P'Q'. Then,

The tensile force on the area (δA) = Stress x Area

(i.e) Tensile Force = $\frac{Yx}{R}$.δA

We know, Moment of force = Force x Perpendicular Distance

Moment of the tensile force about the neutral axis AB (or)

PQ = $\frac{Yx}{R}$. δAx

PQ = $\frac{Y}{R}$. δAx^2

The moment of force acting on both the upper and lower halves of the neutral axis can be got by summing all the moments of tensile and compressive forces about the neutral axis.

The moment of all the forces about the neutral axis = $\frac{Y}{R}$.$\sum x^2$δA

Here I_g = .$\sum x^2$δA = AK^2 is called the geometrical moment of Inertia.

Where, A is the total area of the beam and K is the radius of the Gyration.

Total moment of all the forces (or) Internal bending moment = $\frac{YI_g}{R}$ --------------------(6)

Special cases:

1.Rectangular cross section:

If 'b' is the breadth and 'd' is the thickness of the beam, then

$$\text{Area A} = bd \text{ and } K^2 = \frac{d^2}{12}$$

$$I_g = AK^2 = \frac{bd.d^2}{12} = \frac{bd^3}{12}$$

Substituting the value of in eqn.(6), we can write

Bending moment for a rectangular cross section = $\frac{Ybd^3}{12R}$ --------------------(7)

2. Circular cross section

For a circular cross section if 't' is the radius, then Area A = πr^2

and $$K^2 = \frac{r^2}{4}$$

$$I_g = AK^2 = \frac{\pi r^2 X\ r^2}{4}$$

$$I_g = \frac{\pi r^4}{4}$$

Substituting the value of I_g in eqn.(6), we can write

The Bending moment of a circular cross section = $\frac{\pi Y r^4}{4R}$ --------------------(8)