SNS COLLEGE OF ENGINEERING, COIMBATORE-641 107

DEPARTMENT OF MATHEMATICS

MA 8251 ENGINEERING MATHEMATICS-II
QUESTION BANK- Unit –III
Analytic functions
Part-A

		Unit	Regulat	year
1.	The real part of an analytic function f(z) is constant ,prove that f(z) is a constant function.	3	13	Apr/May 17
2.	Prove that the family of curves $u = c$, $v = k$ cuts orthogonally for an analytic function $f(z) = u + iv$	3	13	Dec/Jan 16
3.	Show that $f(x, y) = \log\sqrt{x^2 + y^2}$ is harmonic.	3	8	Apr /May 17
4.	Is the function $f(z) = \|z\|^2$ analytic . Justify.	3	8	Dec/Jan 16
5.	Is the function f(z) = $\bar{z}$ is analytic	3	13	May/ June 14
6.	Give an example of a function where u and v are harmonic but u+iv is not analytic.	3	13	May/June 16
7.	Give an example of a complex – valued function which is differentiable at a point but not analytic at that point.	3	8	Nov/Dec 14
8.	If $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$, verify whether u is harmonic.	3	8	Nov/Dec 14
9.	Examine whether y+e^xcosy is harmonic.	3	13	Noc /Dec 17
10.	Verify f(z)=z^3 is analytic or not.	3	13	Nov/Dec 14
11.	Find the value of m if u=$2x^2$-my^2+3x is harmonic	3	13	Nov /Dec 16
12.	Find the image of the line x=1 under the transformation w=z^2.	3	13	Noc /Dec 17
13.	Find the image of the circle $\|z\| = 3$ transformation w=2z	3	13	Nov /Dec 16
14.	Define conformal mapping	3	8	Dec/Jan 16
15.	Find the critical points of the transformation $\omega = z^2 - \frac{1}{z^2}$.	3	13	Apr/May 17
16.	Find the invariant points of a function $f(z) = \dfrac{z^3 + 7z}{7 - 6zi}$.	3	13	Dec/Jan 16
17.	Find the critical points of the map w^2=(z-α)(z-β).	3	13	Nov/Dec 14,May/June 16
18.	Find the invariant points of f(z) = z^2	3	13	May/ June 14
19.	Classify the singularities of the function $f(z) = \dfrac{z - \sin z}{z}$	3	8	Dec/Jan 16
	PART-B			
1	If f(z) is a regular function of z,prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\log\|f(z)\| = 0$	3	13	Apr/May 17
2	If f(z) is a regular function of z, then prove that $\nabla^2\|f(z)\|^2 = 4\|f'(z)\|^2$	3	8	Nov/Dec 14,Apr /May 17,
3	If $f(z) = u + iv$ is an analytic function in $z = x + iy$ then prove	3	13	May/June 16

	that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|u\|^2 = 2\|f'(z)\|^2$.			
4	State and prove the necessary conditions for f(z) to be analytic	3	8	Dec/Jan 16
5	Is $f(z)=z^n$ analytic function everywhere?	3	13	Dec/Jan 16
6	Prove that the real and imaginary parts of an analytic function are harmonic functions	3	13	May/ June 14
7	Prove that $w=\frac{z}{z+a}$ where a≠0 is analytic where as $w=\frac{\bar{z}}{\bar{z}+a}$ is not analytic.	3	13	May/June 16
8	If f+u+iv is analytic on a domain D and $\|f\|$ is a constant on D,Prove that f must be a constant on D.	3	8	Nov/Dec 14
9	Prove that f(z)=z^n is analytic for all values of n and find its derivative.	3	13	Noc /Dec 17
10	Find the analytic function $f(z)=u+iv$, given that $2u+3v=e^x(\cos x-\sin y)$.	3	13	Apr/May 17
11	Find an analytic function f(z)= u +iv , given that $2u+3v=\frac{\sin 2x}{\cosh 2y-\cos 2x}$	3	8	Apr /May 17
12	Construct the analytic function $f(z)=u+iv$, given that $u=e^{x^2-y^2}\cos 2xy$. Hence find v	3	8	Dec/Jan 16
13	If $u=x^2-y^2$, $v=\frac{y}{x^2+y^2}$, prove that u and v are harmonic functions but $f(z)=u+iv$ is not an analytic function.	3	13	Dec/Jan 16
14	Show that the function $u=e^{-2xy}\sin(x^2-y^2)$ is a real part of an analytic function. Also find its conjugate harmonic function v and express $f(z)=u+iv$ as function of z.	3	13	Dec/Jan 16
15	Show that u = e^{-x} (x cos y + ysiny) is harmonic function. Hence find the analytic function f(z) = u + iv	3	13	May/ June 14
16	Can $v=\tan^{-1}\left(\frac{y}{x}\right)$ be the imaginary part of an analytic function? If so construct an analytic function $f(z)=u+iv$, taking v as imaginary part and hence find u.	3	13	May/June 16
17	Find the analytic function f=u+iv given that $u(x,y)=e^{2x}(x\sin 2y+y\cos 2y)$.	3	8	Nov/Dec 14
18	Prove that the function u=log$\sqrt{(x^2+y^2}$ is harmonic and hence find its conjugate harmonic.	3	13	Noc /Dec 17
19	Prove that u=x^2-y^2 and $v=\frac{-y}{x^2+y^2}$ are harmonic functions but not harmonic conjugates.	3	13	Nov/Dec 14
20	Given that $u=\frac{\sin 2x}{\cosh 2y-\cos 2x}$, find the analytic function f(z)=u+iv	3	13	Nov/Dec 14
21	Find the analytic function $f(z)=u+iv$, whose real part is $u=e^x(x\cos y-y\sin y)$ Find also the conjugate harmonic of u.	3	13	Nov /Dec 16

22	Show that the transformation $\omega = \frac{1}{z}$ transforms in general ,circles and straight lines into circles and straight lines.	3	13	Apr/May 17
23	Find the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$	3	8	Apr /May 17
24	Discuss the transformation $w = \frac{1}{v}$	3	8	Dec/Jan 16
25	Find the image of the lines $u = a$ and $v = b$ in w-plane into z-plane under the transformation.	3	13	Dec/Jan 16
26	Find the image of $\lvert z+1 \rvert = 1$ under the map w = $\frac{1}{z}$	3	13	May/ June 14
27	Find the image of the circle $\lvert z-2i \rvert=2$ under the transformation w=$\frac{1}{z}$.	3	13	Noc /Dec 17
28	Show that the transformation $\omega = \frac{1}{z}$ transforms in general, circles and straight lines into circles and straight lines.	3	13	Nov /Dec 16
29	If f(z)=u(x,y)+iv(x,y) is an analytic function show that the curves u(x,y)=c_1, v(x,y)=c_2 cut orthogonally.	3	13	Nov /Dec 16
30	Find the bilinear transformation which maps the point -1,0.1 of the z-plane into the points -1 ,-I ,1 of the ω-plane respectively	3	13	Apr/May 17
31	Find the bilinear transformation which maps the points $z = \infty$, $z = i$, $z = 0$ on to the points $w = 0$, $w = i$, $w = \infty$	3	8	Apr /May 17
32	Find the bilinear transformation that maps the points $0, 1, \infty$ of the z – plane in to the points -5, -1, 3 of the w – plane. Also find its fixed points	3	8	Dec/Jan 16
33	Find the bilinear transformation which maps $1, -i, 1$ in z-plane into $0, 1, \infty$ of the w-plane respectively.	3	13	Dec/Jan 16
34	Find the bilinear transformation that maps 1, i and -1 of the z-plane on to 0, 1, ∞ of the w-plane.	3	13	May/ June 14
35	Find the bilinear transformation that transforms the points $z = 1, i, -1$ of the z-plane into the points $w = 2, i, -2$ of the w-plane.	3	13	May/June 16
36	Find the bilinear transformation which maps the points $\infty, 2, -1$ *to* $1, \infty$ *and* 0 respectively.	3	8	Nov/Dec 14
37	Find the bilinear transformation which maps the points z=1,i,-1 in to the points w=i,0,-i. Hence find the image of $\lvert z \rvert<1$.	3	13	Noc /Dec 17
38	Find the bilinear transformation which maps the point z= 1,i.-1 onto the points w= i ,0 ,-i.	3	13	Nov/Dec 14
39	Find the bilinear transformation which maps the point z= 0,1.-1 of the onto the points w= -1 ,0 ,∞ Find also the invariant points of the transformation.	3	13	Nov /Dec 16