## MA 8251 ENGINEERING MATHEMATICS-II QUESTION BANK- Unit -II <br> Vector calculus <br> Part-A

|  |  | Unit | Reg | year |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Find the unit normal vector to the surface $x^{3}+y^{2}-z$ at (1,1,2). | 2 | 13 | Apr/May 17 |
| 2. | Find the unit normal vector of the surface $x^{2}+y^{2}-z=1$ at (1, 1,1). | 2 | 8 | $\begin{gathered} \text { Apr /May } \\ 17 \end{gathered}$ |
| 3. | Find the unit vector normal to the surface $\mathrm{x}^{2}+\mathrm{y}^{2}=\mathrm{z}$ at $(1,-2,5)$ | 2 | 13 | $\begin{gathered} \hline \text { May/ June } \\ 14 \end{gathered}$ |
| 4. | Find the unit normal vector to $\mathrm{xy}=\mathrm{z}^{2}$ at ( $1,1,-1$ ). | 2 | 13 | Nov /Dec 16 |
| 5. | Using Greens theorem in the plane,find the area of the circle $x^{2}+y^{2}=a^{2}$ | 2 | 13 | Apr/May 17 |
| 6. | Using Greens theorem evaluate $\int_{C}(x d y-y d x)$ where C is the circle $x^{2}+y^{2}=1$ in the xy plane | 2 | 13 | Nov/Dec 16 |
| 7. | Prove that $\operatorname{Grad}(1 / r)=\frac{-\vec{r}}{r^{3}}$. | 2 | 13 | Dec/Jan 16 |
| 8. | Prove that $\operatorname{Curl}(\operatorname{grad} \varphi)=0$ | 2 | 13 | May/ June <br> 14 |
| 9. | Evaluate $\nabla^{2} \log r$. | 2 | 13 | $\begin{gathered} \text { May/June } \\ 16 \end{gathered}$ |
| 10 | Find $\nabla\left(\nabla \cdot\left(\left(x^{2}-y z\right) \vec{i}+\left(y^{2}-x z\right) \vec{j}+\left(z^{2}-x y\right) \vec{k}\right)\right)$ at the point $(1,-1,2)$. | 2 | 8 | Nov/Dec 14 |
| 11 | Find Curl $\vec{F}$ if $\vec{F}=x y \vec{i}+y z \vec{j}+z x \vec{k}$ | 2 | 13 | Nov/Dec 14 |
| 12 | Evaluate $\int_{c}(y z \dot{i}+x z \vec{j}+x y \vec{k}) \cdot \overrightarrow{d r}$ where C is the boundary of the surface S . | 2 | 13 | Dec/Jan 16 |
| 13 | Evaluate the integral $\int_{\mathrm{c}} \vec{F} . \mathrm{d} \vec{r}$ if $\vec{F}=\mathrm{xy}^{2} \overrightarrow{\underline{l}}+\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right) \vec{j}$ and C is the curve given by $y=x^{2}-4$ from $(2,0)$ to $(4,12)$. | 2 | 13 | Noc/Dec 17 |
| 14 | What is the greatest rate of $\phi=x y z^{2}$ at (1,0,3) | 2 | 8 | $\begin{gathered} \text { Apr /May } \\ 17 \end{gathered}$ |
| 15 | The temperature of points in space is given by $T(x, y, z)=x^{2}+y^{2}-z$. A mosquito located ( $1,1,2$ )desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move? | 2 | 8 | Dec/Jan 16 |
| 16 | If $\vec{F}=(\mathrm{x}+3 \mathrm{y}) \vec{i}+(\mathrm{y}-2 \mathrm{z}) \vec{j}+(\mathrm{x}+2 \mathrm{kz}) \vec{k}$ has divergence zero, find the unknown value of $k$. | 2 | 13 | Nov/Dec 17 |
| 17 | State Green's theorem in a plane | 2 | $\begin{gathered} 8,1 \\ 3 \end{gathered}$ | Noc /Dec 14,Dec/Jan 16 |
| 18 | State Stokes' theorem | 2 | $\begin{gathered} 8,1 \\ 3 \end{gathered}$ | Noc /Dec 14,May/Jun1 6 |
| Part-B |  |  |  |  |
| 1 | Find the angle between the surfaces $x^{2}+y^{2}+z^{2}=9$ and | 2 | 13 | $\begin{gathered} \hline \text { Nov/Dec } \\ \text { 16,Apr/May } \end{gathered}$ |


|  | $z^{2}=x^{2}+y^{2}-3$ at the point (2,-1,2). |  |  | 17 |
| :---: | :---: | :---: | :---: | :---: |
| 2 | Find the angle between the normals to the surfaces $x^{2}=y z$ at the points( $1,1,1$ ) and $(2,4,1)$. | 2 | 13 | Nov/Dec 14 |
| 3 | Prove that $\operatorname{div}\left(\operatorname{grad} r^{n}\right)=\mathrm{n}(\mathrm{n}+1) r^{n-2}$ <br> Prove that $\operatorname{curl}(\operatorname{grad} \phi)=0$ |  | 8 | $\begin{gathered} \hline \text { Apr /May } \\ 17 \end{gathered}$ |
| 4 | Prove $\nabla^{2}\left(r^{n}\right)=n(n+1) r^{n-2}$ and deduce that $\frac{1}{r}$ satisfies Laplace equation | 2 | 8 | $\begin{gathered} \text { DEC /JAN } \\ 16 \end{gathered}$ |
| 5 | Find the value of n such that the vector $r^{n \vec{r}}$ is both solenoidal and irrotational. | 2 | 13 | May/ June 14 |
| 6 | Prove that CurlCurl $\vec{F}=\operatorname{grad} \operatorname{div} \vec{F}-\nabla^{2} \vec{F}$. | 2 | 13 | May/June $16$ |
| 7 | Prove that $\operatorname{div}(\phi \vec{F})=\phi d i v \vec{F}+\nabla \phi \cdot \vec{F}$. Also, determine the value of n for which $r^{n} \vec{R}$ is solenoidal, where $\vec{R}=x \vec{i}+y \vec{j}+z \vec{K}$ and $r=\|\vec{R}\|$. | 2 | 8 | Nov/Dec 14 |
| 8 | A fluid motion is given by $\vec{V}=(y+z) \dot{i}+(z+x) \vec{j}+(x+y) \vec{k}$. Is this motion is irrotational and is possible for an incompressible fluid? | 2 | 13 | Dec/Jan 16 |
| 9 | If $\nabla \varphi=2 x y z^{3}{ }^{3}+x^{2} z^{3} \vec{j}+3 x^{2} y z^{2} \vec{k}$.find $\varphi(x, y, z)$ given that $\varphi(1,-2,2)=4$ | 2 | 13 | $\begin{gathered} \hline \text { May/June } \\ 16 \end{gathered}$ |
| 10 | Find the constants a,b,c so that $\vec{F}=(x+2 y+a z) \vec{i}+(b x-3 y-z) \vec{j}-(4 x+c y+2 z) \vec{k}$ is irrotational.For those values of a,b,c.Find its scalar potential. | 2 | 13 | Apr/May 17 |
| 11 | Show that $\vec{F}=2 x y z^{3} \bar{i}+x^{2} z^{3} \vec{j}+3 x^{2} y z^{2} \vec{k}$ is irrotational. Find the scalar potential $\phi$ and $\mathrm{F}=\operatorname{grad} \phi$ | 2 | 8 | $\begin{gathered} \hline \text { Apr /May } \\ 17 \end{gathered}$ |
| 12 | Show that $\vec{F}=\left(y^{2}+2 x z^{2}\right) \dot{i}+(2 x y-z) \vec{j}+\left(2 x^{2} z-y+2 z\right) \vec{k}$ is irrotational and hence find its scalar potential | 2 | 8 | $\begin{gathered} \hline \text { Nov/Dec } \\ \text { 14,DEC } \\ \text { /JAN } 16 \end{gathered}$ |
| 13 | Prove that $\vec{F}=\left(\mathrm{x}^{2}-\mathrm{y}^{2}+\mathrm{x}\right) \vec{i}-(2 \mathrm{xy}+\mathrm{y}) \vec{j}$ is a conservative field and find the scalar potential of $\vec{F}$. | 2 | 13 | Noc /Dec 17 |
| 14 | Prove that $\vec{F}=\left(x^{2}-y^{2}+x\right) \vec{i}-(2 x y+y) \vec{j}$ is irrotational and hence find its scalar potential. | 2 | 13 | Nov /Dec 14 |
| 15 | Prove that $\vec{F}=\left(y^{2} \cos x+z^{3}\right) \vec{i}+(2 y \sin x-4) \vec{j}+3 x z^{2} \vec{k}$ is irrotational and find its scalar potential. | 2 | 13 | Nov /Dec 16 |
| 16 | Verify Green's theorem for $\vec{F}=\left(x^{2}+y^{2}\right) \dot{i}-2 x y \vec{j}$ taken around the rectangle bounded by the lines $\mathrm{x}= \pm \mathrm{a}, \mathrm{y}=0$ and $\mathrm{y}=\mathrm{b}$. | 2 | 13 | Dec/Jan 16 |
| 17 | Using Green's theorem in a plane $\int_{C}\left[x^{2}(1+y) d x+\left(x^{3}+y^{3}\right) d y\right]$ where C is the square formed by $x= \pm 1$ and $y= \pm 1$. | 2 | 13 | May/June $16$ |
| 18 | Appply Green's theorem to evaluate $\int_{c}\left(x y-x^{2}\right) d x+x^{2} y$ dy along the closed curve $C$ formed by $y=0, x=1$ and $y=x$. | 2 | 13 | Noc /Dec 17 |
| 19 | Using Greens theorem, evaluate $\int_{C}\left[\left(3 x^{2}-8 y^{2}\right) d x+(4 y-6 x y) d y\right]$ where C is the boundary of the triangle formed by the lines $x=0, y=0, x+y=1$ in the $x y$ plane. | 2 | 13 | Nov /Dec 14 |
| 20 | Verify Stoke's theorem for $\vec{F}=\left(x^{2}-y^{2}\right) \vec{i}+2 x y \vec{j}$,where S is the rectangle | 2 | 13 | May/ June |


|  | in the xy -plane formed by the lines $\mathrm{x}=0, \mathrm{x}=\mathrm{a}, \mathrm{y}=0$ and $\mathrm{y}=\mathrm{b}$. |  |  | $\begin{gathered} \text { 14,Apr/May } \\ 17 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 21 | Using Stoke's theorem to evaluate $\int_{c} \vec{F} d \vec{r}$ where $\vec{F}=(\sin x-y) \dot{i}-\cos x \vec{j}$ and C is the boundary of the triangle whose vertices are $(0,0),\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$ | 2 | 8 | $\begin{gathered} \text { DEC /JAN } \\ 16 \end{gathered}$ |
| 22 | Verify Stoke's theorem for $\vec{F}=\left(x^{2}+y^{2}\right) \vec{i}+2 x y \vec{j}$ where S is the rectangle in the xy -plane formed by the lines $\mathrm{x}=0, \mathrm{x}=\mathrm{a}, \mathrm{y}=0, \mathrm{y}=\mathrm{b}$. | 2 | 8 | Nov/Dec 14 |
| 23 | Verify Divergence theorem for $\vec{F}=4 x z \vec{i}-y^{2} \vec{j}+y z \vec{k}$ taken over the cube bounded by the planes $\mathrm{x}=0, \mathrm{x}=\mathrm{a}, \mathrm{y}=0, \mathrm{y}=\mathrm{a}, \mathrm{z}=0, \mathrm{z}=\mathrm{a}$. | 2 | 13 | $\begin{gathered} \text { Nov/Dec } \\ \text { 16,Apr/May } \\ 17 \end{gathered}$ |
| 24 | Verify Gauss divergence theorem for $\vec{F}=y \bar{i}+x \vec{j}+z^{2} \vec{k}$ for the cylindrical region given by $x^{2}+y^{2}=a^{2}, \mathrm{z}=0, \mathrm{z}=\mathrm{h}$. | 2 | 8 | $\begin{gathered} \text { Apr /May } \\ 17 \end{gathered}$ |
| 25 | Verify Gauss divergence theorem for $\vec{F}=x^{2} \dot{i}+y^{2} \vec{j}+z^{2} \vec{k}$, wher S is the surface of the cuboid formed by the planes $x=0, x=a, y=0, y=b, z=0, z=c$ | 2 | 8 | Nov/Dec 14,DEC /JAN 16 |
| 26 | Verify Gauss divergence theorem for $\vec{F}=\left(x^{2}-y z\right) \dot{i}+\left(y^{2}-x z\right) \vec{j}+\left(z^{2}-x y\right) \vec{k}$. and S is the surface of the rectangular parallelepiped bounded by $x=0, x=a, y=0, y=b, z=0$ and $z=c$. | 2 | 13 | Dec/Jan 16 |
| 27 | Verify Gauss divergence Theorem for $\vec{F}=x^{2 \vec{\imath}}+y^{2 \vec{j}}+z^{2} \vec{k}$ taken over the cube bounded by the planes $\mathrm{x}=0, \mathrm{x}=1, \mathrm{y}=0, \mathrm{y}=1, \mathrm{z}=0, \mathrm{z}=1$ | 2 | 13 | May/ June $14$ |
| 28 | Evaluate $\iint_{\mathrm{s}} \vec{f} \cdot \hat{n} \mathrm{dS}$ where $\vec{F}=\mathrm{z} \vec{l}+\mathrm{x} \vec{\jmath}-3 \mathrm{y}^{2} \mathrm{z} \vec{k}$ and S is the surface of the cylinder $x^{2}+y^{2}=16$ included in the first octant between $\mathrm{z}=0$ and $\mathrm{z}=5$. | 2 | 13 | Noc /Dec 17 |
| 29 | Evaluate $\iint \vec{F} . \hat{n} \mathrm{dS}$ using Gauss divergence theorem for $\vec{F}=\mathrm{x}^{2} \vec{\imath}+\mathrm{y}^{2} \vec{\jmath}+\mathrm{z}^{2} \vec{k}$ taken over the cube bounded by the planes $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0$, $\mathrm{x}=1, \mathrm{y}=1, \mathrm{z}=1$. | 2 | 13 | Noc /Dec 17 |
| 30 | Find the work done in moving particle in the force field $\vec{F}=3 x^{2} \vec{i}+(2 x z-y) \vec{j}+z \vec{k}$ along the curve defined by $x^{2}=4 y$, $3 x^{2}=8 z$ from $\mathrm{x}=0$ to $\mathrm{x}=2$. | 2 | 8 | $\begin{gathered} \text { Apr /May } \\ 17 \end{gathered}$ |
| 31 | Find the values of constants a,b,c so that the maximum value of the directional derivative of $\varphi=a x y^{2}+b y z+c z^{2} x^{3}$ at $(1,2,-1)$ has a magnitude 64 in the direction parallel to z -axis. | 2 | 13 | Dec/Jan 16 |
| 32 | Find the directional derivative of $\phi=4 x z^{2}+x^{2} y z$ at $(1,-2,1)$ in the direction of $2 \vec{i}+3 \vec{j}+4 \vec{k}$ | 2 | 13 | Nov /Dec 16 |
| 33 | Find ' a ' and ' b ' so that the surfaces $a x^{3}-b y^{2} z=(a+3) x^{2}$ and $4 x^{2} y-z^{3}=11$ cut orthogonally at (2, $-1,-3$ ) | 2 | 13 | $\begin{gathered} \text { May/June } \\ 16 \end{gathered}$ |
| 34 | Find the values of constants a,b,c so that the maximum value of the directional derivative of $\varphi=a x y^{2}+b y z+c z^{2} x^{3}$ at $(1,2,-1)$ has a magnitude 64 in the direction parallel to z -axis. | 2 | 13 | Dec/Jan 16 |

