$$L[tf(t)] = -\varphi'(s)$$

Corollary :- If $L[f(t)] = \varphi(s)$ then $L[t^n f(t)] = (-1)^n \varphi^n(s)$.

Proof : W.K.T. $L[tf(t)] = -\varphi'(s)$

$$L[t^2 f(t)] = L[t \cdot tf(t)]$$

$$= -\frac{d}{ds} L[tf(t)]$$

$$= -\frac{d}{ds}\left[\frac{-d}{ds} Lf(t)\right]$$

$$= (-1)^2 \frac{d^2}{ds^2}[Lf(t)]$$

$$= (-1)^2 \frac{d^2}{ds^2} \varphi(s)$$

.....
.....
.....

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \varphi(s) = (-1)^n \varphi^n(s)$$

PROBLEMS BASED ON TRANSFORMS OF DERIVATIVES

Example 1. Find L [t sin 2t]

Solution : W.K.T. $L[t^n f(t)] = (-1)^n \varphi^n(s)$

$$L(t \sin 2t) = -\frac{d}{ds}[L(\sin 2t)] = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$$

$$= -\left[\frac{-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$$

Example 2. Find L [$t^2 e^{-3t}$]

Solution : W.K.T $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[\varphi(s)]$

$$L[t^2 e^{-3t}] = (-1)^2 \frac{d^2}{ds^2} L[e^{-3t}] = \frac{d^2}{ds^2}\left[\frac{1}{s+3}\right]$$

$$= \frac{d}{ds}\left[\frac{-1}{(s+3)^2}\right] = \frac{2}{(s+3)^3}$$