# Unit-4

# DYNAMICS OF PARTICLES

# Particle Kinematics

**Dynamics = Kinematics + Kinetics**

**Kinematics: The *description* of motion (position, velocity, acceleration, time) without regard to forces.**

**Kinetics: Determining the *forces* (based on F=ma) associated with motion.**

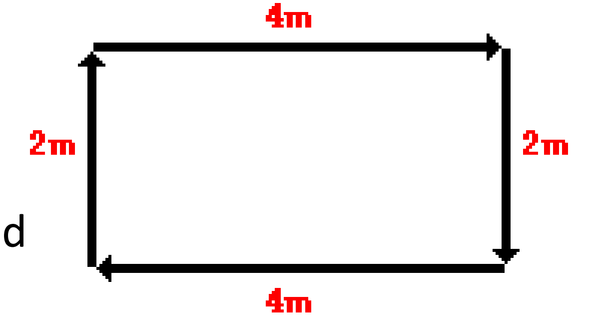
# Distance and Displacement

**Distance** is a scalar quantity that refers to "how much ground an object has covered" during its motion.

**Displacement** is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.

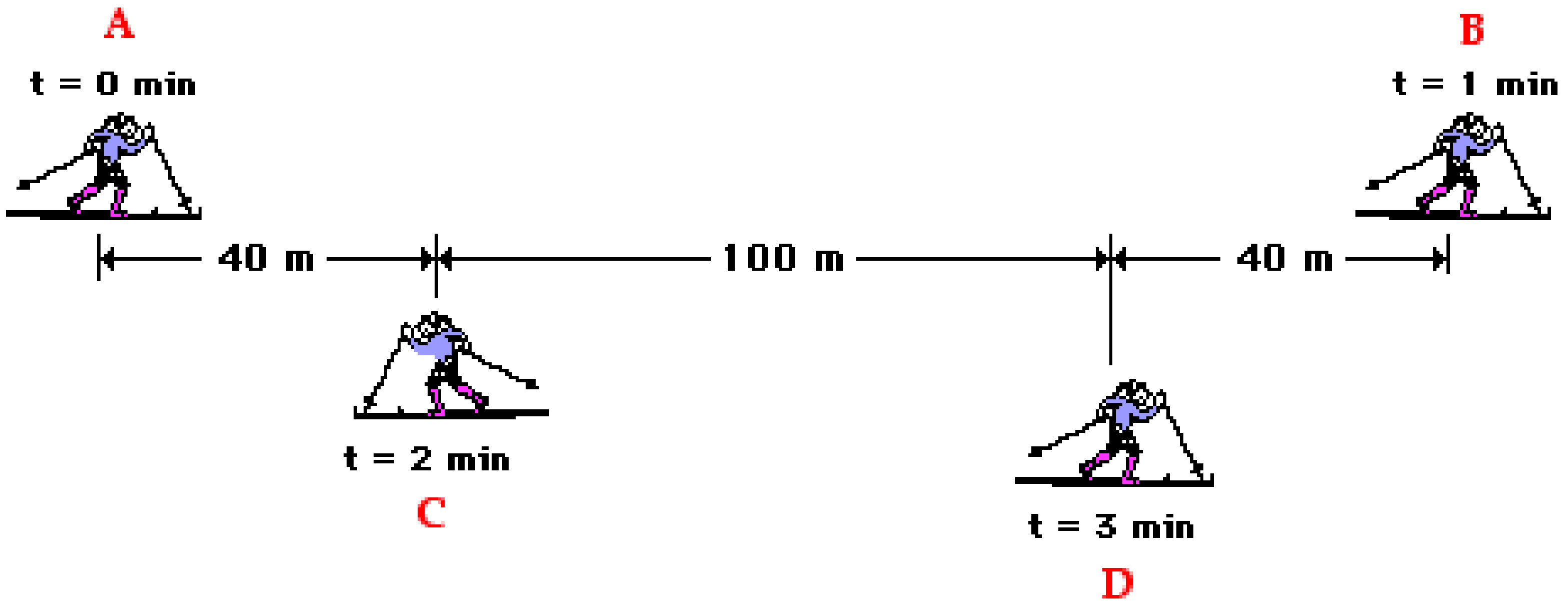
**To test your understanding of this distinction, consider the motion depicted in the diagram below. A person walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North.**

He covered 12 meters of ground
(distance = 12 m)




There is no displacement for his motion
(displacement = 0 m)

**Use the diagram to determine the resulting displacement and the distance traveled by the skier during these three minutes.**



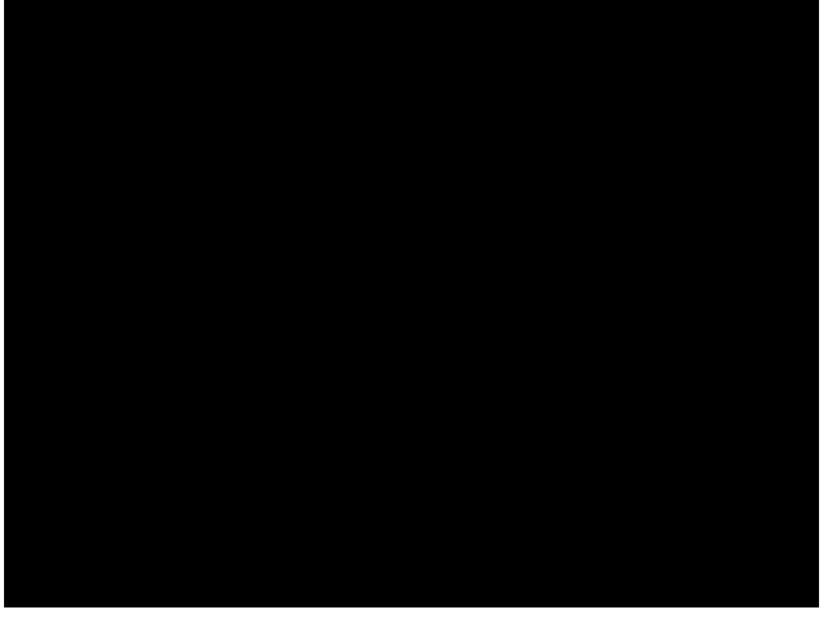

The skier covers a distance of

(180 m + 140 m + 100 m) = **420 m**

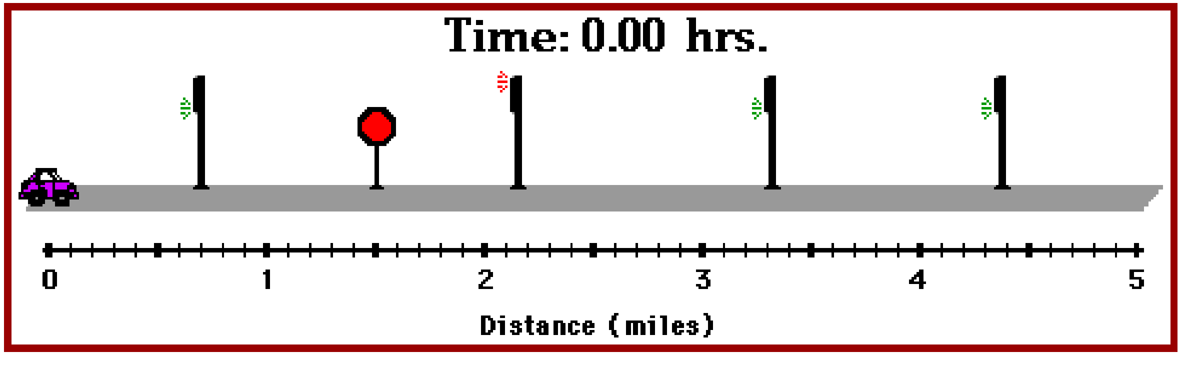
has a displacement of **140 m, rightward**.

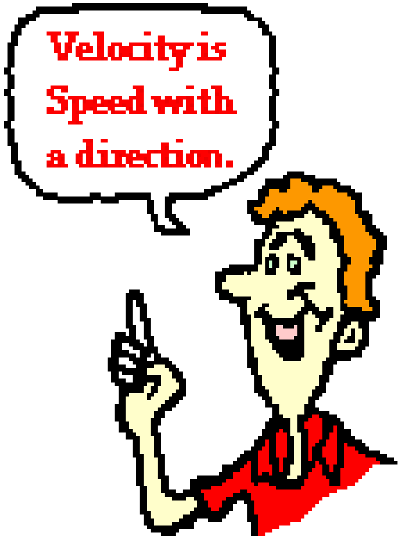
# Speed and Velocity

- **Speed** is a scalar quantity that refers to "how fast an object is moving.“
- Speed can be thought of as the rate at which an object covers distance.
- A fast-moving object has a high speed and covers a relatively large distance in a short amount of time.
- Contrast this to a slow-moving object that has a low speed; it covers a relatively small amount of distance in the same amount of time.
- An object with no movement at all has a zero speed.



- **Velocity** is a vector quantity that refers to "the rate at which an object changes its position."
- Imagine a person moving rapidly - one step forward and one step back - always returning to the original starting position.
- While this might result in a frenzy of activity, it would result in a zero velocity.
- Because the person always returns to the original position, the motion would never result in a change in position.
- Since velocity is defined as the rate at which the position changes, this motion results in zero velocity.



The average speed during the course of a motion is often computed using the following formula:

$$\text{Average Speed} = \frac{\text{Distance Traveled}}{\text{Time of Travel}}$$

In contrast, the average velocity is often computed using this formula

$$\text{Average Velocity} = \frac{\Delta\,\text{position}}{\text{time}} = \frac{\text{displacement}}{\text{time}}$$

An object moving with a constant speed of 6 m/s

| Time (s) | Position (m) |
|---|---|
| 0 | 0 |
| 1 | 6 |
| 2 | 12 |
| 3 | 18 |
| 4 | 24 |

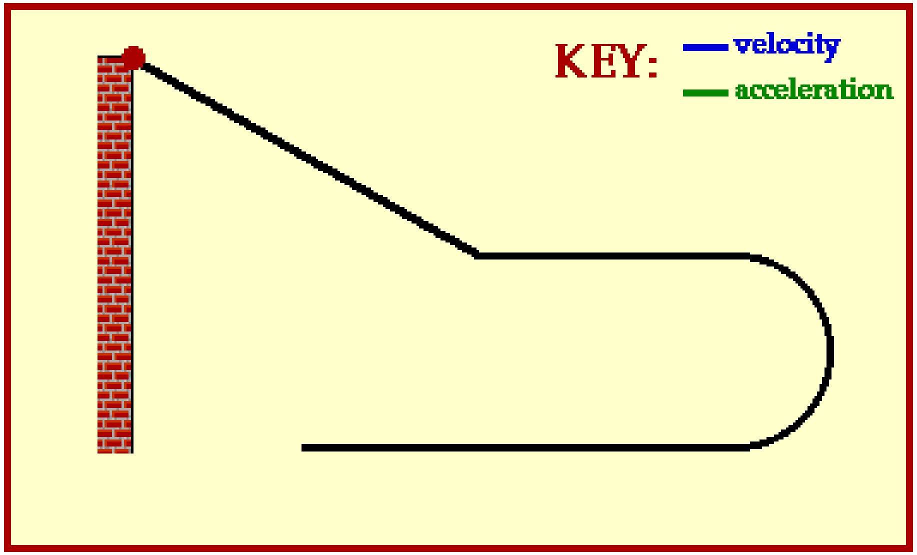
An object moving with a changing speed

| Time (s) | Position (m) |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

**v=ds/dt**

**Acceleration** is a vector quantity that is defined as the rate at which an object changes its velocity. An object is accelerating if it is changing its velocity. **$a = dv/dt$**

1. Which car or cars (red, green, and/or blue) are undergoing an acceleration?
2. Which car (red, green, or blue) experiences the greatest acceleration?




**An object moving with a constant speed of 6 m/s**

| Time (s) | Position (m) |
|---|---|
| 0 | 0 |
| 1 | 6 |
| 2 | 12 |
| 3 | 18 |
| 4 | 24 |

**An object moving with a changing speed**

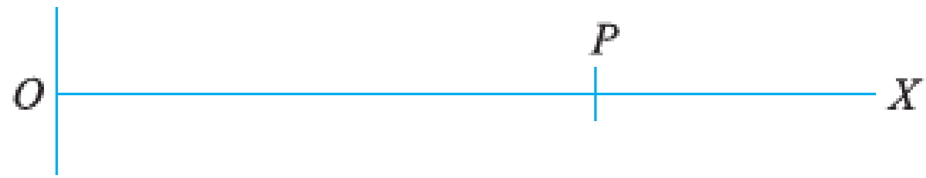
| Time (s) | Position (m) |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

# Kinematic Variables

**Particle kinematics involves describing a particle's position, velocity and acceleration versus time.**

| Kinematic Variables | | |
|---|---|---|
| **Description** | **Vector** | **Scalar** |
| Position | $\vec{r}$ | s |
| Velocity | $\vec{v}$ | v |
| Acceleration | $\vec{a}$ | a |
| Time | t | t |

# MOTION UNDER UNIFORM ACCELERATION




Consider *linear motion of a particle starting from $O$ and moving along $OX$ with a uniform acceleration as shown in Fig. 17.1. Let $P$ be its position after $t$ seconds.

Let $u$ = Initial velocity,

$v$ = Final velocity,

$t$ = Time (in seconds) taken by the particle to change its velocity from $u$ to $v$.

$a$ = Uniform positive acceleration, and

$s$ = Distance travelled in $t$ seconds.

Since in $t$ seconds, the velocity of the particle has increased steadily from ($u$) to ($v$) at the rate of $a$, therefore total increase in velocity

$$= a\,t$$

$$\therefore \quad v = u + a\,t \qquad ...(i)$$

and average velocity

$$= \left(\frac{u+v}{2}\right)$$

We know that distance travelled by the particle,

$$s = \text{Average velocity} \times \text{Time}$$

$$= \left(\frac{u+v}{2}\right) \times t \qquad \dots(ii)$$

Substituting the value of $v$ from equation ($i$),

$$s = \left(\frac{u+u+at}{2}\right) \times t = ut + \frac{1}{2}at^2 \qquad \dots(iii)$$

From equation ($i$), ($i.e.$ $v = u + at$) we find that

$$t = \frac{v-u}{a}$$

Now substituting this vlaue of $t$ in equation ($ii$),

$$s = \left(\frac{u+v}{2}\right) \times \left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a}$$

or $\qquad 2as = v^2 - u^2$

$\therefore \qquad v^2 = u^2 + 2as$

## Newton's Law of Motion

### Types of Motion

The motion can also be termed as "Plane Motion". It is classified into two types. They are

1. Translation
   - Rectilinear
   - Curvilinear
2. Rotation

### Tips for Solving Problems

1. If a body starts from rest, its initial velocity is zero, i.e u=0
2. If a body comes to rest, its final velocity is zero, i.e V=0
3. If a body is projected vertically upwards, the final velocity at the highest point is zero, i.e V=0
4. If a body starts moving vertically downwards, its initial velocity is zero, i.e u=0
5. Equation of motion of body under uniform acceleration due to gravity can be expressed as
   a. For downward motion

$$a = +g \qquad h = ut + \frac{1}{2}gt^2$$

$$v = u + gt \qquad v^2 - u^2 = 2gh$$

The position of a particle which move along a straight line is defined by $x = t^3 - 6t^2 - 15t + 40$ where x is in m, t is in sec. Determine the following

- The time at which the velocity will be zero
- The position and distance travelled by the particle at that time
- Acceleration of the particle at that time
- The distance travelled by the particle t=4 sec and t=6 sec

**Solution:**

Displacement $x = t^3 - 6t^2 - 15t + 40$

We know that,

Velocity, $v = \frac{dx}{dt} = 3t^2 - 12t - 15 \rightarrow (1)$

Also we know that

Acceleration, $a = \frac{dv}{dt} = 6t - 12 \rightarrow \ (2)$

a) Time at which velocity will be zero
By equating eqn (1) to zero

$$3t^2 - 12t - 15 = 0$$

$$t^2 - 4t - 5 = 0$$

$$t = +5 \text{ sec}$$ (t=-1sec is not practically possible)

b) Position and distance travelled when v = 0 when t=5, v=0 (zero velocity)

Position of particle at t=5 sec

$$x_5 = 5^3 - 6(5)^2 - 15(5) + 40$$
$$= 125 - 150 - 75 + 40 = -60\text{m}$$

Initial position of particle at t=0 sec

$$x_0 = 0^3 + 6(0)^2 - 15(0) + 40$$
$$x_0 = 40\text{m}$$

Distance travelled= $x_5 - x_0 = -60 - 40 = -100\text{m}$

i.e 100m in the negative direction

c) Acceleration when v=0

$$\text{v} = 0 \text{ at t} = 5 \text{ sec}$$
$$\text{a} = 6\text{t} - 12$$

$\text{a} = 6(5) - 12 = 18\text{m / sec}^2$

d) Distance travelled by the particle when t=4sec and t=6sec

Position at t=4sec $x_4 = 4^3 - 6(4)^2 - 15(4) + 40 = -52m$

Position at t=6sec $x_6 = 6^3 - 6(6)^2 - 15(6) + 40 = -50m$

Position at t=5sec $x_5 = -60m$

Distance trance travelled when t=5sec to t=6sec

$$= x_6 - x_5$$
$$= -50 - (-60)$$
$$= 10\text{m (Positive Displacement)}$$

Distance travelled when t=4sec to t=5sec

$$= x_5 - x_4$$
$$= (-60) - (-52)$$
$$= 8\text{m (Negative Displacement)}$$

**A train running at 80 km/h is brought to a standing halt after 50 seconds. Find the retardation and the distance traveled by the train before it comes to a halt.**

**Given :**

Initial Velocity, $U = 80$ Km/hr

$$= \frac{80 \times 1000}{3600}$$

$U = 22.22$ m/s

Final Velocity, $V = 0$

time, $t = 50$ sec.

**To find:**

retardation, $a = ?$

distance travelled, $s = ?$

**Solution:**

$$v = u + at$$

$$0 = 22.22 + a(50)$$

$$= -0.44 \text{ m/s}^2 \qquad \text{(Ans)}$$

$$v^2 - u^2 = 2as$$

$$\frac{0-(22.22)^2}{2 \times (-0.44)} = S$$

$$S = 561 \text{ m} \qquad \text{(Ans)}$$

Two trains A and B leave the same station on parallel lines. A starts with a uniform acceleration of $0.15 m/s^2$ and attains the speed of 24 km/hour after which its speed remains constant. B leaves 40 seconds later with uniform acceleration of $0.30 m/s^2$ to attain a maximum of 48 km/hour, its speed also becomes constant thereafter. When will B overtake A

**Solution :**

**Consider the motion of Train A:**

Initial velocity, $u = 0$

Final velocity, $V = 24$ km/hr

$$= \frac{24 \times 1000}{3600} = 6.67 m/s^2$$

Acceleration, $a = 0.15 m/s^2$

T= time taken when the train B will overtake the train A from its start.

$t_A$ = time taken by train A to attain a speed of $6.67 m/s^2$

$$V = u + a\, t_A$$

$$6.67 = 0 + 0.15\, t_A$$

$$t_A = 44.67 \text{ sec.}$$

Distance travelled by train a in 44.67 sec.

$$S_1 = u\,t_A + \frac{1}{2}\,a\,t_A^2$$

$$S_1 = 0 + \frac{1}{2}\,0.15\times(44.67)^2$$

$$S_1 = 150\text{m}$$

Since the train B leaves 40 seconds later, so that the train A has travelled (T+40) sec.

∴ Distance travelled by train A in (T+60) sec,

$$S_A = S_1 + V[(T+60) - t_A]$$

$$S_A = 150 + 6.67[(T+60) - 44.67] \quad \ldots\ldots(1)$$

**Consider the motion of Train B**

Initial velocity, $u = 0$

Final velocity, $V = 48$ km/hr

$$= \frac{48\times1000}{3600} = 13.34 \text{ m/s}$$

Acceleration, $a = 0.30$ m/s²

$t_B$ = time taken by train B to attain a speed of 13.34 m/s.

$$V = u + a\,t_B$$

$$13.34 = 0 + 0.3t_B$$

$$t_B = 44.47 \text{ sec.}$$

Distance travelled by train B in 44.47 sec.

$$S_2 = ut_B + a\, t_B^2$$

$$S_2 = 0 + \frac{1}{2}\ 0.3 \times (44.47)^2$$

$$S_2 = 296.63\text{m}$$

$\therefore$ Distance travelled by train B in T seconds is

$$S_B = S_2 + V(T - t_B)$$

$$S_B = 296.63 + 13.34(T - 44.47) - 44.67] \ .....(2)$$

At the instent, when train B overtake trains will be equal. Hence

$$S_A = S_B$$

$$150 + 6.67\ [(T+60) - 44.67] = 296.63 + 13.34\ (T - 44.47)$$

$$150 + 6.67T + 400.2 - 297.94 = 296.63 + 13.34T - 593.22$$

$$6.67T + 252.26 = 13.34T - 296.59$$

$$6.67T = 548.85$$

$$\mathbf{T = 82.28\ seconds} \quad \textbf{(Ans)}$$

Car A accelerates uniformly from rest on a straight level road. Car B starting from the same point 6 seconds later with initial velocity accelerates at $6 m/s^2$ .It overtakes the car A at 400m from the starting point. What is the acceleration of the car A?

**Given :**

Initial velocity of car A, $u_A = 0$

Initial velocity of car B, $u_B = 0$

acceleration of car B, $a_B = 6 m/s^2$

Distance travelled by car A and car B, $S_A = S_B = 400m$

**To Find :**

Acceleration to car A, $a_a = ?$

**Solution :**

Let '$t_A$' be the time taken by car 'A'.

Since the car 'B' starts 6 seconds later, the time taker car B is, $t_B = t_A - 6$

Consider motion of car 'A'

$$S_A = u_A t_A + \frac{1}{2} a_A t_A^2$$

$$400 = 0 + \frac{1}{2} a_A t_A^2$$

$$a_A t_A^2 = 800 \quad \text{..... (1)}$$

Consider motion of car 'B'

$$S_B = u_B t_B + \frac{1}{2} a_B t_B^2$$

$$400 = 0 + \frac{1}{2} \times 6 (t_A - 6)^2$$

$$\frac{800}{6} = t_A^2 + 36 - 12 t_A$$

$$t_A^2 - 12 t_A - 97.33 = 0$$

Solving we get, $t_A = 17.54$ sec.

Substituting $t_A = 17.54$ sec. in eqn. (1)

$$a_A (17.54)^2 = 800$$

$$a_A = 2.6 m/s^2$$

A stone is dropped from the top of a tower. It strikes the ground after four seconds. Find the height of the tower.

**Given :**

Time, $t = 4$ seconds

Initial Velocity, $u = 0$

Acceleration, $a = 9.81 m/s^2$

**To Find :**

Height of tower, $h = ?$

**Solution :**

We know

$$h = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2} \times 9.81 \times 4^2$$

$$\mathbf{h = 78.48m} \qquad \textbf{(Ans)}$$

A stone is dropped into a well . The sound of the splash is heard 3.63 seconds later.How far below the ground is the surface of water in the well? Assume the velocity of sound as 331m/s

**Given:**

Velocity of sound, v = 350 m/s

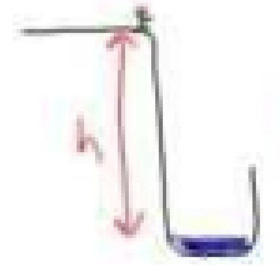
Initial velocity, u = 0

**Solution:**

Let t = time taken by stone to reach bottom of well

Depth of well is

$$h = ut + \frac{1}{2}gt^2$$



$$= 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$h = 4.9\ t^2 \qquad \ldots\ldots\ldots (1)$$

We know,

Time taken by sound to reach the top

$$= \frac{\text{Depth of well}}{\text{Velocity of sound}}$$

$$= \frac{h}{350}$$

$$= \frac{4.9\ t^2}{350}$$

It is given that,

Total time taken = 3 seconds

Total time = time taken by stone to reach bottom of well + time taken by sound to reach the top of well.

$$3 = t + \frac{4.9\ t^2}{350}$$

$$1050 = 350t + 4.9\ t^2$$

$$4.9\ t^2 + 350t - 1050 = 0$$

$$t = \frac{-350 \pm \sqrt{(350)^2 - 4 \times 4.9(-1050)}}{2 \times 4.9}$$

$$= \frac{-350 \pm 378.26}{9.8}$$

$$t = 2.9 \text{ seconds}$$

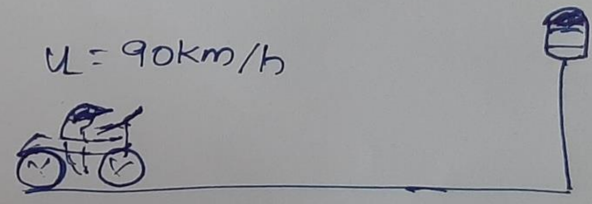
Substituting the value of 't' in equation (1) we get,

$$h = 4.9\ (2.9)^2$$

$$\mathbf{h = 41.21m} \qquad \textbf{(Ans)}$$

A motorist is travelling at 90kmph, when he observes a traffic light 250m ahead of him turns red. The traffic light is timed to stay red for 12sec. If the motorist wishes to pass the light without stopping, just as it turns green. Determine (i) the required uniform deceleration of the motor & ii) the speed of the motor as it passes the traffic light.

**Given data**




Initial velocity $= 90 \times \frac{1000}{3600}$

$= 25 m/s$

$t = 12$ secs

displacement $s = 250m$

Formula's Used

$S = ut + \frac{1}{2} at^2$

$v = u + at$

$v^2 = u^2 + 2as$

i) $S = ut + \frac{1}{2} at^2$

$250 = 25 \times 12 + \frac{1}{2} a \times 12^2$

$a = -0.6944 \ m/s^2$ (-ive sign indicates deceleration)

ii) $V = u + at$

$V = 25 + (-0.6944) \times 12 = 16.67 \ m/s$

$\therefore V = 16.67 \times \frac{3600}{1000}$

$= 60$ kmph (speed of the motorcycle as it passes the traffic light)

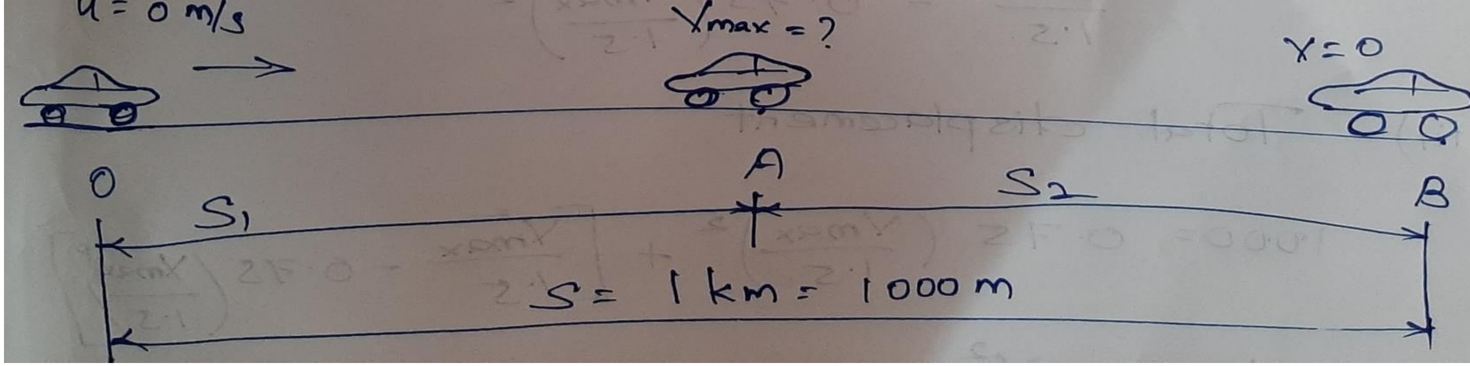
Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, & then stops at the end of the road. The car can accelerate or decelerate at 1.5 $m/s^2$.

Solution

Given data

$a = 1.5\, m/s^2$ $\qquad\qquad\qquad$ $a = -1.5\, m/s^2$

$u = 0\, m/s$



i) Consider the motion of car from O to A

Initial Velocity $u = 0$ ; Final Velocity $v = v_{max}$

Acceleration $a = 1.5\ m/s^2$ ; Time $t = t_1$

Displacement $S = S_1$ ;

$$v = u + at$$

$$V_{max} = 0 + 1.5\,t_1 \quad \therefore t_1 = \frac{V_{max}}{1.5}$$

$$S = ut + \frac{1}{2}at^2$$

$$S_1 = (0)(t_1) + \frac{1}{2} \times (1.5) \times (t_1^2)$$

$$S_1 = 0.75\left(\frac{V_{max}}{1.5}\right)^2$$

ii) Consider the motion of Car from A to B

Initial velocity $u = V_{max}$ ; Final velocity $v = 0$

Acceleration $a = -1.5 \, m/s^2$ ; Time $t = t_2$

Displacement $S = S_2$

$$v = u + at$$

$$0 = V_{max} + (-1.5) \times t_2 \quad \therefore \; t_2 = \frac{V_{max}}{1.5}$$

$$S = ut + \frac{1}{2} at^2$$

$$S_2 = V_{max}\left(\frac{V_{max}}{1.5}\right) + \frac{1}{2}(-1.5)\left(\frac{V_{max}}{1.5}\right)^2$$

$$S_2 = \frac{V_{max}^2}{1.5} - 0.75\left(\frac{V_{max}}{1.5}\right)^2$$

iii) Total displacement

$$1000 = 0.75\left(\frac{V_{max}}{1.5}\right)^2 + \left[\frac{V_{max}^2}{1.5} - 0.75\left(\frac{V_{max}}{1.5}\right)^2\right]$$

$$1000 = \frac{V^2_{max}}{1.5}$$

$$V_{max} = 38.73\ m/s$$

iv) Total time $t = t_1 + t_2$

$$t = \frac{V_{max}}{1.5} + \frac{V_{max}}{1.5} = \frac{38.73}{1.5} + \frac{38.73}{1.5}$$

$$t = 51.64\ sec.$$

Results

$V_{max} = 38.73\ m/s$

$t = 51.64\ sec.$

A train travelling with a speed of 90kmph slow down on account of work in progress, at a retardation of 1.8 kmph per second to 36kmph With this it travels 600m. There after it gain further speed with 0.9 kmph per second till getting orginal speed. Find the delay caused

## Solution

Given data

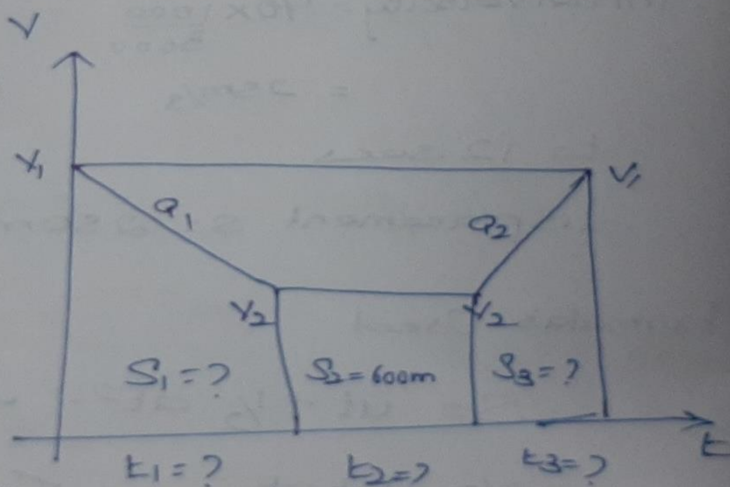
$V_1 = 90 \times 5/18 = 25 m/s$

$a_1 = 1.8 \times 5/18 = 0.5 m/s^2$

$V_2 = 36 \times 5/18 = 10 m/s$

$a_2 = 0.9 \times 5/18 = 0.25 m/s^2$

$S_2 = 600m.$




Formulas used

$S = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$v = u + at$

$S_2 = 600m.$

Formulas used

$S = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$v = u + at.$

Step 1 (Constan acceleration) → (deceleration)

$V = u + at$

$V_2 = u_1 + a_1 t_1$

$10 = 25 + (-0.5) t_1 \qquad t_1 = 30 \text{ secs.}$

$V^2 = u^2 + 2as$

$10^2 = 25^2 + 2(-0.5) S_1$

$10^2 - 25^2 = -1 S_1$

$-525 = -S_1 \qquad \boxed{S_1 = 525 m}$

Step: 2 (Constant Velocity)

$S = ut + \frac{1}{2}at^2$ at constant velocity acceleration is zero

$\therefore S_2 = V_2 t_2$

$600 = 10 \times t_2$

$t_2 = 60$ secs.

Step: 3 (Constant acceleration)

$V = u + at$

$V_3 = V_2 + a_2 t_3$

$25 = 10 + (0.25) \times t_3$

$t_3 = 60$ secs

$v^2 = u^2 + 2as$

$25^2 = 10^2 + 2(0.25) S_3$

$\frac{(625 - 100)}{0.5} = S_3$

$S_3 = 1050 m.$

Total distance traveled $= S_1 + S_2 + S_3$

$= 525 + 600 + 1050 = 2175 m$

Total time taken $= t_1 + t_2 + t_3$

$t = 30 + 60 + 60 = 150 Sec$

If there would have been no work speed will be const

$\therefore V_1 = 25 m/s$ time taken would be $t' = S/V = \frac{2175}{25}$

$t' = 87 Secs$

Time delayed $t - t' = 150 - 87 = 63 Sec$

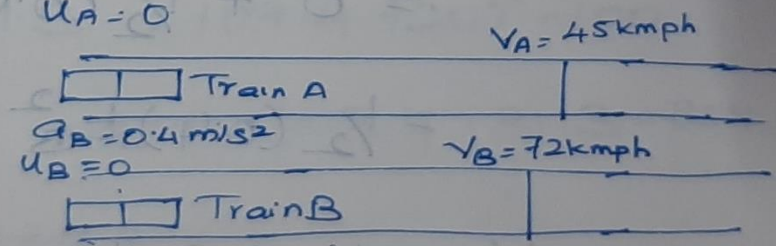
Two electric trains A & B leave the same station parallel lines. The train A starts from rest with a uniform acceleration of 0.2 $m/s^2$ and attains a speed of 45 kmph which is maintained constant afterwards. The train B leaves 1 minute afterwards with a uniform acceleration of 0.4 $m/s^2$ to attain maximum speed of 72 kmph which is maintained constant afterwards. When will the train B overtake train A?

Solution:

Given data

$a_A = 0.2\,m/s^2$

$u_A = 0$




| Train A | Train B |
|---|---|
| $u_A = 0$ | $u_B = 0$ |
| $a_A = 0.2\,m/s^2$ | $a_B = 0.4\,m/s^2$ |
| $V_A = 45$ kmph | $V_B = 72$ kmph |
| $= \frac{45 \times 1000}{3600}$ | $= \frac{72 \times 1000}{3600}$ |
| $= 12.5\,m/s$ | $= 20\,m/s$ |

Formulas Used

$$S = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

| $V_A = 45\ kmph$ | $V_B = 72\ kmph$ |
|---|---|
| $= 45 \times \frac{1000}{3600}$ | $= 72 \times \frac{1000}{3600}$ |
| $= 12.5\ m/s$ | $= 20\ m/s$ |

Formulas Used

$$S = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Let $t_A$ be the time taken by train A to reach maximum speed.

Distance travelled by Train A to reach maximum speed.

$$S_{A_1} = u_A t_A + \frac{1}{2} a_A t_A^2$$

$$= 0 + \frac{1}{2}(0.2) t_a^2$$

$$V_A = u_A + a_A t_A$$

$$12.5 = 0 + (0.2) t_A$$

$$t_A = \frac{12.5}{0.2} \quad 62.5\ sec$$

$$\boxed{t_A = 62.5\ Sec}$$

Sub $t_A = 62.5$ Sec in $S_{A_f}$

$$S_{A_f} = \frac{1}{2} \times (0.2)(62.5)^2$$

$$= 390.6\,m$$

Distance travelled by train B to reach maximum Speed

$$S_{B_f} = u_B t_B + \frac{1}{2} a_B t_B^2$$

$$= 0 + \frac{1}{2}(0.4)(t_B)^2$$

$$= \frac{1}{2}(0.4)\,t_B^2$$

$$V_B = u_B + a_B t_B$$

$$20 = 0 + 0.4\,t_B$$

$$t_B = 20/_4 = 50\text{ Sec.}$$

Sub $t_B = 50$ sec in $S_{B_1}$

$$\therefore S_{B_1} = \frac{1}{2}(0.4)(50)^2 = 500\,m$$

Let 'T' be the time in second when train B will overtake the train A from it start.

$\therefore$ Train A has travelled for (T + 60) seconds

$\Rightarrow$ Total distance travelled by the train A during this time

$$S_A = 390.6 + 12.5[(T+60) - 62.5]\,m \quad \text{--- (1)}$$

$S = ut + \frac{1}{2}at^2$ — $a = 0$ at constant velocity

Total distance travelled by train B

$$S_B = 500 + 20(T-50)\ m \quad \text{—— (2)}$$

When train B overtakes train A the distance travelled by train A & B will be equal.

∴ equating equations (1) & (2)

$$390.6 + 12.5[(T+60) - 62.5] = 500 + 20(T-50)$$

$$12.5T - 31.3 = 109.4 + 20T - 100$$

$$7.5T = 1000 - 109.4 - 31.3 = 859.3$$

$$T = \frac{859.3}{7.5}$$

$$T = 114.6\ \text{secs.}$$

The distance travelled by train A & train B from starting point is 1,792 m

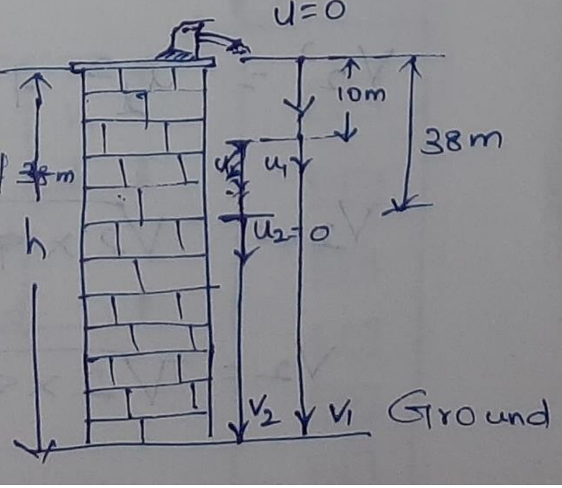
A stone is dropped from the top of a tower when it has fallen a distance of 10m, another stone is dropped from a point 38 m below the top of tower. If both the stones reach the ground at the same time, calculate

i) height of the tower &

ii) the velocity of the stones when they reach the ground.

Solution

Given data




Stone-1

$u_1 = 0$

Stone-2

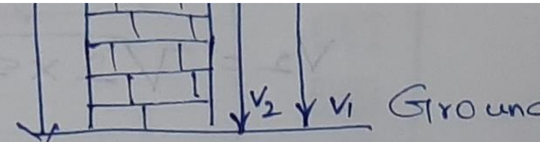
from the height of 38m

38 m

$u_2 = 0$

Motion of Stone (1)

$v^2 = u^2 + 2gh$




## Motion of Stone (1)

$v^2 = u^2 + 2gh$

$u_1^2 = 0^2 + 2 \times 9.81 \times 10$

$u_1 = \sqrt{2 \times 9.81 \times 10}$

$= 14 \text{ m/s}$

$h - 10 = u_1 t_1 + \frac{1}{2} 9.8 t_1^2$ — (1)

$\Rightarrow h = \frac{9.81}{2} t_1^2 + u_1 t_1 + 10$

## Motion of stone (2)

$h - 38 = u_2 t_2 + \frac{1}{2} \times 9.81 t_2^2$

$\Rightarrow h = \frac{9.81}{2} t_2^2 + 38$

$\frac{9.81}{2} t_1^2 + u_1 t_1 + 10 = u_2 t_2 + \frac{9.81}{2} t_2^2 + 38$

$t_1 = 28/14 = 2 \text{ secs.}$

**Formulas used**

$h = ut + \frac{1}{2} gt^2$

$v^2 = u^2 + 2gh$

$v = u + gt$

$g \rightarrow 9.81 \text{ m/s}^2$

h - height

for stone 2 $u_2 = 0$

W.k.t from given data

$t_1 = t_2$

Sub $t = 2$ sec in above equation

$h = 57.62 \, m.$

$V^2 = u^2 + 2gh$

$V_1^2 = u_1^2 + 2 \times 9.81 \; 57.62 \qquad u = 0$

$V_1 = \sqrt{2 \times 9.81 \times 57.62}$

$V_1 = 33.62 \, m/s \; (\downarrow)$ Ans

$V_2^2 = u_2^2 + 2gh_1$

$V_2 = \sqrt{2 \times 9.81 \, (h - 38)}$

$V_2 = \sqrt{2 \times 9.81 \times 19.62}$

$V_2 = 19.62 \, m/s \; (\downarrow)$ Ans

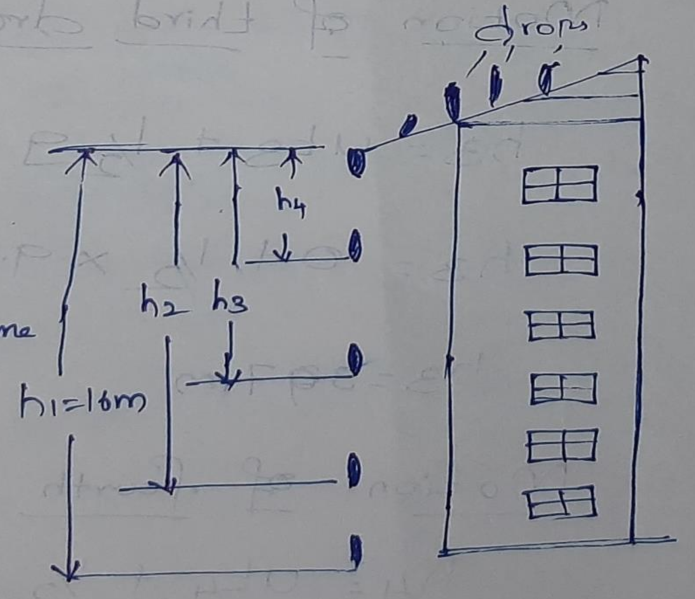
Drops of water fall from the roof of a building 16 m high, at regular interval of time. When first drop strikes the ground, at the same instant fifth drop starts its fall. Find the distance between individual drops in the air the instant first drop reaches the ground.

Solution

Given data

height of the building = 16 m

drops of rain water falls in regular interval of time



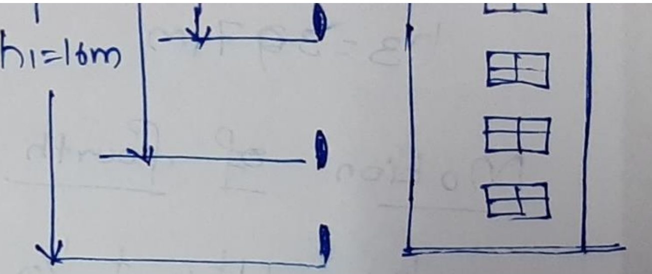

Motion of first drop

$u_1 = 0 \quad h = 16\ m$

$h_1 = u_1 t_1 + \frac{1}{2} g t_1^2$

$h = ut + \frac{1}{2} g t^2$

Motion of first drop




$u_1 = 0 \quad h = 16\,m$

$h_1 = u_1 t_1 + \frac{1}{2} g t_1^2$

$h = ut + \frac{1}{2} g t^2$

$16 = 0 + \frac{1}{2} \times 9.81\, t_1^2$

$$\boxed{t_1 = 1.8\ sec}$$

Let $\Delta t$ be the time interval to start the motion of each drop. In a time interval of $t_1 = 1.8$ sec, four drops have started their motion at reguler interval of time.

$\Delta t = \frac{t_1}{4} = \frac{1.8}{4} = 0.45$ Secs

$t_2 = t_1 - \Delta t = 1.8 - 0.45 = 1.35$ Secs $\quad t_5 = t_4 - \Delta t = 0$ Secs

$t_3 = t_2 - \Delta t = 0.9$ Secs

$t_4 = t_3 - \Delta t = 0.45$ Ses

## Motion of second drop

$$h_2 = ut_2 + \frac{1}{2} g t_2^2$$

$$h_2 = 0 + \frac{1}{2} (9.81) \times (1.35)^2$$

$$h_2 = 8.94 m$$

## Motion of third drop

$$h_3 = ut_3 + \frac{1}{2} g t_3^2$$

$$h_3 = 0 + \frac{1}{2} \times 9.81 \times 0.9^2$$

$$h_3 = 3.97 m$$

## Motion of fourth drop

$$h_4 = ut_4 + \frac{1}{2} g t_4^2$$

$$h_4 = 0 + \frac{1}{2} \times 9.81 \times 0.45^2$$

$$h_4 = 0.99 m$$

Distance b/w individual drop

Distance b/w 1st & 2nd drop $= h_1 - h_2 = 7.06 m$

Distance b/w 2nd & 3rd drop $= h_2 - h_3 = 4.97 m$

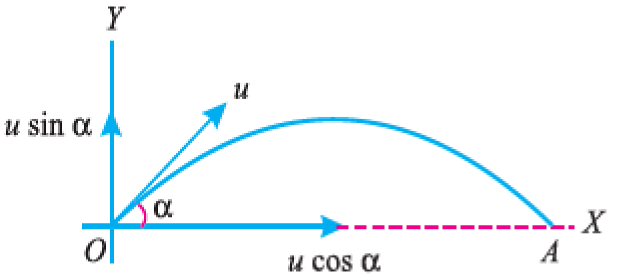
Distance b/w 3rd & 4th drop $= h_3 - h_4 = 2.98 m$

Distance b/w 4th & 5th drop $= h_4 - h_5 = 0.99 m$

# Kinematics of Particles: Plane Curvilinear Motion

**Curvilinear motion** is defined as **motion** that occurs when a particle travels along a curved path.

**Projectile motion** follows a parabolic trajectory.




***Trajectory*. The path, traced by a projectile in the space, is known as trajectory.

***Velocity of projection*. The velocity, with which a projectile is projected, is known as the velocity of projection.

***Angle of projection*. The angle, with the horizontal, at which a projectile is projected, is known as the angle of projection.

***Time of flight*. The total time taken by a projectile, to reach maximum height and to return back to the ground, is known as the time of flight.

***Range*. The distance, between the point of projection and the point where the projectile strikes the ground, is known as the *range*. It may be noted that the range of a projectile may be horizontal or inclined.

## General Equation for Projectile Motion

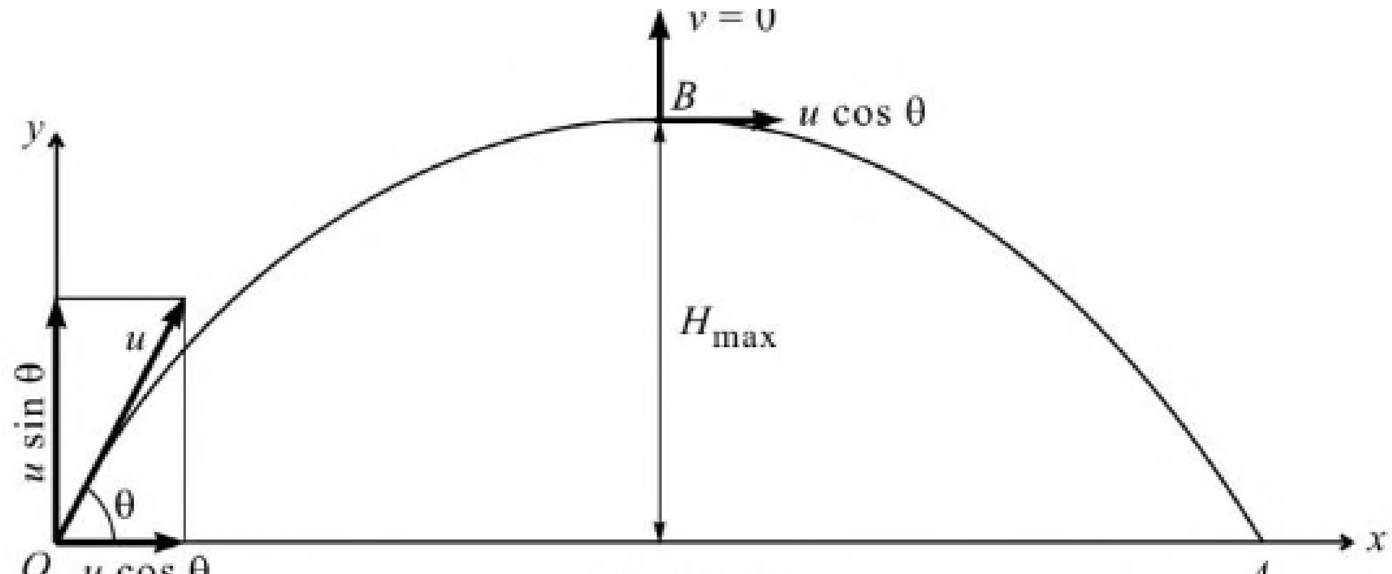



Fig. 9.51(b)

**1.** Time of flight $T = \frac{2u \sin \theta}{g}$

**2.** Horizontal range $R = \frac{2u^2 \sin 2\theta}{g}$

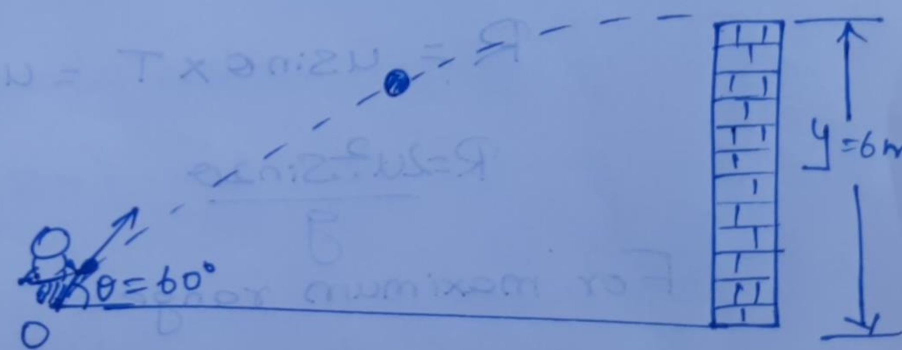
**3.** Maximum height of a projectile $H_{max} = \frac{u^2 \sin^2 \theta}{2g}$

A ball is thrown from horizontal level, such that it clears a wall 6 m high situated at a horizontal distance of 35 m as shown in Fig. If the angle of projection is 60° with respect to the horizontal what should be the minimum velocity of projection?

<u>Solution</u>

$y/h = 6\,m$

$\theta = 60°$




Formula Used

$$Y = x \cdot \tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

$$6 = 35 \times \tan 60° - \frac{9.81 \times 35^2}{2u^2}\left[1+\tan^2(60)\right]$$

$$u = 20.98\,m/s$$

A ball is thrown by a boy in the street is caught by another boy on a balcony 4 m above the ground and 18 m away after 2 sec. Calculate the initial velocity and the angle of projection.

Consider the vertical motion under gravity

$$h = ut + \frac{1}{2} g t^2$$

$$4 = 4 \sin\theta \times 2 - \frac{1}{2}\, 9.81 \times (2)^2$$

$$u \sin\theta = 11.81 \quad \text{— (1)}$$

Consider horizontal motion with constant velocity

$$s = v \times t$$

$$18 = u \cos\theta \times 2$$

$$u \cos\theta = 9 \quad \text{— (2)}$$

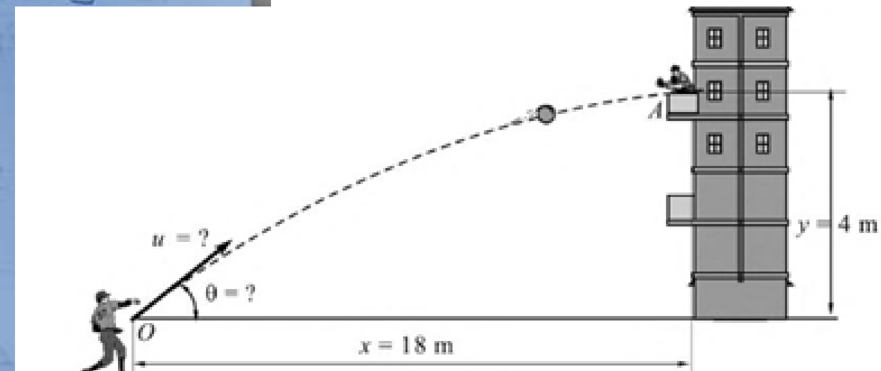
Dividing eqn (1)/(2)

$$\tan\theta = 11.81/9 \qquad \theta = 52.69°$$

Sub $\theta = 52.69$ in eqn (1)

$$u \sin 52.69 = 11.81$$

$$u = 14.85 \text{ m/s}$$



***A projectile is aimed at a mark on the horizontal plane through the point of projection. It falls 12 metres short when the angle of projection is 15° ; while it overshoots the mark by 24 metres when the same angle is 45°. Find the angle of projection to hit the mark. Assume no air resistance.***

Solution

G.D

When angle of projection with the horizontal

$\alpha_1 = 15°$ horizontal Range $R_1 = R - 15°$

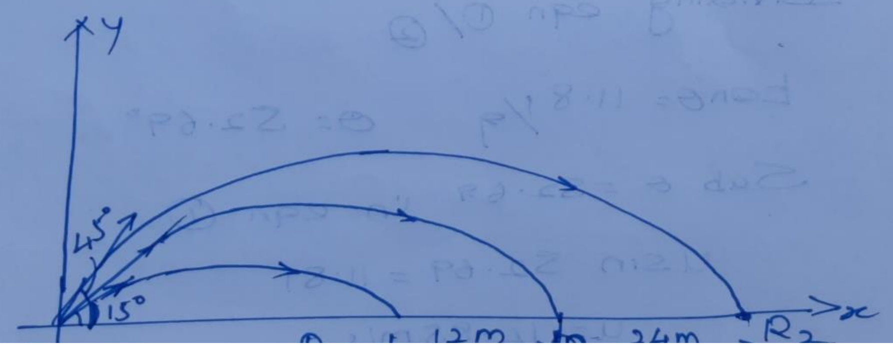
When the angle of projection with the horizontal

$\alpha_2 = 45°$ horizontal Range $R_2 = R + 24$

where R is horizontal Range.

Let u = Velocity of projection

$\alpha$ = Angle of projection to hit the mark.



We know that horizontal range of the projectile when $\alpha = 15°$

$$R_1 = \frac{u^2 \sin 2\alpha_1}{g} = \frac{u^2 \sin(2 \times 15°)}{g}$$

$$(R-12) = \frac{u^2 \sin 30°}{g} = \frac{u^2 \times 0.5}{g} \quad \text{—} ①$$

|||ly

$$R_2 = \frac{u^2 \sin 2\alpha_2}{g} = \frac{u^2 \sin(2 \times 45°)}{g}$$

$$R + 24 = \frac{u^2 \sin 90}{g} = \frac{u^2 \times 1}{g} \quad \text{—} ②$$

Dividing eqn ① / ②

$$\frac{R-12}{R+24} = \frac{0.5}{1} \text{ or } R - 12 = 0.5 R + 12$$

$$R = \frac{24}{0.5} = 48 \text{ m}$$

Substituting the value of $\boxed{R = 48\,m}$ in (1)

$$48 - 12 = \frac{u^2 \times 0.5}{g} = \frac{u^2}{2g}$$

$$u^2 = 36 \times 2g = 72g$$

We know the horizontal distance b/w the point of projection & the mark (R)

$$48 = \frac{u^2 \sin 2\alpha}{g} = \frac{72g \sin 2\alpha}{g} = 72 \sin 2\alpha$$

$$\sin 2\alpha = \frac{48}{72} = 0.667 \quad \text{or} \quad 2\alpha = 41.8^\circ$$

$$\alpha = 20.9^\circ$$

$$u = 26.57\ m/s$$

# NEWTON'S SECOND LAW OF MOTION

$m$ = Mass of a body,

$u$ = Initial velocity of the body,

$v$ = Final velocity of the body,

$a$ = Constant acceleration,

$t$ = Time, in seconds required to change the velocity from $u$ to $v$, and

$F$ = Force required to change velocity from $u$ to $v$ in $t$ seconds.

∴ Initial momentum = $mu$

and final momentum = $mv$

∴ Rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma \qquad \left[\because \frac{v-u}{t} = a\right]$$

According to Newton's Second Law of Motion, the rate of change of momentum is directly proportional to the impressed force.

$$\therefore F \propto ma = kma$$

***A body of mass 7.5 kg is moving with a velcoity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.***

**Solution.**

Given: Mass of body = 7.5 kg

Velocity ($u$) = 1.2 m/s

Force ($F$) = 15 N and time

($t$) = 2 s.

We know that acceleration of the body

$$a = F/m$$

$$= 15/7.5$$

$$= 2 \text{ m/s}^2$$

∴ Velocity of the body after 2 seconds

$v = u + at = 1.2 + (2 \times 2) = 5.2$ m/s **Ans.**

***A vehicle, of mass 500 kg, is moving with a velocity of 25 m/s. A force of 200 N acts on it for 2 minutes. Find the velocity of the vehicle*** :

***(1) when the force acts in the direction of motion, and***

***(2) when the force acts in the opposite direction of the motion***.

**Solution.**

Given : Mass of vehicle ($m$) = 500 kg

Initial velocity ($u$) = 25 m/s

Force ($F$) = 200N

time ($t$) = 2 min = 120 s

**1. *Velocity of vehicle when the force acts in the direction of motion***

**We know that acceleration of the vehicle,**

a = F/m = 200/500

= 0.4 $m/s^2$

∴ Velocity of the vehicle after 120 seconds

$v1 = u + at = 25 + (0{\cdot}4 \times 120) = 73$ m/s **Ans.**

***2. Velocity of the vehicle when the force acts in the opposite direction of motion.***

We know that velocity of the vehicle in this case after 120 seconds,

(when $a = -0.4$ m/s$^2$),

$v2 = u + at$

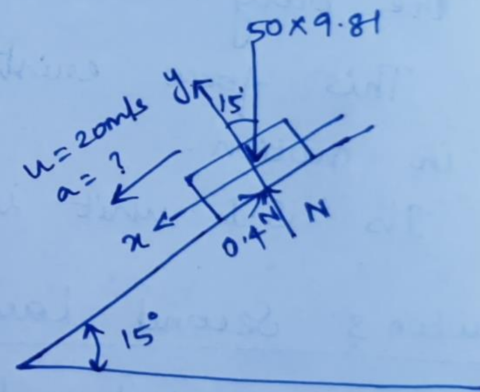
$= 25 + (-0.4 \times 120)$

$= -23$ m/s **Ans.**

Minus sign means that the vehicle is moving in the reverse direction or in other words opposite to the direction in which the vehicle was moving before the force was made to act.

1) A 50 kg block kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_k = 0.4$, determine the distance traveled by the block & the time it will take as it comes to rest.

Sol:




Given data:

Mass (m) = 50 kg

Initial velocity $u$ = 20 m/s

$\mu_k = 0.4$

Sol:

1) Considering the F.B.D of 50 kg block

II) By Newton's Second law, we have

$$\Sigma F_y = m a_y = 0 \quad (\because a_y = 0)$$

$$N - 50 \times 9.81 \cos 15° = 0$$

$$N = 50 \times 9.81 \cos 15°$$

$$\Sigma F_x = m a_x$$

$$50 \times 9.81 \sin 15° - 0.4 \times 50 \times 9.81 \cos 15° = 50\, a$$

$$\boxed{a = -1.25\ m/s^2} \text{ (Retardation)}$$

iii) $u = 20\ m/s$ $a = -1.25\ m/s^2$

$v = 0$ $s = ?$

$t = ?$

$v = u + at$

$0 = 20 + (-1.25)t$

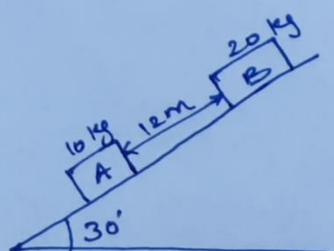
$\boxed{t = 16\ sec}$

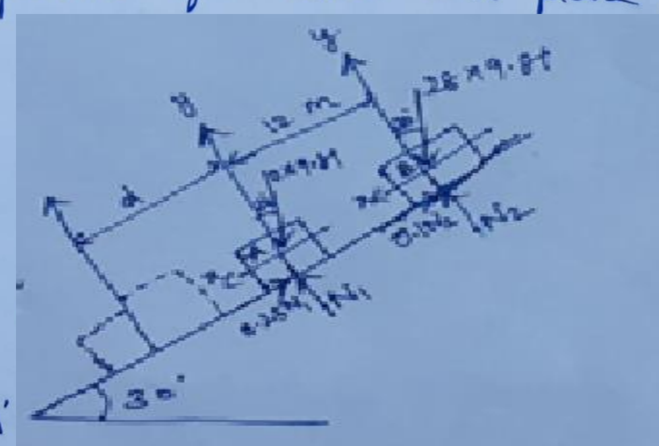
$s = ut + \frac{1}{2}at^2$

$s = 20 \times 16 + \frac{1}{2}(-1.25) \times (16)^2$

$\boxed{s = 160\ m}$

2) Two blocks A (mass 10 kg), B (mass 28 kg) are separated by 12 m as shown in fig. If the blocks start moving find the time 't' when the blocks collide. Assume $\mu = 0.25$ for block A & plane & $\mu = 0.10$ for block B & plane




Sol:

1) Considering the FBD of Block 'A'

**Sol:**

i) Considering the FBD of Block 'A'

By Newton's second law,

$$\Sigma F_x = m a_x$$

$$10 \times 9.81 \sin 30^\circ - 0.25 \times 10 \times 9.81 \cos 30^\circ = 10\, a_A$$

$$a_A = 2.781 \text{ m/s}^2 \ (30^\circ)$$

ii) Considering the F.B.D of Block 'B'

By Newton's Second law,

$$\Sigma F_x = m a_x$$

$$29 \times 9.81 \sin 30^\circ - 0.1 \times 28 \times 9.81 \cos 30^\circ = 28\, a_B$$

$$a_B = 4.055 \text{ m/s}^2 \ (30^\circ)$$

iii) Motion of block A

$$d = 0 + \frac{1}{2} a_A t^2 \longrightarrow (1)$$

iv) Motion of block B

$$d + 12 = 0 + \frac{1}{2} a_B t^2 \longrightarrow (2)$$

v) From eqn (1) & (2), we get

$$\frac{1}{2} \times 2.781 \times t^2 + 12 = \frac{1}{2} \times 4.055 \times t^2$$

$$\boxed{t = 4.34 \text{ sec}}$$

# Thank You