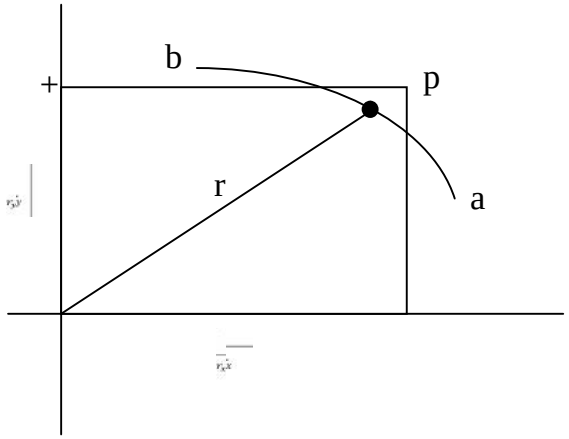
Curvilinear Motion

To define path of motion of a curve the following co-ordinates are followed

1. Rectangular co-ordinates (or) Cartesian coordinates
2. Normal and tangential co-ordinates
3. Polar co-ordinates

Rectangular Co-ordinates



Position vector

$r = r_x i + r_y j$

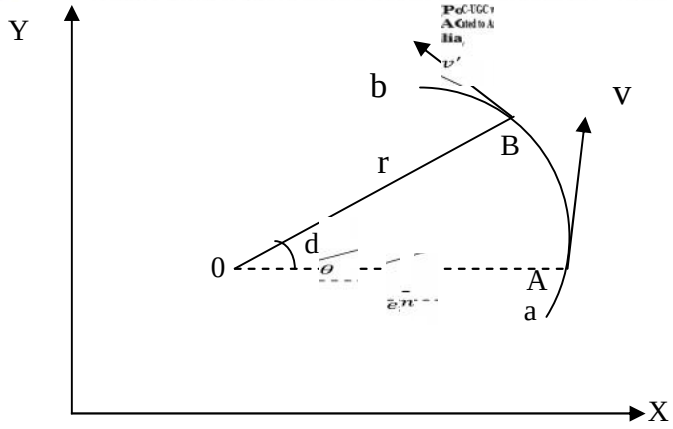
Resultant velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

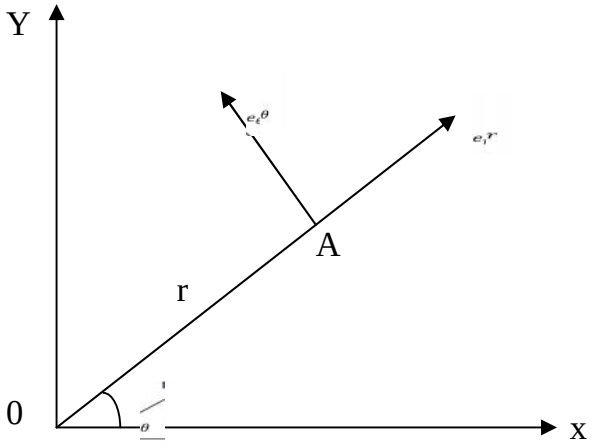
Slope, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Resultant Acceleration $a = \sqrt{a_x^2 + a_y^2}$

Normal and Tangential Co-ordinates



Polar Co-ordinates



Projectiles

The path traced by the freely projected particle which moves under the combined effect of vertical and horizontal motion is called as projectile.

Terminology

1. Velocity of the Projection (u)

The velocity with which the particle is freely projected in the air is termed as velocity of projection. It is denoted by u in m/sec.

2. Angle of Projection (α)

Angle of projection is the angle at which the particle is projected, the angle being measured with the horizontal. It is denoted by α

3. Trajectory

The path described by a body in motion is called as trajectory

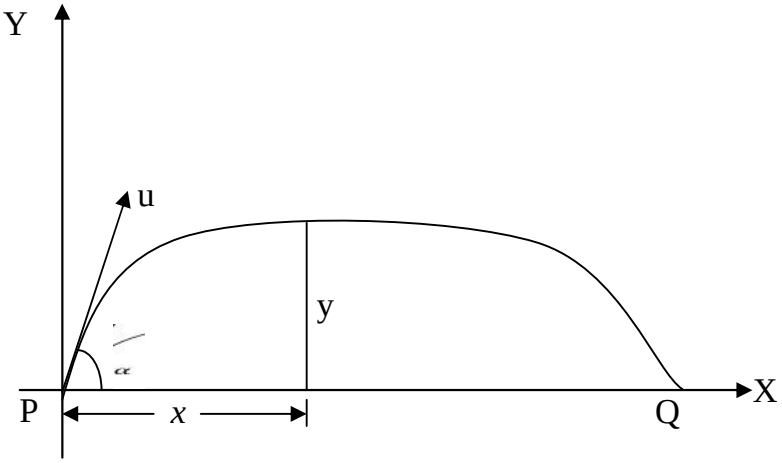
4. Horizontal Range (R)

Horizontal range is the distance between the point of projection and the point where the path of projectile meets the place again drawn through the point of projection.

5. Time of flight (T)

Time of flight is the time travel between the instant at which projectile is projected at the instant when it meets the horizontal plane drawn through the point of projection.

Equation of path of Projectile



Horizontal component of velocity=u cos $\propto$

Vertical component of velocity=u sin $\propto$

From eqn. of motion, we know

$$S = ut + \frac{1}{2}at^2$$

Since the particle is projected upwards

$$a = -g$$

Displacement in y direction,

$$y = (u \sin \propto)t - \frac{1}{2}at^2 \rightarrow (1)$$

Displacement in x direction,

$$x = (u\cos\propto)t - 0$$

$$x = u\cos\propto t$$

$$t = \frac{x}{u}\cos\propto$$

Sub (2) in (1)

$$y = u\sin\propto\left(\frac{x}{u\cos\propto}\right) - \frac{1}{2}(g)\left(\frac{x^2}{u^2\cos^2\propto}\right)$$

$$= x\tan\propto - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\propto$$

$$y = x\tan\propto - \frac{1}{2}g\left(\frac{x^2}{u^2}\right)(1+\tan^2\propto)$$

The above equation is called as the 'Equation of Trajectory'.

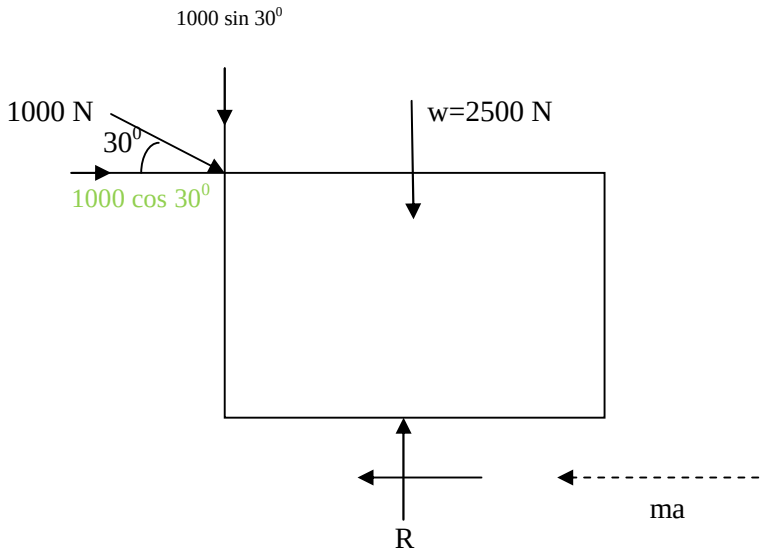
Solution:

Horizontal component of the applied force

= 1000 cos 30= 866.02N

Vertical component of the applied force

=1000 sin 30=500N



Inertia force =ma

$$= \frac{2500}{9.81} \times a \rightarrow (1)$$

Resolving forces horizontally

$$866.02 - F - ma = 0$$

$$ma = -F + 866.02 \rightarrow (2)$$

Resolving forces vertically

$$w + 500 = R$$

$$2500 + 50 = R$$

$$R = 3000N$$

Frictional force $F = \mu R$

$$F = 0.2 \times 3000$$

$$= 600N$$

Sub F in (1)

$$ma = -600 + 866.02$$

$$ma = 266.02 \rightarrow (3)$$

Eq (1) & (3)

$$\frac{2500}{9.81} \times a = 266.02$$

$$a = \frac{266.02}{2500} \times 9.81$$

$$a = 1.044 \text{ m/sec}^2$$

From equation of motion

$$v^2 - u^2 = 2as$$

Initial velocity u=0

$$v^2 = 2as$$

$$v^2 = 2 \times 1.044 \times 30$$

$$v = \sqrt{2 \times 1.044 \times 30}$$

$$v = 7.914 \text{ m/s}$$