Problem 3:

A stone is dropped into a wall with no initial velocity and 4.5 seconds later the splash is heard. Then a second stone is thrown downward into the well with an initial velocity u and the splash is heard 4 seconds later. If the velocity of the sound is constant at 336m/sec. Determine the initial velocity of the second stone.

Solution:

Step 1: Dropping first stone

Initial velocity $u_1 = 0$

Let the stone taken t_1 seconds to reach down the wall.

Since the splash is heard after 4.5 seconds, the time of travel of the second wave is $(4.5 - t_1)$ seconds.

Depth of the well

$$S = u_1 t_1 + \frac{1}{2} g t_1^2$$
$$= 0 + \frac{1}{2} g t_1^2$$
$$= 1/2 \times 9.81 \times t_1^2$$
$$S = 4.905 t_1^2 \rightarrow (1)$$

Distance travelled by sound wave

S=Velocity of sound X Time of travelled

$$S = 336(4.5 - t_1) \rightarrow (2)$$

Substituting

$$4.905t_1^2+336t_1-1512=0$$

$$t_1 = \frac{-336 \pm \sqrt{336^2 - 4(4.905)(-1512)}}{2(4.905)}$$

$$= \frac{-336 \pm 377.57}{9.81}$$

$$t_1 = 4.238 \text{ or } -72.739$$
$$t_1 = 4.238 \text{ sec}$$

Depth of the wall $s = 4.905 \times (4.238)^2 = 88.096\text{m}$

Step 2: Dropping the second stone

Initial velocity $=u_2$

Time taken by sound wave= $(4 - t_2)$sec

$$s = u_2 t_2 + \frac{1}{2} g\, t_2^2$$

$$88.096 = u_2 t_2 + 4.905 \times t_2^2 \rightarrow (3)$$

Distance travelled by the sound

$$s = \text{Velocity of sound} \times \text{Time of travel}$$

$$s = 336\,(4 - t_2)$$

$$\frac{88.096}{336} = 4 - t_2$$

$$t_2 = 4 - \frac{88.096}{336}$$

$$t_2 = 3.738 \text{ sec}$$

Substituting the value of t_2 in (3)

$$88.096 = u_2(3.738) + 4.905(3.738)^2$$

$$3.738u_2 = 88.096 - 4.905(3.738)^2$$

$$u_2 = 5.232 \text{ m/sec}$$