Problem 1: The position of a particle which move along a straight line is defined by $x = t^3 - 6t^2 - 15t + 40$ where x is in m, t is in sec. Determine the following

a. The time at which the velocity will be zero
b. The position and distance travelled by the particle at that time
c. Acceleration of the particle at that time
d. The distance travelled by the particle t=4 sec and t=6 sec

Solution:

Displacement $x = t^3 - 6t^2 - 15t + 40$

We know that,

Velocity, $v = \frac{dx}{dt} = 3t^2 - 12t - 15 \rightarrow (1)$

Also we know that

Acceleration, $a = \frac{dv}{dt} = 6t - 12 \rightarrow (2)$

a) Time at which velocity will be zero
By equating eqn (1) to zero

$$3t^2 - 12t - 15 = 0$$
$$t^2 - 4t - 5 = 0$$
$$t = +5 \text{ sec}$$ (t=-1sec is not practically possible)

b) Position and distance travelled when v = 0 when t=5, v=0 (zero velocity)
Position of particle at t=5 sec

$$x_5 = 5^3 - 6(5)^2 - 15(5) + 40$$
$$= 125 - 150 - 75 + 40 = -60\text{m}$$

Initial position of particle at t=0 sec

$$x_0 = 0^3 + 6(0)^2 - 15(0) + 40$$
$$x_0 = 40\text{m}$$

Distance travelled= $x_5 - x_0 = -60 - 40 = -100\text{m}$
i.e 100m in the negative direction

c) Acceleration when v=0

$$v = 0 \text{ at } t = 5 \text{ sec}$$
$$a = 6t - 12$$

$a = 6(5) - 12 = 18\text{m} / \text{sec}^2$

d) Distance travelled by the particle when
t=4sec and t=6sec
Position at t=4sec $x_4 = 4^3 - 6(4)^2 - 15(4) + 40 = -52m$
Position at t=6sec $x_6 = 6^3 - 6(6)^2 - 15(6) + 40 = -50m$
Position at t=5sec $x_5 = -60m$

Distance trance travelled when t=5sec to t=6sec

$$= x_6 - x_5$$
$$= -50 - (-60)$$
$$= 10\text{m (Positive Displacement)}$$

Distance travelled when t=4sec to t=5sec

$$= x_5 - x_4$$
$$= (-60) - (-52)$$
$$= 8\text{m (Negative Displacement)}$$