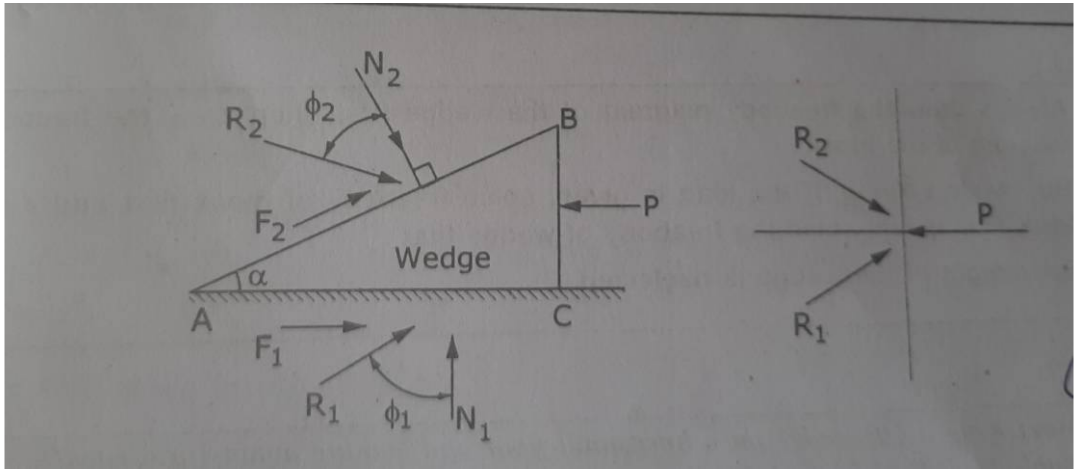
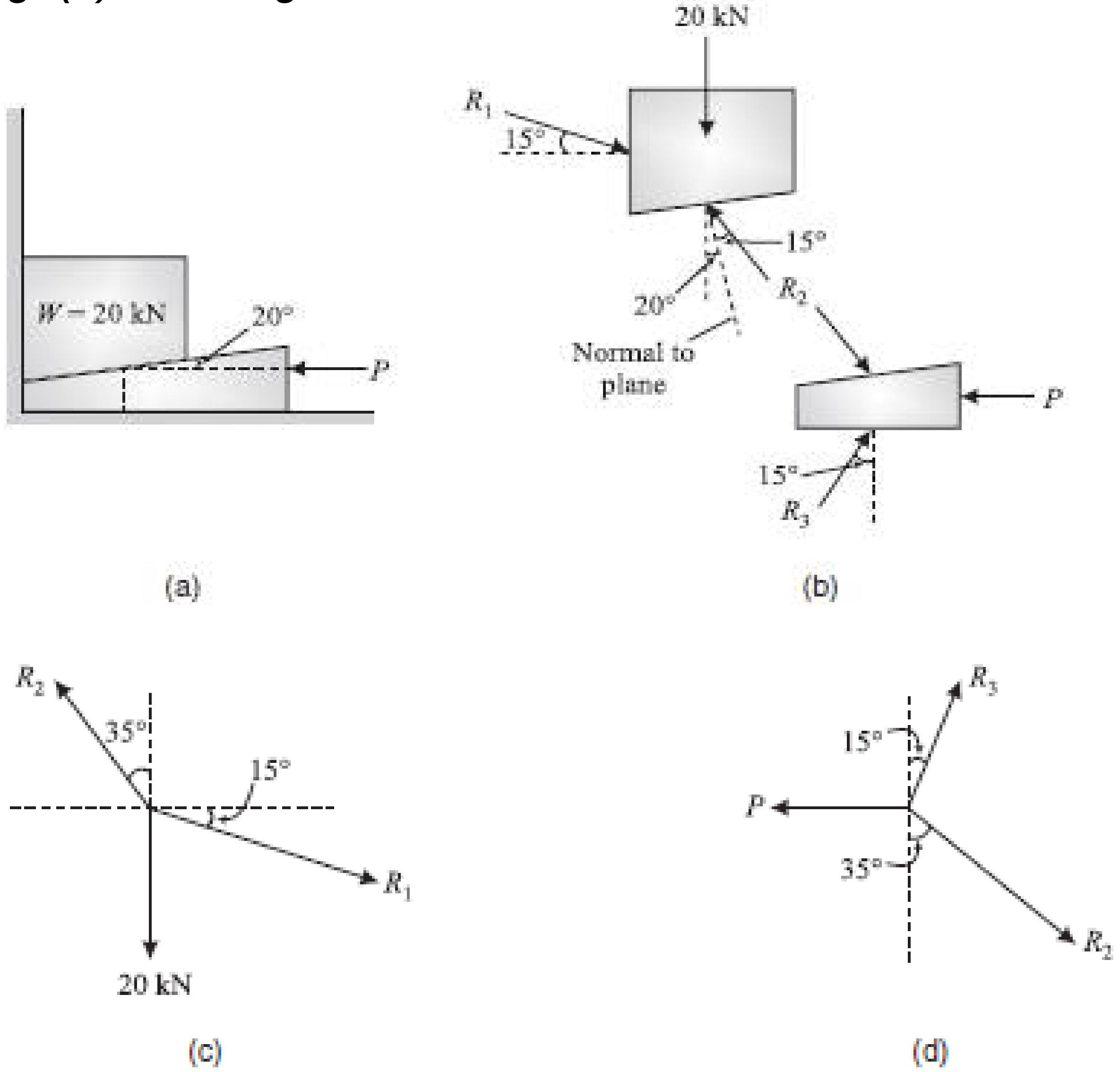
WEDGE

Wedges are small pieces of hard materials with two of their opposite surfaces not parallel to each other.



WEDGE PROBLEMS

Determine the force P required to start the movement of the wedge as shown in Fig. (a). The angle of friction for all surfaces of contact is 15°.



Solution: As wedge is driven, it moves towards left and the block moves upwards. When motion is impending limiting friction develops. Hence resultant force makes limiting angle of 15° with normal. The care is taken to mark 15° inclination such that the tangential component of the resultant opposes the impending motion.

The free body diagrams of the block and wedge are shown in Fig. 5.12(*b*). The forces on block and wedge are redrawn in Figs. 5.12(*c*) and (*d*) so that Lami's theorem can be applied conveniently. Applying Lami's theorem to the system of forces on block

$$\frac{R_1}{\sin(180-15-20)} = \frac{R_2}{\sin(90-15)} = \frac{20}{\sin(15+20+90+15)}$$

i.e.,
$$\frac{R_1}{\sin 145} = \frac{R_2}{\sin 75} = \frac{20}{\sin 140}$$

$\therefore$ $$R_1 = 17.85 \text{ kN}$$

and $$R_2 = 30.05 \text{ kN}$$

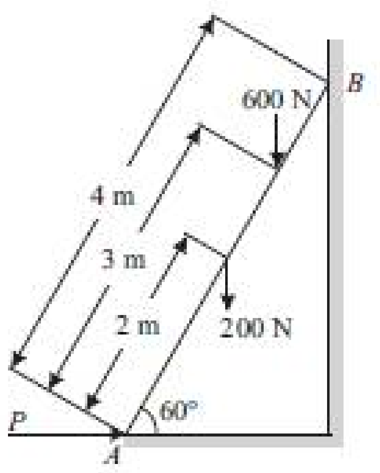
Applying Lami's theorem to system of forces on the wedge, we get

$$\frac{P}{\sin 130} = \frac{R_2}{\sin 105}$$

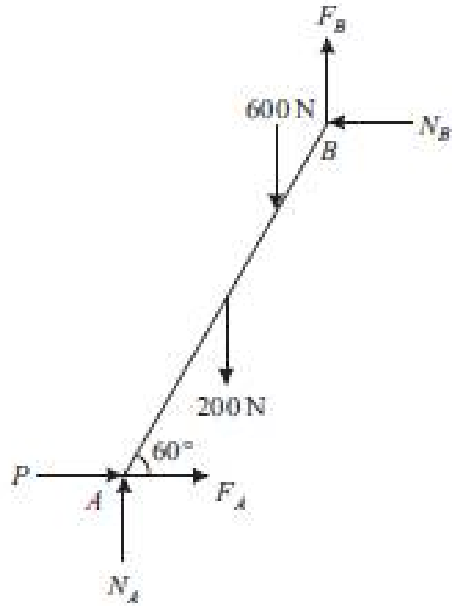
$\therefore$ $$P = 23.84 \text{ kN}$$

LADDER PROBLEMS

A ladder of length 4 m, weighing 200 N is placed against a vertical wall as shown in Fig. 5.14(a). The coefficient of friction between the wall and the ladder is 0.2 and that between floor and the ladder is 0.3. The ladder, in addition to its own weight, has to support a man weighing 600 N at a distance of 3 m from A. Calculate the minimum horizontal force to be applied at A to prevent slipping.



(a)



(b)

Solution: The free body diagram of the ladder is as shown in Fig. 5.14(*b*).

$\sum M_A = 0 \rightarrow$

$$200 \times 2 \cos 60 + 600 \times 3 \cos 60 - F_B \times 4 \cos 60 - N_B \times 4 \sin 60 = 0$$

Dividing throughout by 4 and rearranging the terms, we get

$$0.866\ N_B + 0.5\ F_B = 275$$

From law of friction, $F_B = \mu N_B = 0.2\ N_B$

$$\therefore \quad 0.866\ N_B + 0.5 \times 0.2\ N_B = 275$$

or $N_B = 284.7$ newton.

$\therefore$ $F_B = 56.94$ newton.

$\sum F_V = 0 \rightarrow$

$$N_A - 200 - 600 + F_B = 0$$

$$N_A = 743.06 \text{ newton}, \quad \text{since } F_B = 56.94$$

$$F_A = \mu_A N_A$$

$$= 0.3 \times 743.06 = 222.9 \text{ newton}$$

$\sum F_H = 0 \rightarrow$

$$P + F_A - N_B = 0$$

$$P = N_B - F_A = 284.7 - 222.9$$

$$\mathbf{P = 61.8 \text{ newton}}$$

A ladder of length L rests against a wall, the angle of inclination being 45°. If the coefficient of friction between the ladder and the ground and that between ground and the wall is 0.5 each, what will be the maximum distance on ladder to which a man whose weight is 1.5 times the weight of ladder may ascend before the ladder begins to slip?

Solution: Figure 5.16(*a*) shows the ladder when it is about to slip when the man weighing 1.5 *W* is at a distance, '*a*L' from the end *A*. Its free body diagram is shown in Fig. 5.16(*b*).

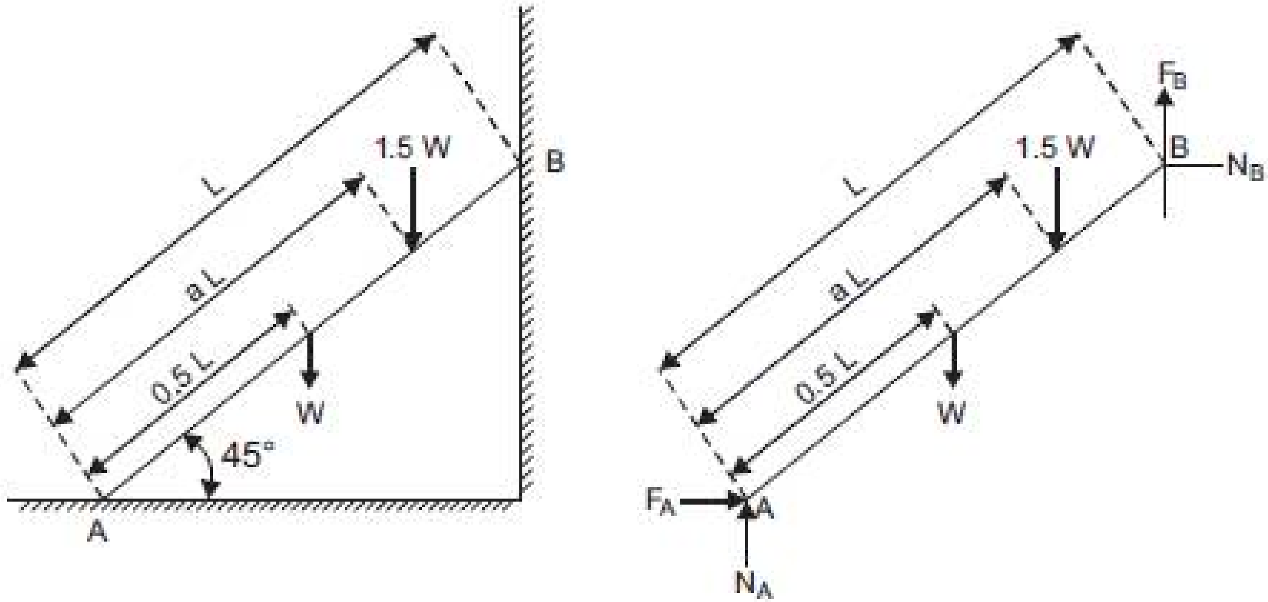
Since ladder is on the verge of slipping, from law of friction,

$$F_A = \mu N_B = 0.5\ N_B \quad ...(1)$$

and

$$F_B = \mu N_B = 0.5\ N_B \quad ...(2)$$

$$\Sigma H = 0 \rightarrow$$



$$F_A - N_B = 0 \quad \text{or} \quad N_B = F_A = 0.5\ N_A \qquad ...(3)$$

$\therefore$

$$F_B = 0.5\ N_B = 0.25\ N_A \qquad ...(4)$$

$$\Sigma V = 0 \rightarrow$$

$$N_A + F_B = W + 1.5\ W$$

i.e.,

$$N_A + 0.25\ N_A = 2.5\ W$$

or

$$N_A = \frac{2.5}{1.5} W = 1.667\ W \qquad ...(5)$$

$$\Sigma M_A = 0 \rightarrow$$

$$-F_B\ L \cos 45° - N_B\ L \sin 45° + 1.5\ W\ aL \cos 45° + W\ 0.5\ L \cos 45° = 0$$

Since $\sin 45° = \cos 45°$, we get

$$F_B + N_B = 1.5\ aW + 0.5\ W$$

$$0.25\ N_A + 0.5\ N_A = 1.5\ aW + 0.5\ W$$

i.e.,

$$0.75 \times 1.667\ W = 1.5\ aW + 0.5\ W$$

i.e.,

$$1.25 = 1.5\ a + 0.5$$

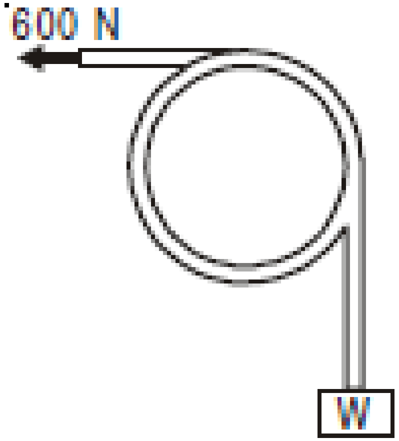
or

$$a = \frac{0.75}{1.5} = 0.5$$

Thus in this case the man can ascend up to '0.5 *L*' of ladder.

BELT FRICTION

A rope making 1 (1 /4) turns around a stationary horizontal drum is used to support a weight W (Fig. 5.18). If the coefficient of friction is 0.3 what range of weight can be supported by exerting a 600 N force at the other end of the rope?



Solution: Angle of contact = $1.25 \times 2\pi = 2.5\pi$

(1) Let the impending motion of the weight be downward.

Then,

$$T_1 = 600 \text{ N};\ T_2 = W$$

$$\frac{W}{600} = e^{\mu 2.5\pi} = e^{0.3 \times 2.5\pi} = e^{0.75\pi}$$

$$W = 6330.43 \text{ N}$$

(2) Let the impending motion of weight be upwards. Then

$$T_1 = W;\quad T_2 = 600 \text{ N}$$

$$T_2 = T_1 e^{\mu\theta}$$

$$600 = W e^{0.75\pi}$$

$$W = 56.87 \text{ N}$$

Assignment Questions

A 3000 N block is placed on an inclined plane as shown in Fig. 5.24. Find the maximum value of W for equilibrium if tipping does not occur. Assume coefficient of friction as 0.2. [Ans. 2636.15]

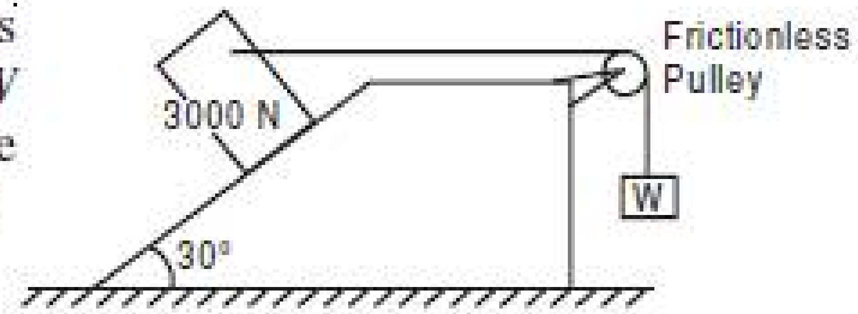


Fig. 5.24

Find whether block A is moving up or down the plane in Fig. 5.25 for the data given below. Weight of block A = 300 N. Weight of block B = 600 N. Coefficient of limiting friction between plane AB and block A is 0.2. Coefficient of limiting friction between plane BC and block B is 0.25. Assume pulley as smooth.

[Ans. The block A is stationary since F developed $< F_{min}$]

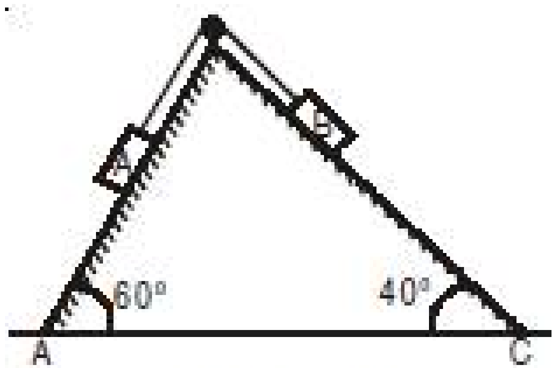


Fig. 5.25

Thank You