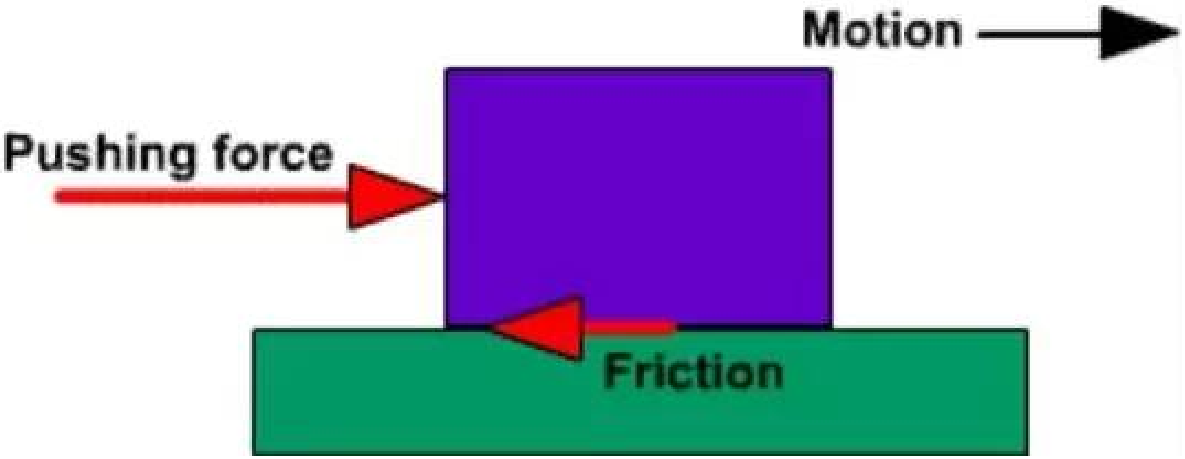
Friction

- When a body moves or tends to move over another body, a force opposing the motion develops at the contact surfaces.
- The force which opposes the movement or the tendency of movement is called the **frictional force** or simply **friction**

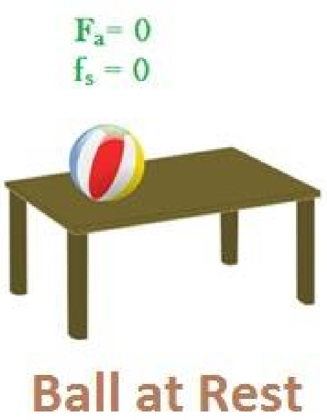


Friction

- Friction is the force distribution at the surface of contact between two bodies that prevents or impedes sliding motion of one body relative to the other.
- This force distribution is tangent to the contact surface and has, for the body under consideration, a direction at every point in the contact surface that is in opposition to the possible or existing slipping motion of the body at that point.

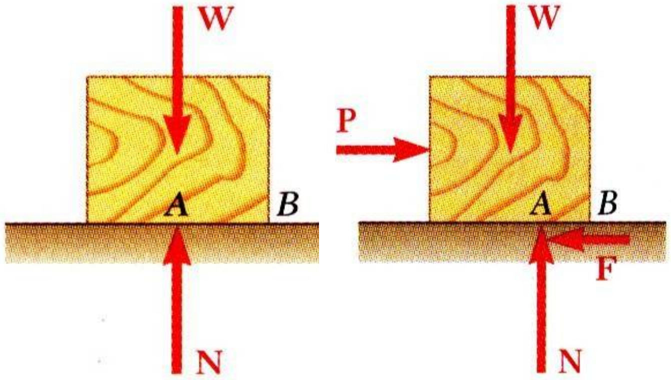
There are 3 types of Friction:

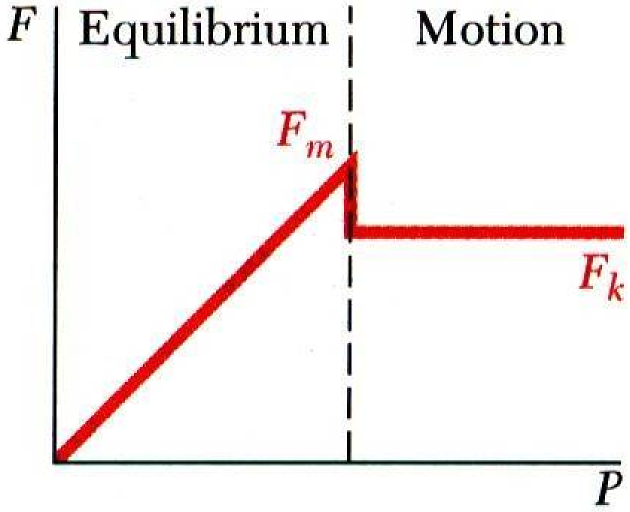
Static friction



Kinetic friction

Rolling friction





•Block of weight *W* placed on horizontal surface. Forces acting on block are its weight and reaction of surface *N*.

•Small horizontal force *P* applied to block. For block to remain stationary, in equilibrium, a horizontal component *F* of the surface reaction is required. *F* is a *static-friction force*.

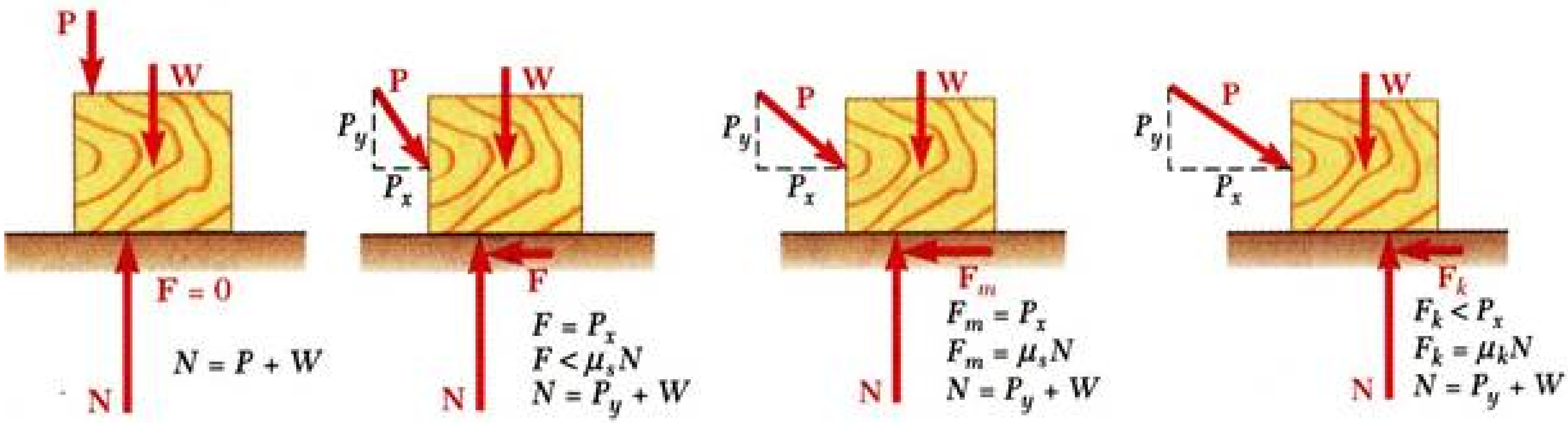
•As *P* increases, the static-friction force *F* increases as well until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

•Further increase in P causes the block to begin to move as *F* drops to a smaller *kinetic-friction force* F_k.

$$F_k = \mu_k N$$

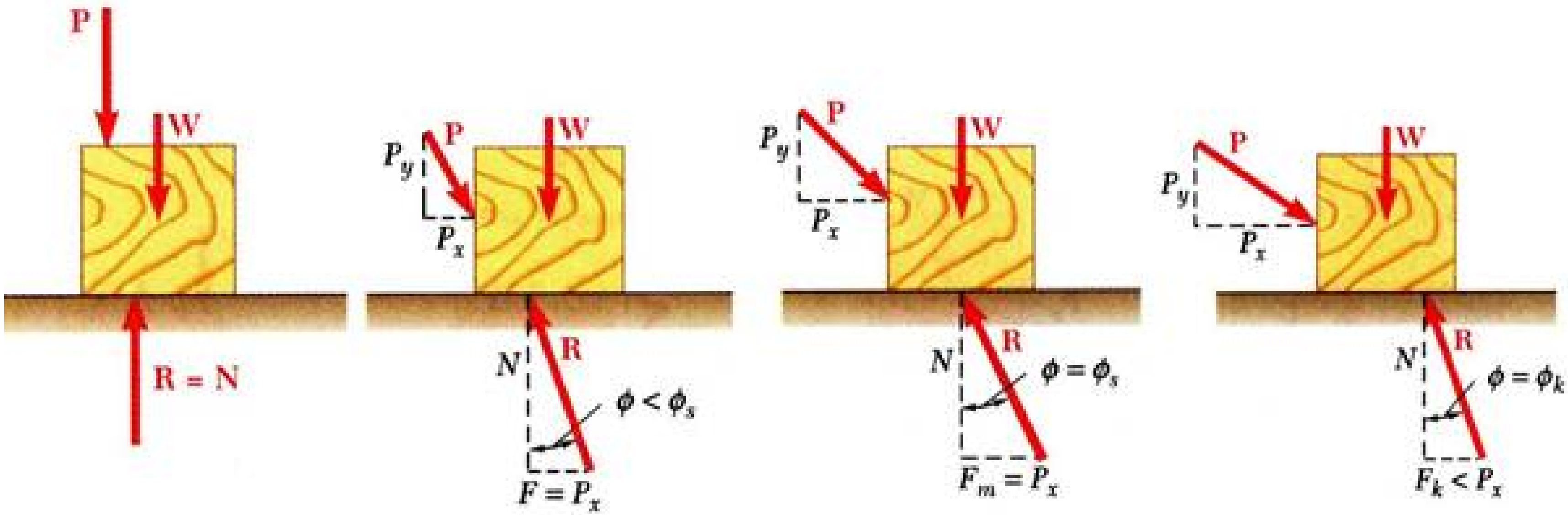
- Four situations can occur when a rigid body is in contact with a horizontal surface:



- No friction, ($P_x = 0$)
- No motion, ($P_x < F_m$)
- Motion impending, ($P_x = F_m$)
- Motion, ($P_x > F_m$)

Angles of Friction

- It is sometimes convenient to replace normal force. *N* and friction force *F* by their resultant ***R***:



- No friction
- No motion
- Motion impending
- Motion

$$\tan \phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

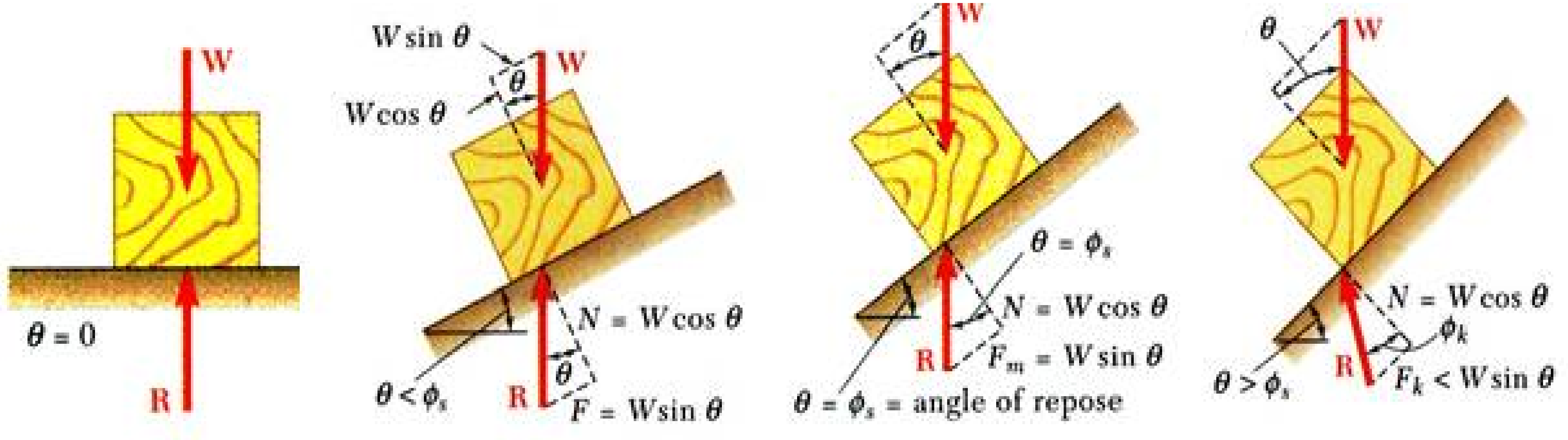
$$\tan \phi_s = \mu_s$$

$$\tan \phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan \phi_k = \mu_k$$

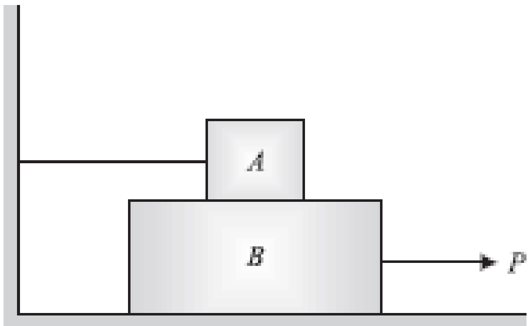
Angles of Friction

- Consider block of weight *W* resting on board with variable inclination angle θ.

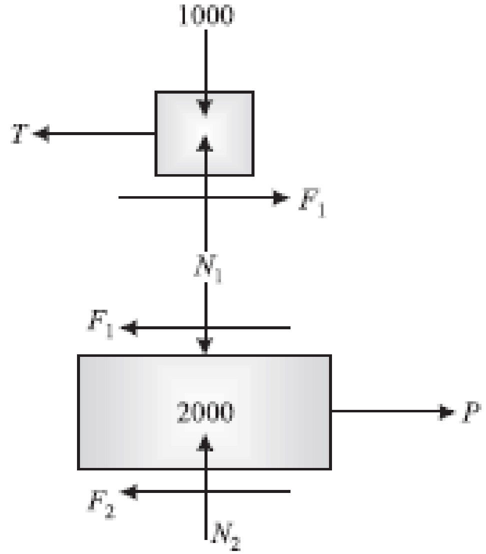


- No friction
- No motion
- Motion impending
- Motion

Block A weighing 1000 N rests over block B which weighs 2000 N as shown in Fig. 5.5(a). Block A is tied to a wall with a horizontal string. If the coefficient of friction between A
and B is 1/4 and that between B and the floor is 1/3, what value of force P is required to create
impending motion if (a) P is horizontal, (b) P acts 30° upwards to horizontal?



(a)



(b)

Now consider the equilibrium of block A.

$$\sum F_V = 0 \rightarrow$$

$$N_1 - 1000 = 0 \quad \text{or} \quad N_1 = 1000 \text{ newton.}$$

Since F_1 is limiting friction,

$$\frac{F_1}{N_1} = \mu_1 = \frac{1}{4}$$

$$\therefore \quad F_1 = \frac{1}{4} \times 1000 = 250 \text{ newton.}$$

$$\sum F_H = 0 \rightarrow$$

$$F_1 - T = 0 \quad \text{or} \quad T = F_1, \quad i.e. \quad T = 250 \text{ newton.}$$

Consider the equilibrium of block B.

$$\sum F_V = 0 \rightarrow$$

$$N_2 - N_1 - 2000 = 0.$$

$$\therefore \quad N_2 = N_1 + 2000 = 1000 + 2000 = 3000 \text{ newton.}$$

Since F_2 is limiting friction,

$$F_2 = \mu_2 N_2 = \frac{1}{3} \times 3000 = 1000 \text{ newton.}$$

$$\sum F_H = 0 \rightarrow$$

$$P - F_1 - F_2 = 0$$

$$\therefore \quad \boldsymbol{P} = F_1 + F_2 = 250 + 1000 = \mathbf{1250 \text{ newton.}}$$

(b) **When *P* is inclined:** Free body diagrams for this case are shown in Fig. 5.5(*c*).

Considering equilibrium of block *A*, we get

$$\sum F_V = 0 \quad \rightarrow \quad N_1 = 1000 \text{ newton.}$$

$$\therefore \quad F_1 = \frac{1}{4} \times 1000 = 250 \text{ newton.}$$

$$\sum F_H = 0 \quad \rightarrow \quad T = F_1 = 250 \text{ newton.}$$

Consider the equilibrium of block *B*.

$$\sum F_V = 0 \quad \rightarrow$$

$$N_2 - 2000 - N_1 + P \sin 30 = 0$$

or $$N_2 + 0.5P = 3000, \text{ since } N_1 = 1000 \text{ newton.}$$

From law of friction

$$F_2 = \mu_2 N_2 = \frac{1}{3} \times (3000 - 0.5P)$$

$$= 1000 - \frac{0.5}{3} P.$$

$$\sum F_H = 0 \rightarrow$$

$$P \cos 30 - F_1 - F_2 = 0$$

$$\therefore \quad P \cos 30 - 250 - \left(1000 - \frac{0.5}{3} P\right) = 0$$

$$\therefore \quad P\left(\cos 30 + \frac{0.5}{3}\right) = 1250$$

$$\therefore \quad \mathbf{P = 1210.4 \text{ newton}}$$

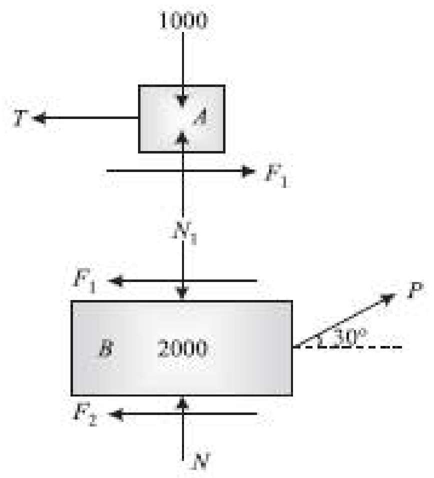
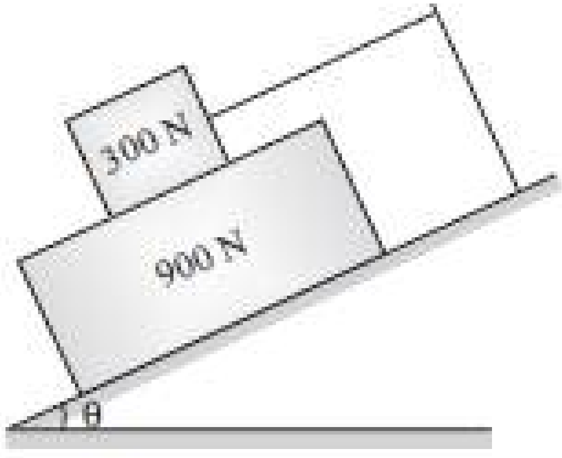


Fig. 5.5(c)

What should be the value of θ in Fig. 5.6(a) which will make the motion of 900 N block down the plane to impend? The coefficient of friction for all contact surfaces is 1/3.



Solution: 900 N block is on the verge of moving downward. Hence frictional forces F_1 and F_2 [Ref. Fig. 5.6(*b*)] act up the plane on 900 N block. Free body diagrams of the blocks are as shown in Fig. 5.6(*b*).

Consider the equilibrium of 300 N block.

Σ Forces normal to plane = 0 →

$$N_1 - 300 \cos\theta = 0 \quad \text{or} \quad N_1 = 300 \cos\theta \qquad ...(i)$$

From law of friction,

$$F_1 = \frac{1}{3} N_1 = 100 \cos \theta \qquad \text{...}(ii)$$

For 900 N block:

Σ Forces normal to plane = 0 →

$$N_2 - N_1 - 900 \cos \theta = 0$$

or

$$N_2 = N_1 + 900 \cos \theta$$

$$= 300 \cos \theta + 900 \cos \theta$$

$$= 1200 \cos \theta.$$

From law of friction,

$$F_2 = \mu_2 N_2 = \frac{1}{3} \times 1200 \cos \theta = 400 \cos \theta.$$

Σ Forces parallel to the plane = 0 →

$$F_1 + F_2 - 900 \sin \theta = 0$$

$$100 \cos \theta + 400 \cos \theta = 900 \sin \theta$$

$$\therefore \quad \tan \theta = \frac{500}{900}$$

$$\therefore \quad \theta = 29.05°$$

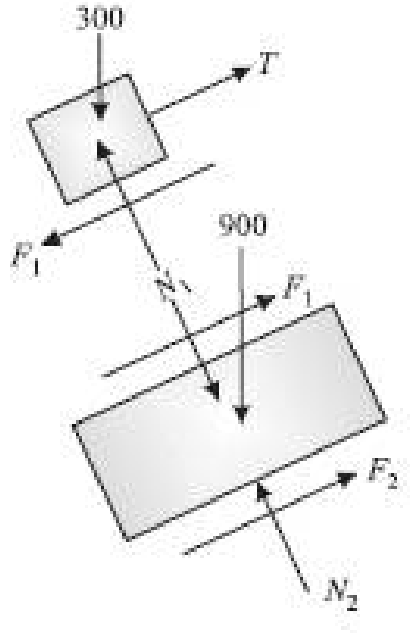
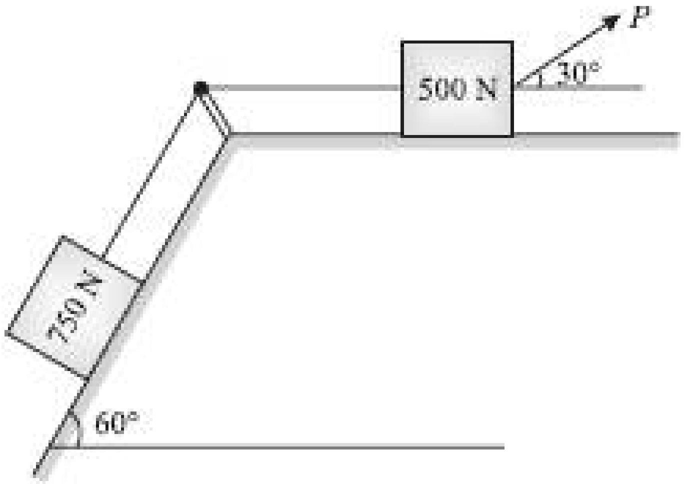
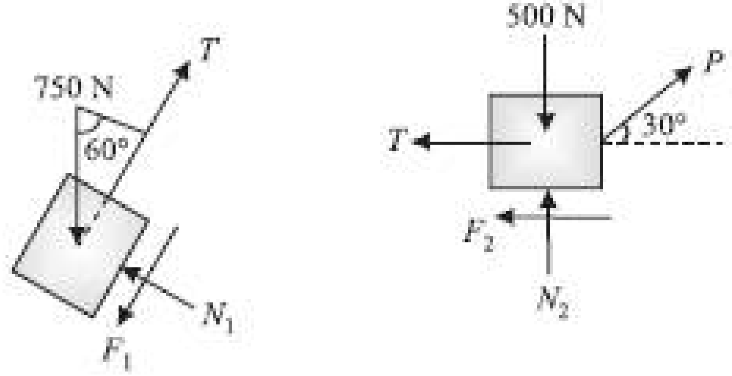


Fig. 5.6(b)

What is the value of P in the system shown in Fig. to cause the motion to impend?
Assume the pulley is smooth and coefficient of friction between the other contact surfaces is 0.2.



(a)



(b)

Solution: Free body diagrams of the blocks are as shown in Fig. 5.9(*b*). Consider the equilibrium of 750 N block.

Σ Forces normal to the plane = 0 $\rightarrow$

$$N_1 - 750 \cos 60 = 0 \quad \therefore \; N_1 = 375 \text{ newton} \qquad ...(i)$$

Since the motion is impending, from law of friction,

$$F_1 = \mu N_1 = 0.2 \times 375 = 75 \text{ newton} \qquad ...(ii)$$

Σ Forces parallel to the plane $= 0 \rightarrow$

$$T - F_1 - 750 \sin 60 = 0$$

$\therefore$ $$T = 75 + 750 \sin 60 = 724.5 \text{ newton.} \qquad ...(iii)$$

Consider the equilibrium of 500 N block.

$$\sum F_V = 0 \rightarrow$$

$$N_2 - 500 + P \sin 30 = 0$$

i.e., $$N_2 + 0.5P = 500 \qquad ...(iv)$$

From law of friction,

$$F_2 = \mu N_2 = 0.2\ (500 - 0.5P) = 100 - 0.1P \qquad ...(v)$$

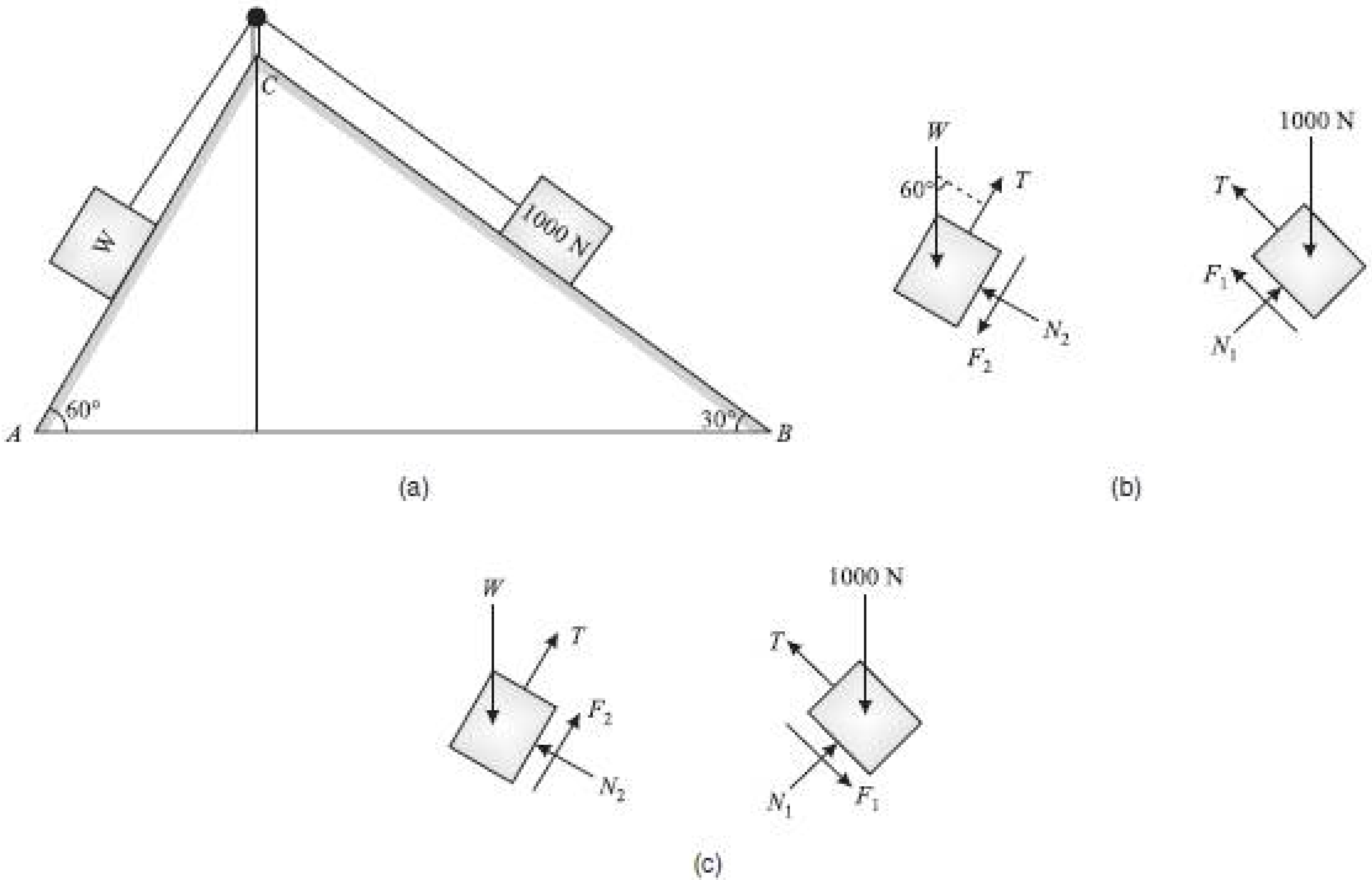
$$\sum F_H = 0 \rightarrow$$

$$P \cos 30 - T - F_2 = 0$$

i.e., $$P \cos 30 - 724.5 - 100 + 0.1P = 0$$

$\therefore$ $$P = 853.5 \text{ N}$$

Two identical planes AC and BC, inclined at 60° and 30° to the horizontal meet at C as shown in Fig. 5.10. A load of 1000 N rests on the inclined plane BC and is tied by a rope passing over a pulley to a block weighing W newtons and resting on the plane AC. If the coefficient of friction between the load and the plane BC is 0.28 and that between the block and the plane AC is 0.20, find the least and greatest values of W for the equilibrium of the system.



(a) Least value of *W*:

In this case motion of 1000 N block is impending down the plane and block *W* has impending motion up the plane. Hence free body diagrams for the blocks are as shown in Fig. 5.10(*b*). Considering the equilibrium of 1000 N block,

Σ Forces normal to the plane = 0 $\rightarrow$

$$N_1 - 1000 \cos 30 = 0 \quad \therefore N_1 = 866.0 \text{ newton} \qquad ...(i)$$

From the law of friction

$$F_1 = \mu_1 N_1 = 0.28 \times 866.0 = 242.5 \text{ newton} \qquad ...(ii)$$

Σ Forces parallel to the plane = 0 $\rightarrow$

$$T - 1000 \sin 30 + F_1 = 0$$

$$T = 500 - 242.5 = 257.5 \text{ newton} \qquad ...(iii)$$

Now consider the equilibrium of block weighing *W*.

Σ Forces normal to the plane = 0 $\rightarrow$

$$N_2 - W \cos 60 = 0 \quad i.e., \quad N_2 = 0.5\ W \qquad ...(iv)$$

From law of friction

$$F_2 = \mu_2 N_2 = 0.2 \times 0.5\ W = 0.1\ W \qquad ...(v)$$

Σ Forces parallel to the plane = 0 $\rightarrow$

$$T - F_2 - W \sin 60 = 0$$

Substituting the values of T and F_2 from eqns. (*iii*) and (*v*), we get

$$257.5 - 0.1\ W - W \sin 60 = 0$$

$$W = \frac{257.5}{0.1 + \sin 60} = 266.6 \text{ N.}$$

(b) For the greatest value of W:

In such case 1000 N block is on the verge of moving up the plane and W is on the verge of moving down the plane. For this case free body diagrams of the blocks are as shown in Fig. 5.10(*c*).

Considering the block of 1000 N,

Σ Forces normal to plane = 0 →

$$N_1 - 1000 \cos 30 = 0 \quad \therefore N_1 = 866.0 \text{ newton} \qquad ...(vi)$$

From law of friction,

$$F_1 = \mu_1 N_1 = 0.28 \times 866.0 = 242.5 \text{ N} \qquad ...(vii)$$

Σ Forces parallel to the plane = 0 →

$$T - 1000 \sin 30 - F_1 = 0$$

$$\therefore \quad T = 500 + 242.5 = 742.5 \text{ newton} \qquad ...(viii)$$

Considering the equilibrium of block weighing W,

Σ Forces normal to plane = 0 →

$$N_2 - W \cos 60 = 0 \quad \text{or} \quad N_2 = 0.5\,W \qquad ...(ix)$$

$$\therefore \quad F_2 = \mu_2 N_2 = 0.2 \times 0.5\,W = 0.1\,W \qquad ...(x)$$

Σ Forces parallel to plane = 0 →

$$T - W \sin 60 + F_2 = 0 \qquad ...(xi)$$

Substituting the values of T and F_2 from eqns. (*viii*) and (*x*), we get,

$$742.5 - W \sin 60 + 0.1\,W = 0$$

or

$$W = \frac{742.5}{\sin 60 - 0.1} = \mathbf{969.3 \text{ newton}}$$

The system of blocks are, in equilibrium for *W* = 266.6 N to 969.3 N.

WEDGE

Wedges are small pieces of hard materials with two of their opposite surfaces not parallel to each other.