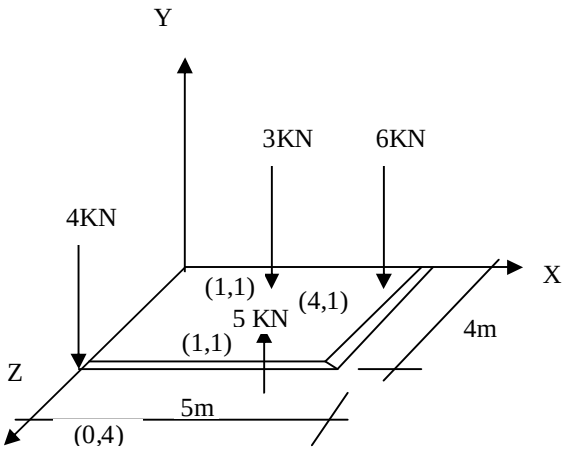
Problem 26: A slab of 4m * 5m carries from parallel forces as shown in fig. (locate the resultant force by scalar and vector approach.)



Solution:

Scalar approach

Magnitude of Resultant force $R = -4 + 5 - 3 - 6 = -8\,KN$

$$R = 8\,KN(\downarrow)$$

Talking moment of forces about x-axis

$$\sum m_x = -(4 \times 4) - (3 \times 1) + (5 \times 2)$$

$$= -15\,KNm$$

$$= 15\,KNm(Anticlockwise)$$

Talking moment about z-axis

$$\sum m_z = (4 \times 0) + (3 \times 1) + (6 \times 4) - (5 \times 3)$$

$$= 12\,KNm\ (clockwise)$$

Let 'R' be the resultant force passing through E of co-ordinates x,z as shown in fig.

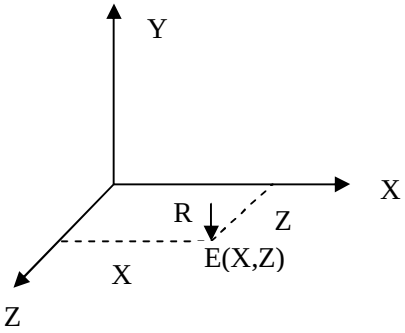
Moment of resultant force about

x-axis

$$\sum m_x = R \times Z$$

$$15 = 8\,Z$$

$$Z = 1.875\,m$$



lll[ly] moment of resultant force about Z axis

$$\sum m_z = R \times x$$

$$12 = 8\,x$$

$$x = 1.5\,m$$

Vector method:

Force vector of 4 KN is -4 j and co-ordinates A (0,0,4)

Force vector of 5 KN is 5 j and co-ordinates B (3,0,2)

Force vector of 3 KN is -3 j and co-ordinates C (1,0,1)

Force vector of 6 KN is -6 j and co-ordinates D (4,0,1)

The position vectors are

$$\overrightarrow{r_{OA}} = 4\,K$$

$$\overrightarrow{r_{OB}} = 3\,i + 2k$$

$$\overrightarrow{r_{OC}} = i + k$$

$$\overrightarrow{r_{OB}} = 4i + k$$

$\therefore$ Resultant vector, $\vec{R} = \overrightarrow{F_A} + \overrightarrow{F_B} + \overrightarrow{F_C} + \overrightarrow{F_C}$

$$= -4j + 5j - 3j - 6j$$

$$= -8j$$

Resultant moment, $\vec{m} = \overrightarrow{r_{OA}} \times F_A + \overrightarrow{r_{OB}} \times F_B + \overrightarrow{r_{OC}} \times F_C + \overrightarrow{r_{OD}} \times F_D$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 4 \\ 0 & -4 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 0 & 5 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & -3 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 0 & 1 \\ 0 & -6 & 0 \end{vmatrix}$$

$$= [i[0-(-16)] + j(0) + k(0)] + [i(-10) - j(0) + k(15)] + [i(3) - j(0) + k(-3)] + [i(6) - i(0) + k(-24)]$$

$$= 16\,i + (10\,i + 15k) + (3i - 3k) + (6i - 24k)$$

$$\vec{m} = 15\,i + 12\,k \rightarrow (1)$$

Let the resultant vector passing through a point P of co-ordinates (x,y,z) hence

$$\overrightarrow{r_{OP}} = xi + yj + 2k$$

From varignon's theorem

$$\overrightarrow{m_O} = \overrightarrow{r_{OP}} \times \vec{R} = \begin{vmatrix} i & j & k \\ x & y & z \\ 0 & -8 & 0 \end{vmatrix}$$

$$15\,i - 12\,k = i(8z) = j(0) + k(-8x)$$

$$15\,i - 12\,k = 8\,zi - 8\,kx$$

$$\therefore \quad 15 = 8\,z \quad ; \quad -12 = -8z$$

$$z = 1.875\,m \quad ; \quad x = 1.5\,m$$

Co-oridnates of resultant R is (1.5,0,1.875)m

Types of support in 3D force system:

Cable

Plane or smooth curved surface

Roller

Ball and socket

Plane or curved rough surface

Clamped or fixed

Single smooth pin