Sub Code: GE 8292 **Semester: II**
Subject: ENGINEERING MECHANICS **Unit – I: BASICS AND STATICS OF PARTICLES**

PART A

1. Define Coplanar & concurrent forces. **(AU JUN'10, DEC'10, DEC'12)**

(a) Coplanar forces : If the line of action of all forces lie on same plane, then the forces are said to be coplanar forces.

(b) Concurrent forces : If the line of action of all forces meet at common point, then the forces are said to be concurrent forces.

2. What is the different between a resultant force and equilibrant force? **(AU DEC'10,JUN'12)**

Resultant Force: If a number of forces acting simultaneously on a particle, then these forces can be replaced by a single force which would produce the same effect as produced by all forces. This single force is called as resultant force.

Equilibrant force : The force which bring the set of forces in equilibrium is known as equilibrant force. It is equal in magnitude and opposite in direction of the resultant force.

3. State the necessary and sufficient conditions for static equilibrium of a particle in two dimensions. **(AU JUN'12, DEC'11)**

i) The algebric sum of horizontal components of all forces in Plane is Zero i.e., $\Sigma F_x = 0$

ii) The algebric sum of all vertical components of all forces in plane is Zero i.e., $\Sigma F_y = 0$

4. What is unit vector? **(AU JUN'09)**

A Vector, whose magnitude is unity is called as unit vector. A unit Vector is denoted by n.

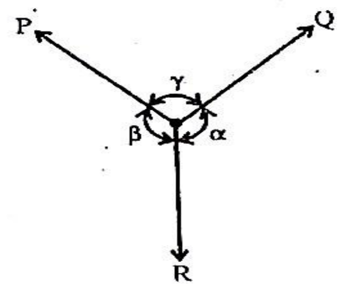
ex : For vector AB, $n = \dfrac{\overrightarrow{AB}}{|AB|}$

5. State Lame's theorem**. (AU JUN'12,DEC 10)**

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of angle between the other two".

Mathematically

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



6. A force $\vec{F} = 9\hat{\imath} + 6\hat{\jmath} - 15\hat{k}$ acts through the origin. What is the magnitude of the force and the angle it makes with X, Y and Z axis? **(AU DEC'09,JUN 09)**

Given : $F = 9i + 6j - 15k$

To find: $|\vec{F}| = F = ?$

$\theta_x = ?, \quad \theta_y = ?, \quad \theta_z = ?$

Solution :

$F_x = 9$ N, $F_y = 6$ N, $F_z = -15$N

$$|\vec{F}| = F = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

$$= \sqrt{9^2 + 6^2 + (-15)^2} = 18.49 \text{ N}$$

$$\text{Cos}\theta x = \frac{Fx}{F} = \frac{9}{18.49} = 0.4867$$

$$\theta x = \text{Cos}^{-1}(0.4867) = 60.87$$

$$\text{Cos}\theta_y = \frac{F_y}{F} = \frac{6}{18.49} = 0.3244$$

$$\theta_y = \text{Cos}^{-1}(0.3244) = 71.06^\circ$$

$$\text{Cos}\theta_z = \frac{Fz}{F} = \frac{-15}{18.49} = -0.8112^\circ$$

$$\theta_z = \text{Cos}^{-1}(-0.8112) = 144.2^\circ$$

7. State Varignon's Theorem? **(AU MAY'11)**

If a number of coplanar forces are acting simultaneously on a body , the algebraic sum of the moments of all the forces about any point is equal to the moment of resultant force about the same point.

8. Find the magnitude of the resultant of the two concurrent forces of magnitude 60 kN and 40 kN with an included angle of 70° between them. **(AU MAY'11)**

Given :

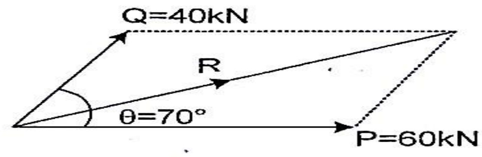
Force, $P = 60$ kN

Force, $Q = 40$ kN

Included angle, $\theta = 70°$

To Find :

Resultant, $R = ?$



Solution :

By parallelogram law of forces,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{60^2 + 40^2 + 2 \times 60 \times 40 \times \cos 70°}$$

R = 82.7/kN **(Ans)**

9. A force of magnitude 500N is passing through the origin and a point A (0.2, 1, 0) m. write the couple form of the force. **(AU DEC'11)**

Given :

Force, $|\vec{F}| = F = 500N$

Point A (0.2, 1, 0) m

To Find :

Force vector $\vec{F} = ?$

Solution :

The given force passing through origion O (0,0,0) and A (0.2,1;0)

Position vector $\overrightarrow{OA} = \vec{A} - \vec{O}$

$= (02i+j+ok) - (0i+0j+ok)$

$\overrightarrow{OA} = 0.2i + j$

Magnitude of $\overrightarrow{OA} = |\overrightarrow{OA}| = OA$

$= \sqrt{(0.2)^2 + (1)^2}$

$OA = 1.02$

Unit vector along OA, $n_{DA} = \dfrac{\overrightarrow{OA}}{|\overrightarrow{OA}|}$

$= \dfrac{0.2i + j}{1.02} = 0.196i + 0.98j$

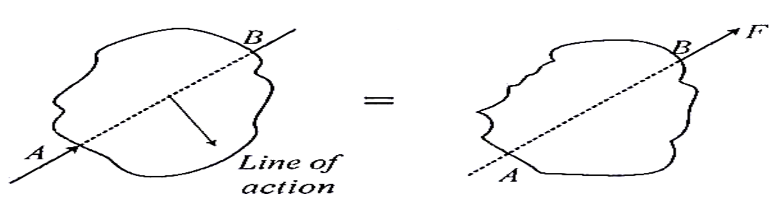
Force vector, $\vec{F} = F.n_{OA}$

$= 500\ (0.196i + 0.98j)$

$\vec{F} = \mathbf{98i + 490j}$ **(Ans)**

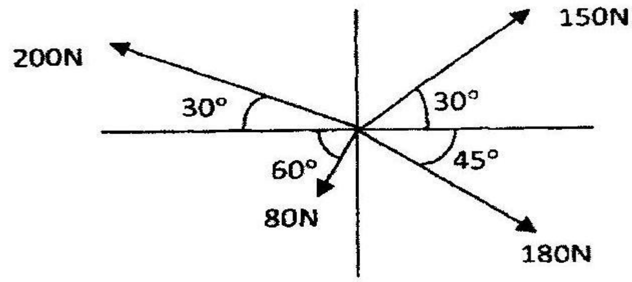
10. State the principal of transmissibility of forces with simple sketch. **(AU JUN'09, DEC'11)**

The conditions of equilibrium of motion of a rigid body remains unchanged, if a force acting at a given point of the rigid body is replaced by a force of same magnitude and direction, but acting at a different point provided that the two forces have the same line of action.



PART B

1. Determine the resultant of the concurrent force system shown in the following Figure. **(AU JUN'10, DEC'10, DEC'12)**



Solution:

Resolving forces horizontally,

$\Sigma H = 150\cos30 - 200\cos30 - 80\cos60 + 180\cos45$

$= 130 - 173.2 - 40 + 127.28$

$\Sigma H = 44.08$ N

Resolving forces vertically

$\Sigma V = 150\sin30 + 200\sin30 - 80\sin60 - 180\sin45$

$= 75 + 100 - 69.28 - 127.28$

$= \mathbf{-21.56\ N}$

Magnitude of Resultant

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= \sqrt{(44.08)^2 + (21.56)^2}$

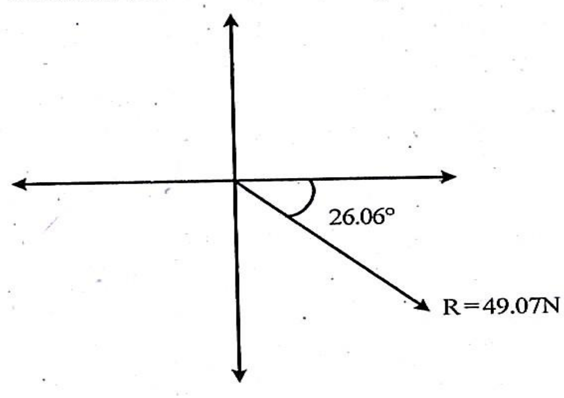
$\mathbf{R = 49.07\ N}$

Direction of resultant is

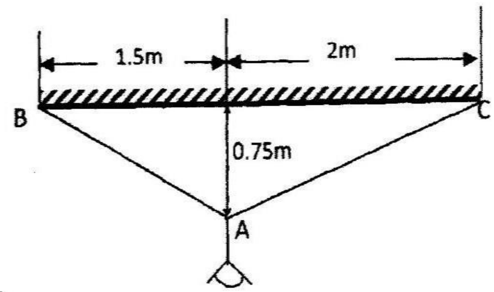
$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{21.56}{44.08}\right) = \mathbf{26.06\ (IV\ Quadrant)}$$

The resultant force is shown in figure.



2. The following figure shows a 10 kg lamp supported by two cables AB and AC. Find the tension in each cable. **(AU JUN'10, DEC'10, DEC'12)**



Given:

mass of lamp = 10 kg

weight, w = 10×9.81 = 98.1 N

To find:

1) Tension in cable AB, T_{AB} = ?

2) Tension in cable AC, T_{AC} = ?

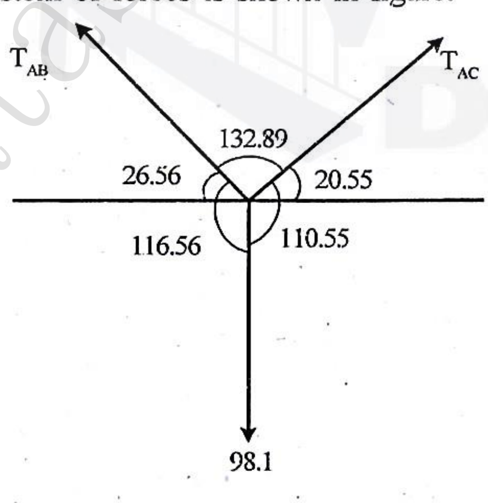
Solution:

From the given figure,

In angle ABD, $\tan\theta_1 = \frac{0.75}{1.5} = 26.56$

In angle ACD, $\tan\theta_2 = \frac{0.75}{2} = 20.55$

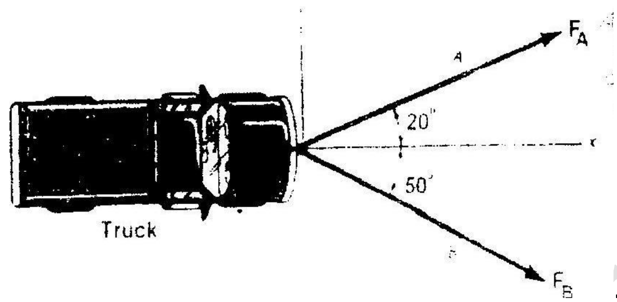
The system of forces is shown in figure.



Applying Lami's theorem

$$\frac{T_{AC}}{\sin 116.56} = \frac{T_{AB}}{\sin 110.55} = \frac{98.1}{\sin 132.89}$$

3. The truck is to be towed using two ropes. Determine the magnitudes of forces F_A and F_B acting on each rope in order to develop a resultant force of 950N directed along the positive X-axis. **(AU MAY'11, JUN'12)**



Given :

Inclination of F_A, $\theta_A = 20°$, with x-axis.

Inclination of F_B, $\theta_B = 50°$, with x-axis

To Find :

$F_A = ?$; $F_B = ?$

Solution :

It is given that resultant force R = 950N acting along positive x-axis. Hence, $\Sigma H = 950N$ and $\Sigma V = 0$.

Resolving forces Horizontally,

$$\Sigma H = F_A \cos 20° + F_B \cos 50°$$

$$950 = 0.9396\ F_A + 0.6427\ F_B \quad ..... (1)$$

Resolving force vertically,

$$\Sigma V = F_A \sin 20° - F_B \sin 50°$$

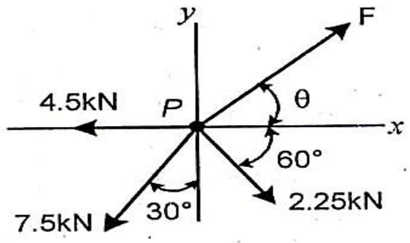
$$0 = 0.342\ F_A - 0.766\ F_B \quad ..... (2)$$

Solving equations (1) & (2), we get,

$\mathbf{F_A = 774.53N}$ **(Ans)**

$\mathbf{F_B = 345.75N}$ **(Ans)**

4. Determine the magnitude and angle θ and F so that particle shown in figure, is in Equilibrium **(AU MAY'11, JUN'12)**



Given :

Inclination of force, F = θ, with x-axis

Inclination of force 4.5KN = 0 with x-axis

Inclination of force 7.5KN with x-axis = (90–30) = 60°

Inclination of force 2.25KN with x-axis = 60°

To Find :

F = ? ; θ = ?

Solution :

It is given that the particle 'P' is in equilibrium. Hence

Hence, $\Sigma H = 0$ and $\Sigma V = 0$.

Resolving forces horizontally,

$$\Sigma H = F\cos\theta - 4.5\cos0° - 7.5\cos60° + 2.25\cos60°$$

$$0 = F\cos\theta - 4.5 - 3.75 + 1.125$$

$$F\cos\theta = 7.125 \qquad \text{..... (1)}$$

Resolving forces vertically,

$$\Sigma V = F\sin\theta + 4.5\sin0° - 7.5\sin60° - 2.25\sin60°$$

$$0 = F\sin\theta + 0 - 6.495 - 1.948$$

$$F\sin\theta = 8.443 \qquad \text{..... (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{F\sin\theta}{F\cos\theta} = \frac{8.443}{7.125}$$

$$\tan\theta = 1.185$$

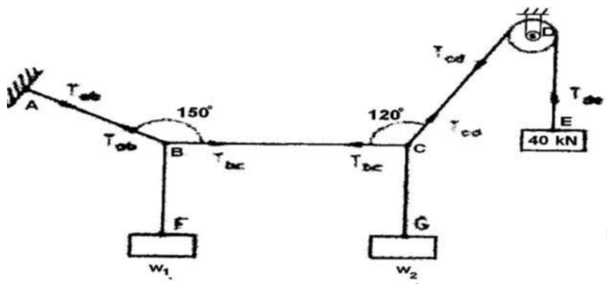
$$\boldsymbol{\theta = 49.84°} \qquad \textbf{(Ans)}$$

Put θ = 49.84° in eqn. (1), we get

$$F\cos(49.84) = 7.125$$

$$\mathbf{F = 11\ KN} \qquad \textbf{(Ans)}$$

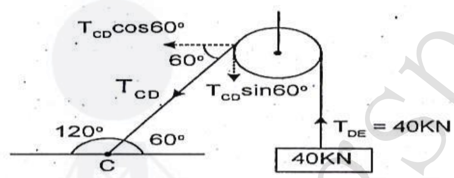
5. ABCDE is a light string whose end A is fixed. The weights W_1 and W_2 are attached to the string at B & C and the string passes round a small smooth wheel at D carrying a weight 40KN at the free end E. In the position of equilibrium, BC is horizontal and AB and CD make angles 150° and 120° with horizontal. **(AU DEC'12)**



Find (i) the tensions in AB, BC and DE of the given string (ii) magnitudes of W_1 and W_2.

Solution :

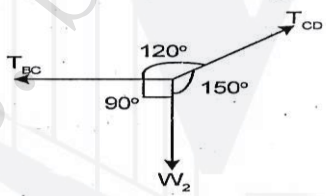
Let us consider the pulley first the various forces acting are shown below:



At point D,

$$T_{DE} = 40 \text{ kN}$$
$$\Sigma Fy = 0$$
$$-T_{CD} \sin 60° \; T_{DE} = 0$$
$$-0.866\, T_{CD} + 40 = 0$$
$$\mathbf{T_{CD} = 46.18 \text{ kN}} \qquad \textbf{(Ans)}$$

Now consider joint C, the various forces acting at C is shown below



Applying Lami's Theorem at joint C,

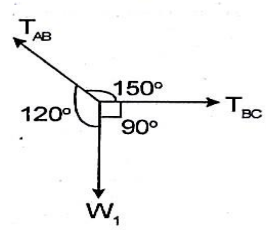
$$\frac{T_{CD}}{\sin 90°} = \frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°}$$

$$\frac{46.18}{\sin 90°} = \frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°}$$

$$T_{BC} = \frac{46.18 \times \sin 150}{\sin 90°} = \mathbf{23 \text{ kN}} \qquad \textbf{(Ans)}$$

$$W_2 = \frac{46.18 \times \sin 120}{\sin 90°} = \mathbf{40 \text{ kN}} \qquad \textbf{(Ans)}$$

Now consider joint B, the various forces acting at 'B' is shown in fig. Applying Lami's Theorem,

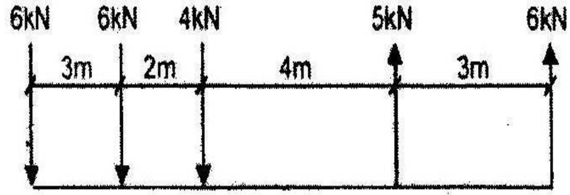


$$\frac{T_{AB}}{\sin 90°} = \frac{W_2}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

$$T_{AB} = \frac{T_{BC} \times \sin 90°}{\sin 120°} = \mathbf{26.56 \text{ kN}} \qquad \textbf{(Ans)}$$

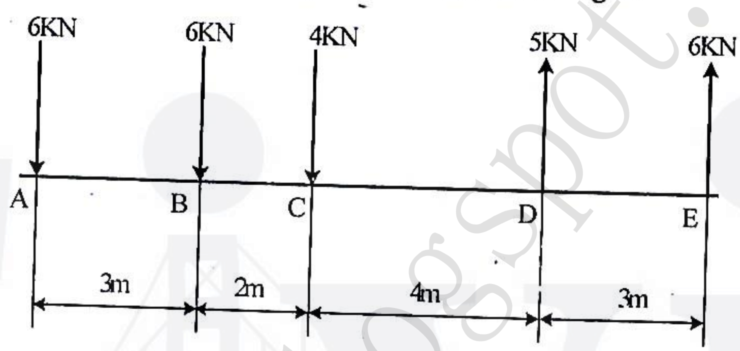
$$W_1 = \frac{T_{BC} \times \sin 150}{\sin 120°} = \mathbf{13.28 \text{ kN}} \qquad \textbf{(Ans)}$$

6. Find the magnitude and position of the resultant of the system of forces shown in Figure below. **(AU JUN'09,DEC ' 10)**



Solution:

The given force system is shown in the figure



To find magnitude of resultant:-

Resultant, $R = -6 - 6 - 4 + 5 + 6$

$R = 5$ kN (acting downwards)

To find the position of resultant:-

Let the resultant 'R' acts at a distance 'x' from point 'A'.

By Varignon's theorem

Algebric sum of all moments about 'A' = Moments of resultant about 'A'

$$6\times0 - 6\times3 - 4\times5 + 5\times9 + 6\times12 = R \times x$$

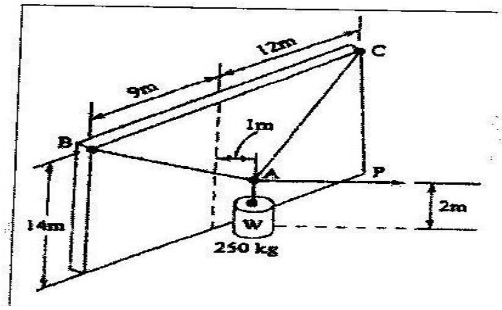
$$0 - 18 - 20 + 45 + 72 = 5\times x$$

$$79 = 5x$$

$$x = 15.8 \text{ m}$$

Hence, resultant $R = 5$kN acts at a distance $x = 15.8$ m from point 'A'

7. A horizontal force P normal to the wall holds the cylinder in the position shown in figure below. Determine the magnitude of P and the tension in each cable. **(AU DEC'12)**



Given:

Weight, W = 250kg = 250 x 9.81 = 2452.5N

To find:

i) Magnitude of P = ?

ii) Tension in each cable

Solution:

Let T_{AB} and T_{AC} be the forces along the cables AB and AC respectively. The co-ordinates of various points are A(1, 2, 0), B(0, 14, 9), C(0, 14, −12).

Tension in Cable AB:

$$\text{Position Vector } \overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= (0i + 14j + 9k) - (i + 2j + 0k)$$
$$\overrightarrow{AB} = -i + 12j + 9k$$
$$\text{Magnitude of } \overrightarrow{AB} = |\overrightarrow{AB}|$$
$$= AB = \sqrt{(-1)^2 + 12^2 + 9^2}$$
$$= 15.03$$

Unit vector along AB,

$$n_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$
$$= \frac{-i + 12j + 9k}{15.03}$$
$$= -0.0665\ i + 0.7984\ j + 0.5988\ k$$

Tension along AB, $\overrightarrow{T_{AB}} = T_{AB} \cdot n_{AB}$

$$= T_{AB}(-0.665\ i + 0.7984j + 0.5988k)$$
$$= -0.665T_{AB}i + 0.7984T_{AB}j + 0.5988T_{AB}k$$

Tension in Cable AC:

$$\text{Position Vector } \overrightarrow{AC} = \vec{C} - \vec{A}$$
$$= (0i + 14j - 12k) - (i + 2j + 0k)$$
$$\overrightarrow{AC} = -i + 12j - 12k$$

Magnitude of $\overrightarrow{AC} = |\overrightarrow{AC}|$

$$= AC = \sqrt{(-1)^2 + 12^2 + (-12)^2}$$

$$AC = 17$$

Unit vector along AC,

$$n_{AC} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$$

$$= \frac{-i + 12j - 12k}{17}$$

$$= -0.0588\,i + 0.7058\,j - 0.7058k$$

Tension along AC, $\overrightarrow{T_{AC}} = T_{AC} \cdot n_{AC}$

$$= T_{AC}(-0.0588\,i + 0.7058\,j - 0.7058k)$$

$$= -0.0588T_{AC}i + 0.7058T_{AC}j - 0.7058T_{AC}k$$

Force through weight,

$$\overrightarrow{W} = W(-j) = -2452.5j$$

Force through P,

$$\vec{P} = P \cdot i$$

using equations of equilibrium

$\Sigma F_x = 0$

$$-0.665\,T_{AB} - 0.0588\,T_{AC} + P = 0 \quad \text{.........} \ (1)$$

$\Sigma F_y = 0$

$$0.7984\,T_{AB} + 0.7058\,T_{AC} - 2452.5 = 0 \quad \text{.........} \ (2)$$

$\Sigma F_z = 0$

$$0.5988\,T_{AB} - 0.7058\,T_{AC} = 0 \quad \text{.........} \ (3)$$

Solving equations (1), (2), & (3) we get

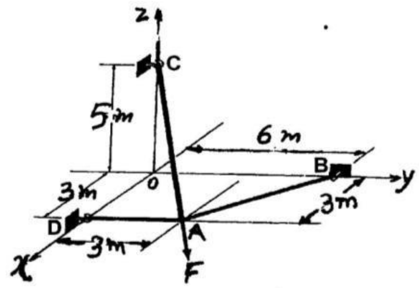
$$\mathbf{T_{AB} = 1755.8\ N}$$

$$\mathbf{T_{AC} = 1489.1\ N}$$

$$\mathbf{P = 204.3\ N}$$

(Ans)

8. Figure below shows three cables AB, AC, AD that are used to support the end of a sign which exerts a force of $\vec{F} = \{250i + 450j - 450k\}N$ at A. Determine the force develop in each cable. **(AU DEC'11)**



Solution :

Let F_{AB}, F_{AC} and F_{AD} be the forces acting along cables AB, AC and AD respectively.

From the geometry of figure, the co-ordinates of various points are

A (3,0,3), B (6,0,0), C (0,5,0), D (0,0,3)

Force Acting along AB

Position vector of AB is, $\overrightarrow{AB} = \vec{B} - \vec{A}$

$= (6i - 0i + 0k) - (3i + 0i + 3k)$

$\overrightarrow{AB} = 3i - 3k$

Magnitude of $\overrightarrow{AB} = |\overrightarrow{AB}| = AB = \sqrt{3^2 + (-3)^2} = 4.24$

Unit vector along AB is $n_{AB} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$= \dfrac{3i - 3k}{4.24}$

$n_{AB} = (0.7075i - 0.7075k)$

Force in wire AB, $\overrightarrow{F_{AB}} = F_{AB} . n_{AB}$

$= F_{AB}\ (0.7075i - 0.7075k)$

$= 0.7075\ F_{AB}\ i - 0.07075\ F_{AB} k$

Force acting along AC

Position vector of AC is $\overrightarrow{AC} = \vec{C} - \vec{A}$

$= (0i + 5j + 0k) - (3i + 0j + 3k)$

$\overrightarrow{AC} = -3i + 5j - 3k$

Magnitude of $\overrightarrow{AC} = |\overrightarrow{AC}| = AC = \sqrt{(-3)^2 + 5^2 + (-3)^2} = 6.55$

Unit vector acting along AC is $n_{AC} = \dfrac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$

$= \dfrac{3i + 5j - 3k}{6.55}$

$n_{AC} = -0.458i+0.763j -0.458k.$

Force in wire AC, $\overrightarrow{F_{AC}} = F_{AC}.n_{AC}$

$= F_{AC}(-0.458i+0.763j-0.458k)$

$\overrightarrow{F_{AC}} = -0.458\ F_{AC}i+00763\ F_{AC}j-0.458\ F_{AC}\ k$

Force acting along AD

Position vector of AD is $\overrightarrow{AD} = \overrightarrow{D} - \overrightarrow{A}$

$= (0i+0j+3k)-()3i+0j+3k)$

$\overrightarrow{AD} = -3i$

Magnitude of $\overrightarrow{AD} = |\overrightarrow{AD}| = AD = \sqrt{(-3)^2} = 3$

Unit vector acting along AD is $n_{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{-3°}{3} = -1°$

Force in wire AD, $\overrightarrow{F_{AD}} = F_{AD}.n_{AD}$

$\overrightarrow{F_{AC}} = -F_{AD}i$

It is given that force at A is

$\vec{F} = 250i + 450j - 150k$

Applying equations of equilibrium,

$\Sigma Fx = 0.7075\ F_{AB} - 0.458\ F_{AC} - F_{AD} + 250 = 0$ ..... (1)

$\Sigma Fy = 0.763\ F_{AC} + 450 = 0$ ..... (2)

$\Sigma Fz = -0.7075\ F_{AB} - 0.458\ F_{AC} - 150 = 0$ ..... (3)

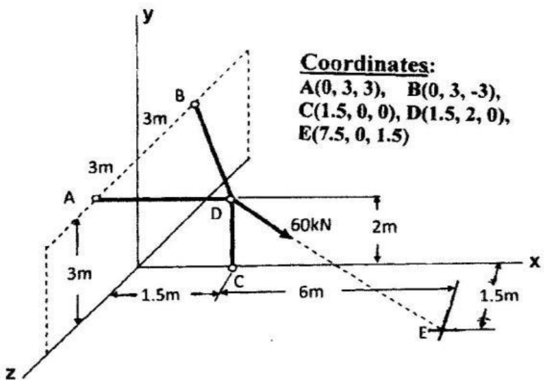
Solving equations (1), (2) & (3) we get,

$\mathbf{F_{AC} = 589.77N}$ **(Ans)**

$\mathbf{F_{AB} = 593.8N}$ **(Ans)**

$\mathbf{F_{AD} = 400N}$ **(Ans)**

9. In the figure shown, three wires are joined at D.



Tow ends A and B are on the wall and the other end C on the ground. The wire CD is vertical. A force of 60 KN is applied at 'D' and it passes through a point E on the ground as shown in figure. Find the forces in all the three wires. **(AU JUN'10, JUN'12)**

Given :

Co-ordinates are

A (0, 3, 3), B (0,3, –3), C (1.5, 0,0), D (1.5, 2, 0) and E (7.5, 0, 1.5)

Force acting along DE, $F_{DE} = 60KN$

To Find :

$F_{DA} = ?$

$F_{DB} = ?$

$F_{DC} = ?$

Solution :

Let F_{DA}, F_{DB}, F_{DC} and F_{DE} be the forces acting along DA, DB, DC and DE respectively.

Force in the wire DA

Force in the wire DA is directed from D to A.

Position vector of DA is $\overline{DA} = \overline{A} - \overline{D}$

$= (0i + 3j = 3k) - (1.5i + 2j + 0k)$

$\overline{DA} = -1.5i + j + 3k$

Magnitude of $\overline{DA} = |\overline{DA}| = DA = \sqrt{(-1.5)^2 + (1)^2 + 3^2}$

$DA = 3.5$

Unit vector along, DA, $n_{DA} = \frac{\overline{DA}}{DA}$

$= \frac{1.5i + j + 3k}{3.5}$

$n_{DA} = -0.428i + 0.285j + 0.857k$

Force in wire DA, $\overline{F_{DA}} = F_{DA} \cdot n_{DA}$

$= F_{DA} (-0.428i + 0.285j + 0.857k)$

$\overline{F_{DA}} = -0.428 F_{DA}\, i + 0.285 F_{DA}\, j + 0.857 F_{DA}\, k.$

Force in the wire DB :-

Position vector of DB is $\overline{DB} = \overline{B} - \overline{D}$

$= (0i+3j-3k) - (1.5j + 2j + 0k)$

$\overline{DB} = -1.5i + j - 3k$

Magnitude of $\overline{DB} = |\overline{DB}| = DB = \sqrt{(-1.5)^2+1^2+(-3)^2}$

$DB = 3.5$

Unit vector along, DB, $n_{DB} = \frac{\overline{DB}}{DB}$

$= \frac{1.5i + j + -3k}{3.5}$

$n_{DB} = -0.428i + 0.285j + 0.857k$

Force in wire DB, $\overline{F_{DB}} = F_{DB} \cdot n_{DB}$

$n_{DB} = (-0.428i + 0.285j + 0.857k)$

$\overline{F_{DB}} = 0.428\ F_{DB}\ i + 0.285\ F_{DB}\ j + 0.857\ F_{DB}\ k.$

Force in the wire DC:

Position vector of DC is $\overline{DC} = \overline{C} - \overline{D}$

$= (1.5i + 0j - 0k) - (1.5i + 2j + 0k)$

$\overline{DC} = -0i - 2j + 0k$

Magnitude of $\overline{DC} = |\overline{DC}| = DC = \sqrt{0^2+(-2)^2+0^2} = 2$

Unit vector along DC, $n_{DC} = \frac{\overline{DC}}{DC}$

$= \frac{0i + 2j + 0k}{2}$

$n_{DC} = -j$

Force in wire DC $\overline{F_{DC}} = F_{DC} \cdot n_{DC}$

$\overline{F_{DC}} = -F_{DC}\ j$

Force acting along DE :

Position vector of DE is $\overline{DE} = \overline{E} - \overline{D}$

$= (7.5i + 0j + 1.5k) - (1.5i + 2j + 0k)$

$\overline{DE} = 6i - 2j + 1.5k$

Magnitude of $\overline{DE} = |\overline{DE}| = DE = \sqrt{6^2+(-2)^2+(1.5)^2}$

Unit vector along DE, $n_{DE} = \frac{\overline{DE}}{DE}$

$= \frac{6i - 2j + 1.5k}{6.5}$

$n_{DE} = 0.923i - 0.307j + 0.2307k$

Force acting along DE, $\overline{F_{DE}} = F_{DE} \cdot n_{DE}$

$\overline{F_{DE}} = 60\ (0.923i - 0.307j + 0.2307k)$

$\overline{F_{DE}} = 55.38i - 18.42j + 13.84k$

Applying equations of equilibrium,

$\Sigma F_x = 0$

$-0.428\ F_{DA} - 0.428 F_{DB} + 55.38 = 0$

$F_{DA} + F_{DB} = 129.39$ ..... (1)

$\Sigma F_y = 0$

$0.285\ F_{DA} + 0.285 F_{DB} - F_{DC} - 0.307 = 0$

$F_{DA} + F_{DB} - 3.5\ F_{DC} = 1.077$ ..... (2)

$\Sigma F_z = 0$

$0.857\ F_{DA} - 0.857 F_{DB} + 13.84 = 0$

$F_{DA} - F_{DB} = -16.149$ ..... (3)

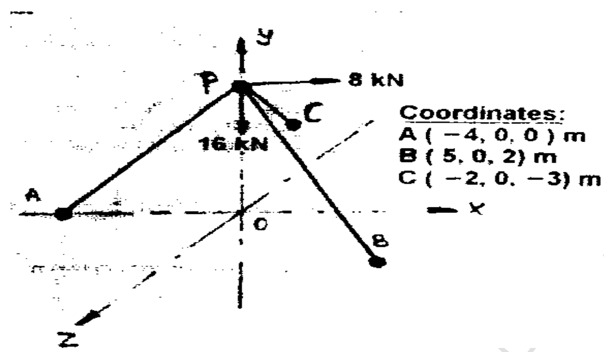
Solving equations (1), (2) and (3), we get

$\mathbf{F_{DA} = 56.62}$ **KN** **(Ans)**

$\mathbf{F_{DB} = 72.76}$ **KN** **(Ans)**

$\mathbf{F_{DC} = 36.65}$ **KN** **(Ans)**

10. Two forces act upon a tripod at 'P' as shown in figure. The force 8 kN is parallel to x-axis and the force 16kN is parallel to y – axis. **(AU MAY'11,JUN '12)**



Determine the forces acting at the legs of tripod if the legs rest on ground at A, B, and C whose coordinates with respect to O are given .The height of the P above the origin is 10m.

Given :

Co-ordinates

A (–4, 0, 0)

B (5, 0, 2)

C (–2, 0, –3)

To Find :

Forces acting in legs of tripod

Solution :

Let F_{PA}, F_{PB} and F_{PC} be the forces acting along the legs A, B, and C of tripod respectively.

The Co-ordinate of P (0, 10, 0)

Let $F_1 = 8$ kN

and $F_2 = 16$ kN

Since $F_1 = 8$ kN acting a long x-axis, its unit vector is i.

Hence, $\overline{F_1} = F_1.i = 8i$

Since F_2=16 kN acting parallel to y-axis, in downword direction, its unit vector is $-j$

$\therefore \overline{F_2} = F_2.-j = -16j$

Force acting along PA :-

Position vector of PA is $\overline{PA} = \overline{A} - \overline{P}$

$= (-4i+0j+0k)-(0i+10j+k)$

$\overline{PA} = -4i - 10j$

Magnitude of $\overline{PA} = |\overline{PA}| = PA = \sqrt{(-4)^2 + (-10)^2} = 10.77$

Unit Vector along $PA = n_{PA} = \frac{\overline{PA}}{|\overline{PA}|}$

$= \frac{-4i-10j}{10.77} = -0.371i - 0.928j$

Force in leg PA, $\overline{F_{PA}} = F_{PA}.n_{PA}$

$= F_{PA}\ (-0.371i\ -0.928j)$

$\overline{F_{PA}} = -0.371\ F_{PA}i - 0.928\ F_{PA}j$

Force acting along PB:-

Position vector of PB is $\overline{PB} = \overline{B} - \overline{P}$

$= (5i+10j+2k) - (0i+10j+0k)$

$\overline{PB} = 5i - 10j \ ! \ 2k$

Magnitude of $\overline{PB} = |\overline{PB}| = \overline{PB} = \sqrt{(5)^2 + (-10)^2 + 2^2} = 11.35$

Unit Vector along $PB = n_{PB} = \dfrac{\overrightarrow{PB}}{|\overrightarrow{PB}|}$

$$= \frac{5i - 10j + 2k}{11.35} = 0.44i - 0.88j + 0.176k$$

Force in leg PB, $\overrightarrow{F_{PB}} = F_{PB}.n_{PB}$

$$= F_{PB}\ (0.44i - 0.88j + 0.176k)$$

$\overrightarrow{F_{PB}} = 0.44\ F_{PB}i\ - 0.88\ F_{PB}j\ + 0.176 F_{PB}k$

Force acting along PC:-

Position Vector of PC is $\overrightarrow{PC} = \vec{C} - \vec{P}$

$$= (-2i + 0j - 3k) - (0i + 10j + 0k)$$

$\overrightarrow{PC} = -2i - 10j - 3k$

Magnitude of $\overrightarrow{PC} = |\overrightarrow{PC}| = PC = \sqrt{(-2)^2 + (-10)^2 + (-3)^2} = 10.63$

Unit Vector along PC is $n_{PC} = \dfrac{\overrightarrow{PC}}{|\overrightarrow{PC}|}$

$$= \frac{2i - 10j + 3k}{10.63} = -0.188i - 0.94j - 0.282k$$

Force in leg PC, $\overrightarrow{F_{PC}} = F_{PC}.n_{PC}$

$$= F_{PC}\ (-0.188i - 0.94j - 0.282k)$$

$\overrightarrow{F_{PC}} = -0.18^{\circ} F_{PC}i - 0.94 F_{PC}j\ - 0.282\ F_{PC}k$

Applying equations of equilibrium,

$\Sigma Fx = 0$

$8 - 0.371\ F_{PA} + 0.44\ F_{PB} - 0.94 F_{PC} = 0$ ..... (1)

$\sum Fy = 0$

$-16\ - 0.928\ F_{PA} - 0.88 F_{PB} - 0.282\ F_{PC} = 0$ ..... (2)

$\sum Fz = 0$

$0.176\ F_{PB} - 0.282 F_{PC} = 0$ ..... (3)

Solving above equations we get,

F_{PA} = 77.58 kN **(Ans)**

F_{PB} = 53.03 kN **(Ans)**

F_{PC} = 33.09 kN **(Ans)**

11. Forces R, S, T, U are collinear. Forces R and T act from left to right. Forces S and U act right to left. Magnitudes of the forces R, S, T, U are 40N, 45N, 50N and 55N respectively. Find the resultant of R, S, T, U. **(AU JUN'09)**

Given:

Force R, $F_R = 40$ N

Force S, $F_S = 45$ N

Force T, $F_T = 50$ N

Force U, $F_U = 55$ N

To Find:

Resultant, R = ?

Solution:

The given forces are shown in the figure.

$$\text{Resultant, } R = F_R + F_T - F_S - F_U$$
$$= 40 + 50 - 45 - 55$$
$$R = -10\text{ N} \qquad \textbf{(Ans)}$$

Resultant force R = 10 N will act from right to left.

11. State the parallelogram law of forces **(AU JUN'09, DEC 12)**

When two forces are acting at a point, then parallelogram law is used to find the resultant force.

It states that, "If two forces, acting simultaneously on a particle be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection".

12. Distinguish between scalar and vector quantities. **(AU JUN'09, DEC 10)**

Scalar Quantity :

Scalar Quantities are those which are completely defined by their magnitude only.

Ex : 2 kg of mass, 10 m/s speed, 50°C temperature

Vector Quantity :

The Quantities which are defined by their magnitude and direction is known as vector quantity.

Ex : 20 N force acting vertically downwards

9.8 m/S^2 acceleration directed towards centre of earth.