Sub Code: GE 6253 **Semester: II**
Subject: ENGINEERING MECHANICS **Unit – II:EQUILIBRIUM OF RIGID BODIES**

PART A

1. Explain free body diagram .**AU Dec'09,Jun'10)**

Free body diagram is defined as the diagram of the isolated body, containing all the applied forces and reactive forces acting on it.

2. State Varignon's theorem.**(AU JUN'09,DEC'09 , MAY'11,DEC'12)**

It states that, "If a number of coplanar forces are acting simultaneously on a particle, the algebric sum of moments of all forces about any point is equal to the moment of their resultant force about the same point."

In other words,

Sum of clockwise moments=sum of anti-clockwise moments.

3. List out the steps to be followed to draw the Free Body Diagram of a rigid body.**(AU Dec'11,JUN'12)**

1. Draw a diagram of body completely isolated from all other bodies. The isolated body may consist of entire system (or) any portion of the system.
2. Remove all the supports and represent the reaction exerted by those supports on the body.
3. Indicate the magnitude and direction of known applied load and indicate the unknown load by a symbol.
4. Indicate the suitable dimensions (including slopes, angles) or the co-ordinates of key points which defines the configuration of force systems.
5. The weight of the free body is indicated with a vertical downward arrow.
6. The above procedure is repeated for all objects of system in equilibrium.

4. State the necessary and sufficient conditions for equilibrium of rigid bodies in two dimensions.**(AU JUN'09,Dec'11)**

   1. Summation of horizontal components of all forces should be zero. i.e ., $\Sigma H = 0$.
   2. Summation of vertical components of all forces should be zero. i.e., $\Sigma V = 0$.
   3. Summation of all moment of forces must be zero i.e., $\Sigma M = 0$.

5. Write the conditions of equilibrium of a system of parallel forces acting in a plane.**(AU Dec'10,JUN'12)**

   The conditions of equilibrium of a system of parallel forces acting in a plane are $\Sigma F_x=0$; $\Sigma F_y=0$; $\Sigma M=0$

6. State the general condition of equilibrium of particle.**(AU MAY'11)**

   i) The algebric sum of horizontal components of all forces in Plane is Zero i.e., $\Sigma SF_x = 0$
   ii) The algebric sum of all vertical components of all forces in plane is Zero i.e., $\Sigma SF_y = 0$

7. Why the couple moment is said to be a free vector?**(AU JUN'10)**

   The couple is a pure turning effect which may be moved anywhere in its own plane without change its effect on the body. Hence, couple moment is said to be free vector.

8. Distinguish between a couple and a moment.**(AU Jun'10,DEC'12)**

Couple	Moment
❖ Two equal and parallel forces acting in opposite direction constitute a couple.	❖ Moment is the turning effect produced by a force on the body on which it acts.
❖ It doesnot depend on any point or axis.	❖ It depends on point or axis about which moment is taken.

PART B

1. A force (10i+20j-5k)N acts at a point P (4,3,2) m. Determine the moment of this force about the point Q(2,3,4) m in the vector form, Also find the magnitude of the moment andits angles with respect to x,y,z axes.**(AU Dec'10,JUN'12)**

Given :

Force, $\vec{F} = (10i + 20j - 5k)N$

P (4,3,2) m

Q (2,3,4) m

To Find :

Moment, $\overrightarrow{M_Q} = ?$

Magnitude of moment, $|\overrightarrow{M_Q}| = M = ?$

$\theta_x = ?,\ \theta_y = ?\ \theta_z = ?$

Solution :

Position vector, $\overrightarrow{QP} = \vec{P} - \vec{Q}$

$= (4i + 3j + 2k) - (2i + 3j + 4k)$

$\overrightarrow{QP} = 2i + 0j - 2k$

Moment about point Q, $\overrightarrow{M_Q} = \overrightarrow{QP} \times \vec{F} = \begin{vmatrix} i & j & k \\ 2 & 0 & 2 \\ 10 & 20 & -5 \end{vmatrix}$

$= i\,(0 + 40) - j\,(-10 + 20) + k\,(40 - 0)$

$\overrightarrow{M_Q}$ **= (40i − 10j + 40k) N-m.** **(Ans)**

Here, $M_x = 40$N-m ; $M_y = -10$ N−m, $M_z = 40$ N-m

Magnitude of moment $= |\overrightarrow{M_Q}| = M = \sqrt{(40)^2 + (-10)^2 + (40)^2}$

$M = 57.44$ N−m

Angle made by $\overrightarrow{MO}$ with x-axis

$$\cos\theta x = \frac{Mx}{M}$$

$$\theta x = \cos^{-1}\left(\frac{40}{57.44}\right)$$

$$\theta x = \mathbf{45.86^\circ}$$ **(Ans)**

Similarly,

$$\cos\theta y = \frac{My}{M}$$

$$\theta y = \cos^{-1}\left(\frac{-10}{57.44}\right)$$

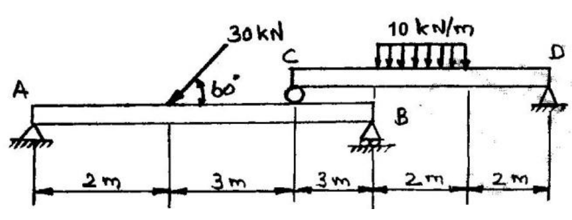
$$\theta y = \mathbf{100.02^\circ}$$

(Ans)

$$\theta z = \cos^{-1}\left(\frac{Mz}{M}\right) = \cos^{-1}\left(\frac{40}{57.44}\right)$$

$$\theta z = \mathbf{45.86^\circ}$$ **(Ans)**

2. Two beams AB and CD are shown in figure. A and D are hinged supports. B and C are roller supports.



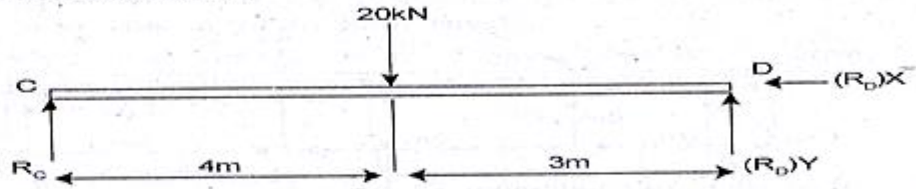
(i) Sketch the free body diagram of the beam AB and determine the reactions at the supports A & B.

(ii) Sketch the free body diagram of beam AB and determine the reactions at the supports C and D. **(AU Dec'10,DEC'12)**

Solution :

Let R_A, R_B, R_C and R_D be the reactions at supports A, B, C and D respectively.

Consider the beam CD. The free body diagram of beam CD is shown below.



The uniformly distributed load of 10 kN/m for a length of 2 m is assumed as equivalent point load of (10×2=20kN) and acting at a distance 4 m form C.

Using equations of equilibrium,

$\Sigma F_x = 0$

$(R_D)_x = 0$ **(Ans)**

$\Sigma F_y = 0$

$R_C - 20 + (R_D)_y = 0$

$R_C + (R_D)_y = 20$ ..... (1)

Taking moments about C,

$\Sigma M_C = 0$

$-20\times4 + (R_D)_y \times 7 = 0$

$-80 + 7(R_D)_y = 0$

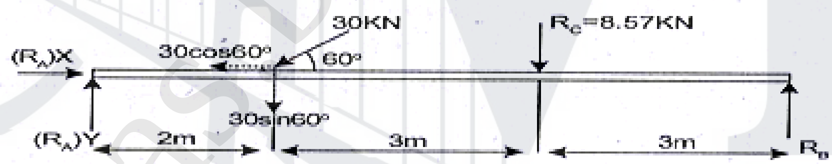
$(R_D)_y = 11.43$ kN **(Ans)**

Put $(R_D)_y = 11.43$ in eqn. (1), we get

$R_C + 11.43 = 20$

$R_C = 8.57$ kN **(Ans)**

Now consider the beam AB. The free body diagram of beam AB is shown below



Using equations of equilibrium,

$\Sigma F_x = 0$

$(R_A)_x - 30\cos60 = 0$

$(R_A)_x = 15$ kN

$\Sigma F_y = 0$

$(R_A)_y - 30\sin60 - 8.57 + R_B = 0$

$(R_A)_y + R_B = 34.55$ ..... (2)

Taking moments about A,

$\Sigma M_A = 0$

$-30\sin60\times2 - 8.57\times5 + R_B\times8 = 0$

$-51.96 - 42.85 + 8R_B = 0$

$R_B = 11.85$ kN **(Ans)**

Put $R_B = 11.85$ in eqn. (2), we get

$(R_A)_y + 11.85 = 34.55$

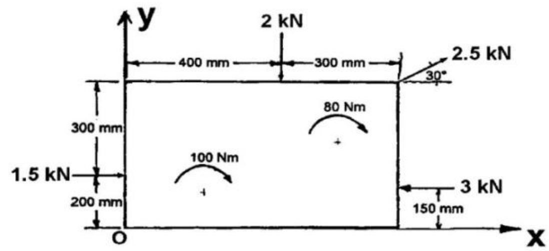
$(R_A)_y = 22.7$ kN

$R_A = \sqrt{(R_A)_x^2 + (R_A)_y^2}$

$= \sqrt{(15)^2 + (22.7)^2}$

$R_A = 27.2$ kN **(Ans)**

3. A force couple system acting on a rectangular plate is shown in figure below.



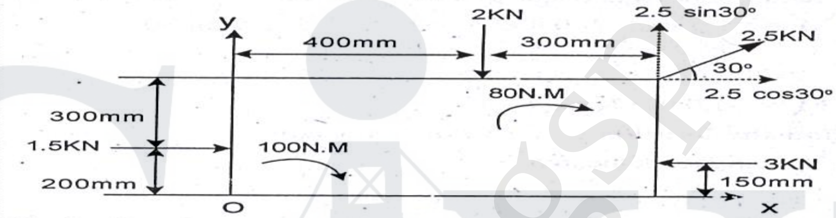
(i) Find the equivalent force couple system at the origin O.

(ii) Find the single resultant force and its location on x – axis. **(AU Dec'11, JUN 10)**

Solution:

To find equivalent force couple system at origin 'O' we must know,

- Single Resultant Force
- Single Moment Through Point 'O'



Single Resultant Force, R :

Resolving Forces along x-axis

$$\sum H = 1.5 - 3 + 2.5\ \cos 30°$$

$$\sum H = 0.665\text{kN}$$

Resolving Forces along y-axis

$$\sum V = -2 + 2.5\ \sin 30°$$

$$\sum V = -0.75\text{kN}$$

$$\text{Resultant, } R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$= \sqrt{(0.665)^2 + (-0.75)^2}$$

$$R = 1\ \text{kN}$$

$$\text{Direction of resultant } \theta = \tan^{-1}\left(\frac{\sum V}{\sum H}\right)$$

$$= \tan^{-1}\left(\frac{-0.75}{0.665}\right)$$

$$\theta = 48.43° \text{ (IV Quadrent)}$$

Single moment through origin 'O'

Taking moments about point 'O' we get

$$M_O = -1.5\times0.2 - 2\times0.4 - 2.5\ \cos30°\times0.5 + 2.5\ \sin30° \times 0.7 + 3 \times 0.15 - 100\times10^3 - 80\times10^3$$

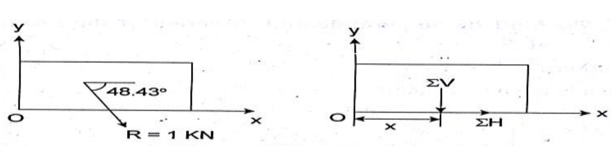
$$= -0.3 - 0.8 - 1.08 + 0.875 + 0.45 - 100 - 80$$

$$M_O = -1.035 \text{ kN–m } (\nearrow)$$

To find the location of resultant on x-axis

By varignon's theorem,

$$\left(\begin{matrix}\text{sum of moment of all} \\ \text{forces about 'O'}\end{matrix}\right) = \left(\begin{matrix}\text{Moment of resultant} \\ \text{about 'O'}\end{matrix}\right)$$



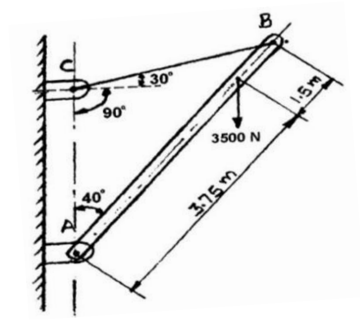
$$-1.035 = \Sigma H\times o - \Sigma V \times x$$

$$x = \frac{1.035}{\Sigma V}$$

$$x = \frac{1.035}{0.75}$$

$$x = 1.38 \text{ m} \qquad \text{(Ans)}$$

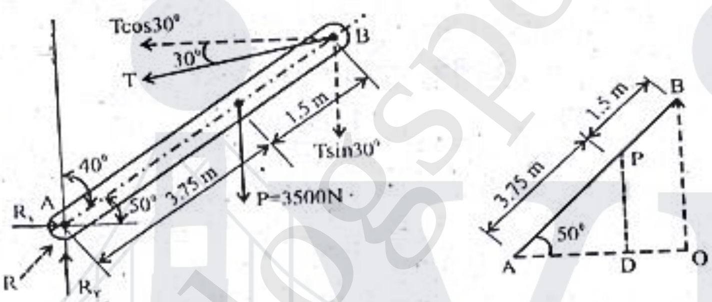
4. A load of 3500 N is acting on the boom, which is held by a cable BC as shown in figure below, the weight of the boom can be neglected. **(AU Dec'11)**



(i) Sketch the free body diagram of the boom.
(ii) Determine the tension in cable BC.
(iii) Find the magnitude and direction of the reaction at A

Solution :

a. Free body diagram:



Let R_x and R_y be horizontal and vertical component of reaction 'R' at point A.

Let 'T' be the tension in the string BC.

From the Geometry of figure as in fig .

In D^{le} APD,

$$\cos 50^\circ = \frac{AD}{AP} \Rightarrow AD = 3.75 \cos 50^\circ = 2.41\text{m}$$

In Δ^{le} ABO,

$$\sin 50^\circ = \frac{BO}{AB} \Rightarrow BO = 5.25 \times \sin 50^\circ = 4.02\text{m}$$

$$\cos 50^\circ = \frac{AO}{AB} \Rightarrow AO = 5.55 \cos 50^\circ = 3.37\text{m}$$

Using equations of equilibrium,

$$\Sigma F_x = 0$$

$R_x - T\cos30° = 0$

$R_x = 0.866T$ ......(1)

$\Sigma F_y = 0$

$R_y - 3500 - T\sin 30^0 = 0$

$Ry = 3500 + 0.5\ T$ ......(2)

$\Sigma M = 0$

Taking moments about A,

$-3500 \times AD - T\sin30° \times AO + T\cos30° \times BO = 0$

$-3500 \times 2.41 - T\sin30° \times 3.37 + T\cos30° \times 4.02 = 0$

T = 4695.4 N **(Ans)**

Substituting T = 4695.4 in (1) & (2), we get

$R_x = 4066.3$ N

$R_y = 5847.7$ N

$\therefore$ magnitude of reaction $R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{(4066.3)^2 + (5847.7)^2}$

R = 7122.52N **(Ans)**

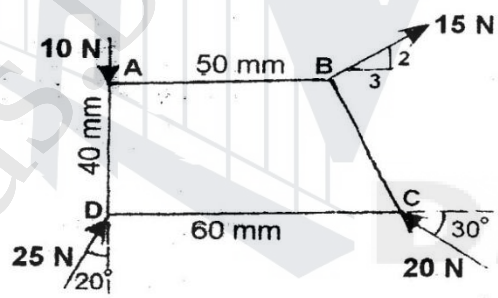
$\therefore$ Direction of reaction is

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{5847.7}{4066.3}\right)$$

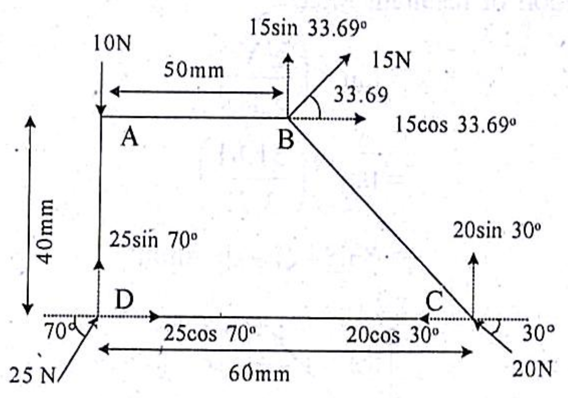
θ = 55.18° **(Ans)**

5. Replace the given system of forces acting on a plate ABCD shown in figure by a Force-couple system acting at the point A.**(AU MAY'11, DEC '12)**



Solution :

The forces acting on plate ABCD is shown in figure.



The inclination of 15N force is given by

$$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$$

To find resultant force:-

Resolving Forces horizontally,

$$\Sigma H = 25\cos 70° - 20\cos 30° + 15\cos 33.69$$
$$= 8.55 - 17.32 + 12.48$$
$$\Sigma H = 3.71 \text{ N}$$

Resolving forces vertically,

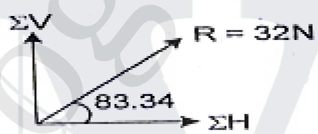
$$\Sigma V = -10 + 25\sin 70° + 20\sin 30° + 15\sin 33.69$$
$$= -10 + 23.49 + 10 + 8.32$$
$$\Sigma V = 31.81 \text{ N}$$

Magnitude of Resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(3.71)^2 + (31.81)^2}$$
$$R = 32 \text{ N}$$

Direction of resultant force

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$
$$= \tan^{-1}\left(\frac{31.81}{3.71}\right)$$
$$\alpha = 83.34 \text{ (I - quadrant)}$$



To find location of resultant force, w.r.t A :-

Let the perpendicular distance of resultant force from 'A' be 'x'.

Moment of all forces about 'A'

$$\Sigma M_A = 15 \sin 33.69° \times 50 + 25 \cos 70° \times 40$$
$$+ 20 \sin 30° \times 60 - 20 \cos 30° \times 40$$
$$= 46 + 342 + 600 - 692.82$$
$$\Sigma M_A = 665.18 \text{ N-mm } (\circlearrowleft)$$

Since ΣM_A is anti-clockwise, the resultant force should also produce anticlockwise moment about 'A'

To satisfy the direction of resultant force and the nature of anti-clockwise moment, it is taken on right side of 'A"

By Varignon's theorem

Sum of moments about A = moment of resultant force about 'A'

$$\Sigma M_A = R \times x$$
$$x = \frac{\Sigma M_A}{R} = \frac{665.18}{32} = 20.78\text{mm}$$

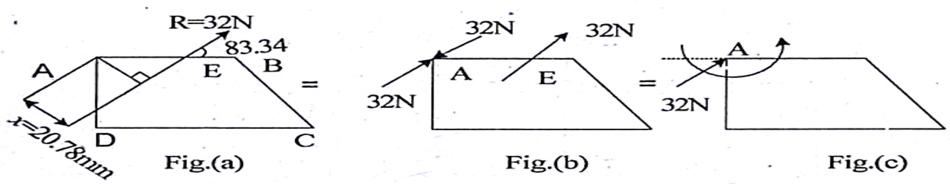


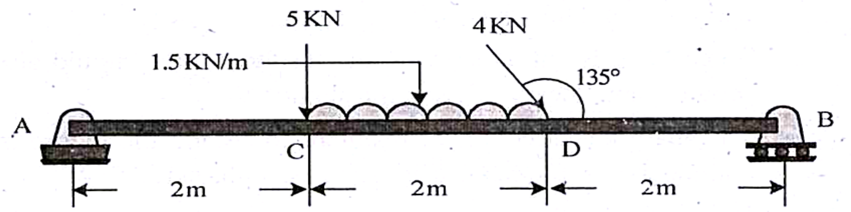
Fig.(a) Fig.(b) Fig.(c)

To find force - couple system at point A:-

The resultant force acting at point E is shown in fig (a).

To reduce the resultant force into force - couple system at A, apply two equal and opposite collinear forces at A, parallel to resultant and of same magnitude (32N) as shown in fig (b).

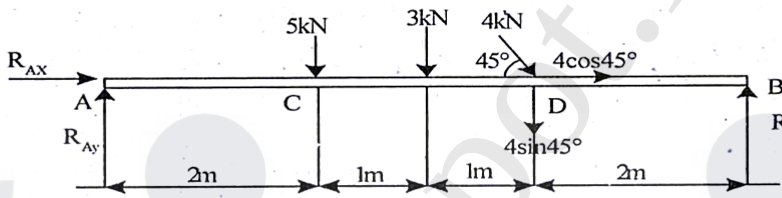
6. A simply supported beam AB of 6m span is loaded as shown A is a hinged support; B is a roller support. Determine the reactions at A and B. **(AU MAY'11)**



Solution :

Let R_{AX} and R_{AY} be horizontal and verticle component of reaction R_A at hinged support A.

Let R_B be the vertical component at B due to roller support.



The uniformly distributed load of 1.51kN/m for a length of 2m is assumed as equivalent point load of (1.5×2= 3kN) and acting at a distance $\frac{2}{2}$ = 1m from 'C'.

Using equations of equilibrium

Taking moments about A,

$\Sigma M_A = 0$

$-5\times2-3\times3-4\sin 45°\times4 + R_B\times6 = 0$

$-10-9-11.31+6R_B = 0$

$\mathbf{R_B = 5.05kN}$ **(Ans)**

$\Sigma F_x = 0$

$R_{Ax}+4\cos 45° = 0$

$R_{Ax} = -3.98$ kN

$\Sigma F_y = 0$

$R_{Ay}-5-3-4\sin 45°+ 4R_B = 0$

$R_{Ay} = 5.778$kN

Magnitude of Reaction, RA $= \sqrt{R_{AX}{}^2 + R_{Ay}2}$

$= \sqrt{(3.98)^2 + (5.77)^2}$

$\mathbf{R_A = 7\ kN}$ **(Ans)**

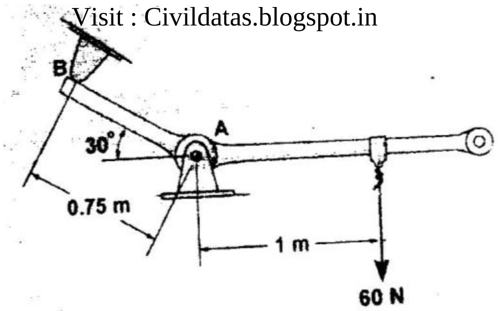
Direction of R_A is

$$\theta = \tan^{-1}\left(\frac{R_{Ay}}{R_{Ax}}\right)$$

$$\theta = \tan^{-1}\left(\frac{5.778}{3.98}\right)$$

$\theta = \mathbf{55.4}$ **(Ans)**

7. A force of 60 N acts on a lever as shown in the figure.

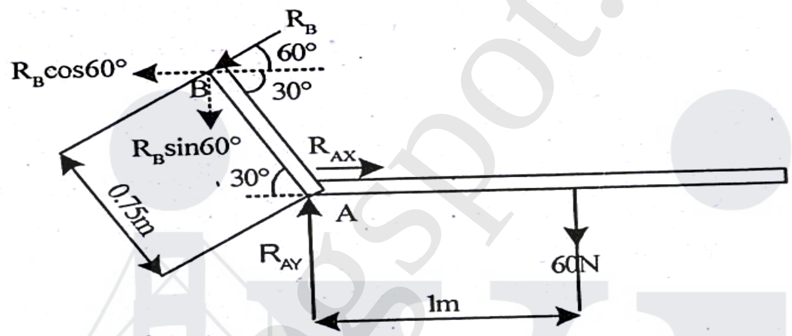


Determine the reactions at A and B.

Solution :

Let R_{Ax} and R_{Ay} be horizontal and vertical component of reaction R_A

Let R_B be the reaction at B due to knife edge support.



Using equations of equilibrium.

Taking moments about A,

$$\Sigma M_A = 0$$

$$R_B\cos60^\circ \times 0.75\sin30^\circ + R_B\sin60^\circ \times 0.75\cos30^\circ - 60\times1 = 0$$

$$0.1875\ R_B + 0.5625\ R_B - 60 = 0$$

$$\mathbf{R_B = 80\ N} \qquad \textbf{(Ans)}$$

$$\Sigma F_x = 0$$

$$-R_B\cos60 + R_{Ax} = 0$$

$$R_{Ax} - 80\cos60 = 0$$

$$R_{Ax} = 40N$$

$$\Sigma F_y = 0$$

$$-R_B\cos60^\circ + R_{Ay} - 60 = 0$$

$$R_{Ay} - 80\sin 60^\circ - 60 = 0$$

$$R_{Ay} = 129.28N$$

Magnitude of reaction $R_A = \sqrt{R^2_{AX} + R^2_{Ay}}$

$$= \sqrt{(40)^2 + (129.28)^2}$$

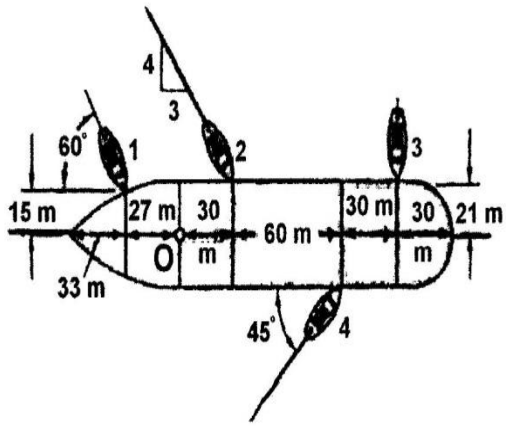
$$\mathbf{R_A = 135.32N} \qquad \textbf{(Ans)}$$

Direction of R_A is $\theta = \tan^{-1}\left(\frac{R_{Ay}}{R_{Ax}}\right)$

$$= \tan^{-1}\left(\frac{129.28}{40}\right)$$

$$\mathbf{\theta = 72.8^\circ} \qquad \textbf{(Ans)}$$

8. Four tugboats are used to bring an ocean large ship to its pier. Each tugboat exerts a 22.5 kNforce in the direction as shown in the figure **(AU Jun'10)**



(i) Determine the equivalent force-couple system at O.

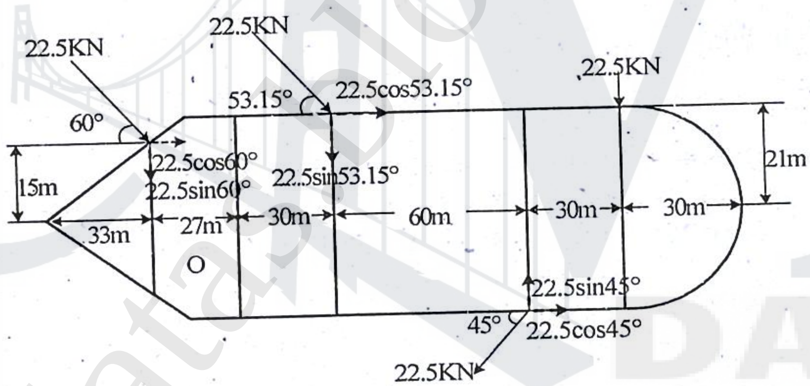
(ii) Determine a single equivalent force and its location along the longitudinal axis of the ship.

Solution:

The inclination of tug boat 2 is

$$\tan\theta_2 = \frac{4}{3} = \theta_2 = 53.13$$

The components of force acting on large ship is shown in the figure.



To find single equivalent force:

Resolving forces horizontally,

$$\Sigma H = 22.5 \cos 60 + 22.5 \cos 53.15 + 22.5 \cos 45$$

$$= 11.25 + 13.49 + 15.9 = 40.64 \text{ kN}$$

Resolving forces vertically

$$\Sigma V = -44.08 \text{ kN}$$

Magnitude of resultant, R is

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

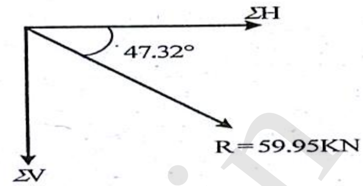
$$= \sqrt{(40.64)^2 + (-44.08)^2} = \mathbf{59.95\ kN}$$

Direction of resultant is

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{44.08}{40.64}\right) = \mathbf{47.32}\ \text{(IV Quadrant)}$$

The line of action of resultant makes an angle 47.32 with the horizontal as shown in the figure.



To find location of resultant:

Let 'x' be the perpendicular distance of resultant force from point 'O'.

Algebric sum of moments of all forces about 'O'

$$\Sigma M_O = 22.5 \sin 60 \times 27 - 22.5 \cos 60 \times 15 - 22.5 \cos 53.15 \times 21 - 22.5 \sin 53.15 \times 30 - 22.5 \times 120 + 22.5 \cos 45 \times 21 + 22.5 \sin 45 \times 90$$

$$= 526.11 - 168.75 - 283.36 - 540.14 - 2700 + 334.1 + 1431.89$$

$$\Sigma M_O = -1400.15 \text{ kN-m (clockwise)}$$

Since ΣM_O is clockwise, the resultant force should also produce clockwise moment about O.

Hence, to satisfy the direction of resultant and the nature of clockwise moment, the resultant force should act on right side of point 'O' as shown in figure (a).

By varignon's theorem,

Sum of moments about 'O' = moment of resultant about 'O'

$$\Sigma M_O = R \times x$$

$$x = \frac{\Sigma M_O}{R} = \frac{1400.15}{59.95} = 23.35 \text{ m}$$

To find equivalent force couple system at 'O':

The give force system is reduced to equivalent force-couple. System at 'O' by the following procedure.

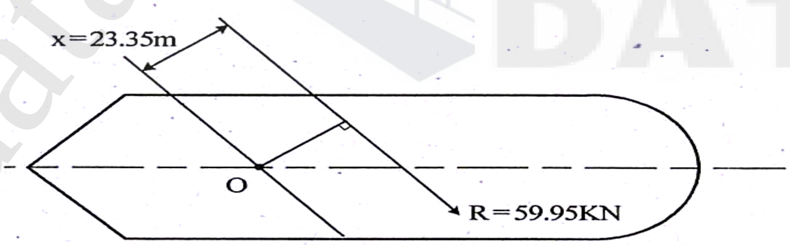


Fig (a)

1. Introduce two equal and opposite forces of magnitude R = 59.95 KN at 'O' as shownin figure.(b)

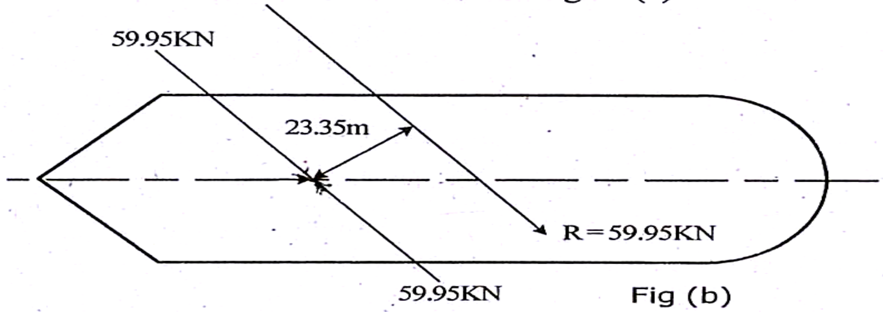


Fig (b)

2) Now, the 59.95 KN acting upwards at O and the downward resultant force forms a clockwise couple about 'O' whose magnitude is

Magnitude of couple = 59.95×23.35

= 1399.8 KN.m **(Ans)**

Figure (c) shows the equivalent force -couple at 'O'.

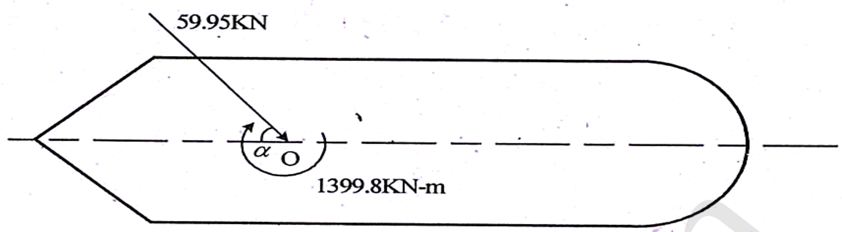
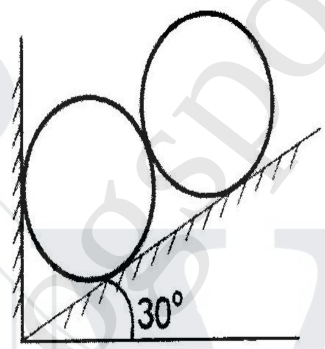


Fig (c)

9. Two identical rollers, each of weight 500N, are supported by an inclined plane making an angle of 30^o to the horizontal and a vertical wall as shown in the figure.**(AU Jun'10,DEC'12)**



(i) Sketch the free body diagrams of the two rollers.

(ii) Assuming smooth surfaces, find the reactions at the support points.

Given:

Weight of two identical rollers, W = 500 N

To Find:

Reactions at supports.

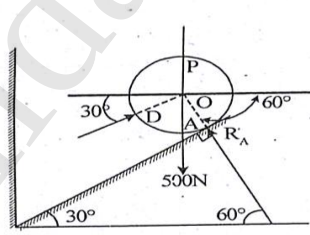
Solution:

Let R_A, R_B and R_C be the reactions at supports.

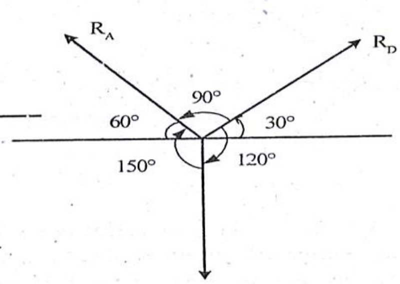
Considering the roller P

The freebody diagram is shown in the figure.

Free body diagram



Force diagram



Applying Lami's theorem

$$\frac{R_A}{\sin 120} = \frac{R_B}{\sin 150} = \frac{500}{\sin 90}$$

$$\frac{R_A}{\sin 120} = \frac{500}{\sin 90} \Rightarrow \mathbf{R_A = 433\ N}$$

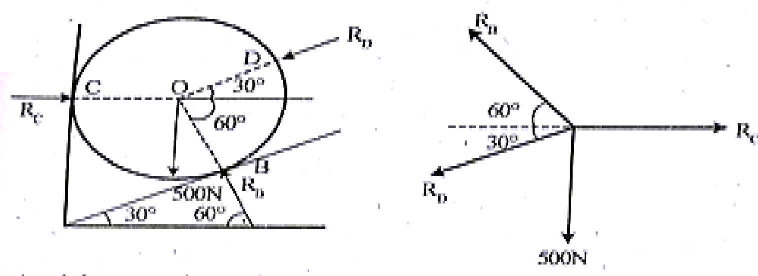
$$\frac{R_D}{\sin 150} = \frac{500}{\sin 90} \Rightarrow \mathbf{R_D = 250\ N}$$

Considering rollers Q,

The free body diagram is shown in the figure.

Free body Diagram

Force Diagram



Applying equations of equilibrium,

$\Sigma F_x = 0$

$$R_C - R_B \cos 60 - R_D \cos 30 = 0$$

$$R_C - 0.5\ R_B - 250 \times 0.866 = 0$$

$$R_C - 0.5\ R_B = 216.5 \quad \text{....(1)}$$

Resolving forces vertically,

$\Sigma F_y = 0$

$$R_B \sin 60 - R_D \sin 30 - 500 = 0$$

$$0.866\ R_B - 250 \times 0.5 - 500 = 0$$

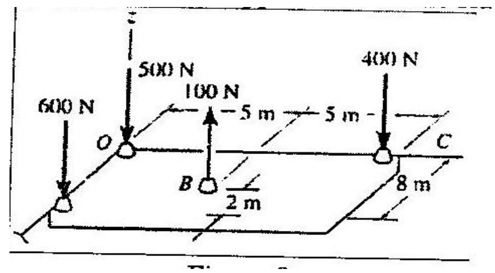
$$R_B = 721.7\ N$$

Put $R_B = 721.7$ in equation (1)

$$R_C - 0.5\ (721.7) = 216.5$$

$$R_C = 577.35\ N$$

10. The slab in figure below is subjected to parallel forces .Determine the magnitude and direction of resultant force equivalent to the given force system and locate its point of application on the slab.**(AU Dec'09)**



To find:

i. Magnitude and direction of resultant force

ii. Location of resultant

Solution:

i. To find magnitude & direction of resultant force:

Resultant, R $= \Sigma F = -600 + 100 - 400 - 500$

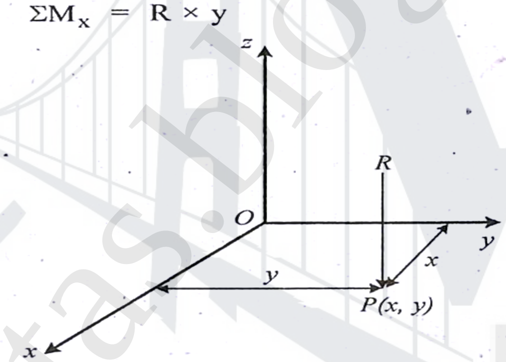
$= -1400 \text{ N} = 1400 \text{ N} (\downarrow)$ **(Ans)**

ii. To find location of resultant, R:

Let Resultant, 'R' acting at the point, P (x, y) in x – y plane.

Taking moment about O' along x-axis and equating the same with moment of resultant,

$$\Sigma M_x = R \times y$$



$$600 \times 0 - 100 \times 5 - 400 \times 10 + 500 \times 0 = -1400 \times y$$

$$500 - 4000 = -1400y$$

$$y = 2.5\text{m} \quad \textbf{(Ans)}$$

Taking moment about O' along y-axis and equating the same with moment of resultant,

$$\Sigma M_y = R \times x$$

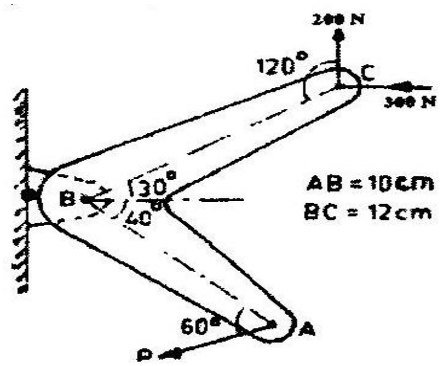
$$600 \times 8 - 100 \times 6 - 400 \times 0 + 500 \times 0 = 1400 \times x$$

$$4800 - 600 = 1400x$$

$$x = 3\text{m} \quad \textbf{(Ans)}$$

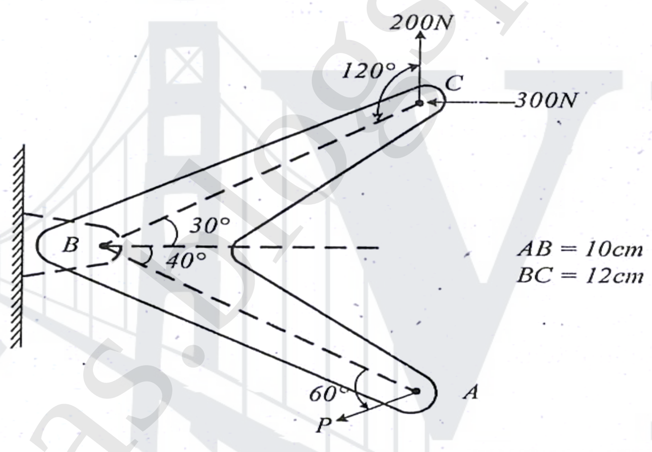
$\therefore$ The resultant R = 1400N acts on x-y plane at (3, 2.5) in downward direction.

11. The lever ABC of a component of machine is hinged at B and is subjected to a system of coplanar forces as shown in figure below. Neglecting friction, find the magnitude of the force (p) to keep the lever in equilibrium. Also determine the magnitude and direction of reaction at B. **(AU Dec’09)**



Solution:

The free body diagram of the lever ABC is shown in the figure below.



To find magnitude of force (P):

Since, the lever is in equilibrium, taking moments about B and equate to zero.

$$\Sigma M_B = 0$$

$$200 \times 12 \cos 30^o + 300 \times 12 \sin 30^o - P \cos 20^o \times 10 \sin 40^o - P \sin 20^o \times 10 \cos 40^o = 0$$

$$2078.46 + 1800 - 6.04 P - 2.62 P = 0$$

$$3878.46 = 8.66 P$$

$$P = \mathbf{447.8\ N} \quad \textbf{(Ans)}$$

To find magnitude of reaction at B:

using, $\Sigma F_x = 0$

$$-300 + R_{BX} - P\cos 20^o = 0$$

$$R_{BX} - 300 - 447.8 \times \cos 20 = 0$$

$$R_{BX} = 720.8\ N$$

using, $\Sigma F_y = 0$

$$200 + R_{BY} - P\sin 20^o = 0$$

$$R_{BY} + 200 - 447.8 \times \sin 20 = 0$$

$$R_{By} = 46.84\ N$$

Reaction at B, $R_B = \sqrt{R_{BX}^2 + R_{BY}^2}$

$$= \sqrt{(720.8)^2 + (46.84)^2}$$

$R_B = 722.3\ N$ **(Ans)**

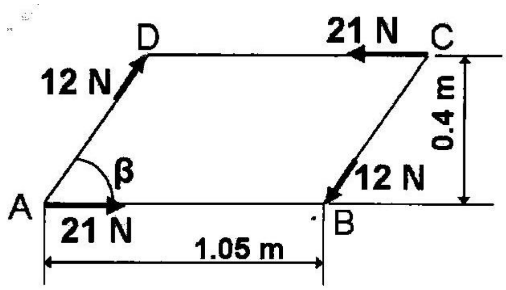
Direction of R_B is

$$\theta = \tan^{-1}\left(\frac{R_{BY}}{R_{BX}}\right)$$

$$= \tan^{-1}\left(\frac{46.84}{720.8}\right)$$

θ = **3.71^o** **(Ans)**

12. A plate ABCD in the shape of a parallelogram is acted upon by two couples, as shown in the figure

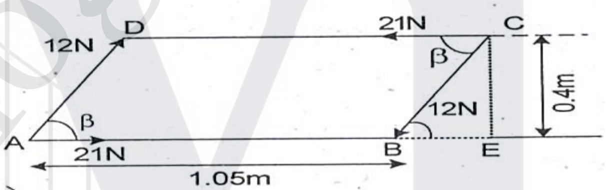


Determine the angle β if the resultant couple is 1.8 N.m clockwise.**(AU Dec'10,JUN'12)**

Solution :

In angle CBE,

$$\tan\ \beta = \frac{0.4}{BE}$$



$$BE = \frac{0.4}{\tan\beta}$$

$$AE = AB + BE$$

$$= \left(1.05 + \frac{0.4}{\tan\beta}\right)$$

Now taking moments about 'A' we get,

$21 \times 0.4 + 12\ \cos\beta \times 0.4 - 12\ \sin\beta \times AE$

It is given that resultant couple = 1.8 N.m

$$21 \times 0.4\ \ 12\cos\beta \times 0.4 - 12\ \sin\beta\left(1.05 + \frac{0.4}{\tan\beta}\right) = -\ 1.8$$

$$8.4 + 4.8\ \cos\beta - 12.6\ \sin\beta - 4.8\ \cos\beta = -1.8$$

$$12.6\ \sin\beta = 10.2$$

$$\sin\beta = \frac{10.2}{12.6}$$

$$\therefore\ \beta = 54^\circ\ \ \textbf{(Ans)}$$