$\left(1 \times 4 \times \frac{4}{2}\right) - [8 \times (12 - 5.33)] - (V_B \times 12) = 0$

$$12\, V_B = 8 + 53.36$$

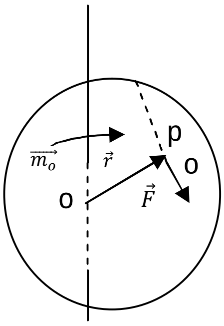
$$V_B = 5.11\, KN\ (\uparrow)$$

Substitute $V_B\ in\ (1)$

$V_A = 6.89\, KN$

EQUILIBRIUM OF RIGID BODIES IN THREE DIMENSIONS

Moment of a figure about a point



$$If\ \overrightarrow{r}\ and\ \overrightarrow{f}\ are\ giving\ by$$

$$\overrightarrow{r} = xi + yj + zk$$

and

$$\vec{f} = f_x i + f_y j + f_z k$$

Then moment

$$\vec{m} = \vec{r} \times \vec{f}$$

But $\vec{m} = m_x i + m_y j + m_z k$

Writing $\vec{m} = \vec{r} \times \vec{f}$

$$\vec{m} = \begin{vmatrix} i & j & k \\ x & y & z \\ f_x & f_y & f_z \end{vmatrix}$$

$$\vec{m} = i(f_z y - f_y z) + j(f_x z - f_z x) + k(f_y x + f_x y)$$

$$m_x = f_z y - f_y z;\ m_y = f_x z - f_z x;\ m_z = f_y x - f_x y$$

Magnitude of moment , $m = \sqrt{m_x^2 + m_y^2 + m_z^2}$

Direction of moment $\vec{m}$

Let the moment of $\vec{m}$,makes angles Φ_x, Φ_y, Φ_z about x,y and z axes

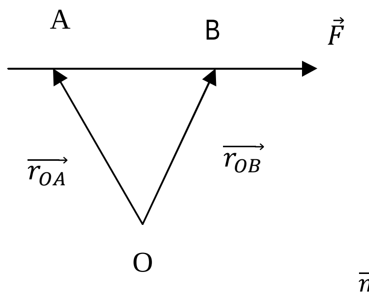
Then

$$\cos \Phi_x = \frac{m_x}{m} \quad \Rightarrow \quad \Phi_x = \cos^{-1}(\frac{m_x}{m})$$

lll[ly] $\quad \Phi_y = \cos^{-1}\frac{m_y}{m} \quad and\ \Phi_z = \cos^{-1}(\frac{m_z}{m})$

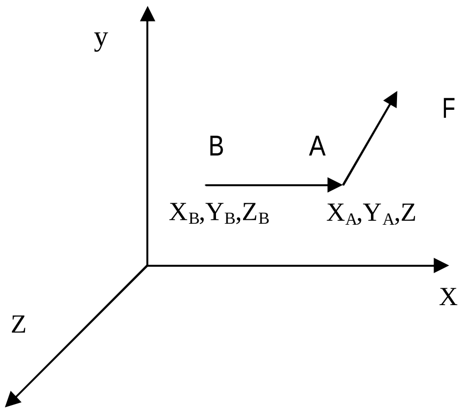
Note:

1.The point P may be taken any where on the line of action of $\vec{f}$



$$\overrightarrow{m_O} = \overrightarrow{r_{OA}} \times \vec{F} = \overrightarrow{r_{OB}} \times \vec{F}$$

2. In case , if moment about any arbitrary point B, of force $\vec{F}$ acting at A is required , the relative position vector of A , with respect to B should be used (write this as $\frac{r_A}{B}$)



Case 1:When position vector of A and B are known

$$\overrightarrow{m_B} = \overrightarrow{r_{\frac{A}{B}}} \times \vec{F}$$

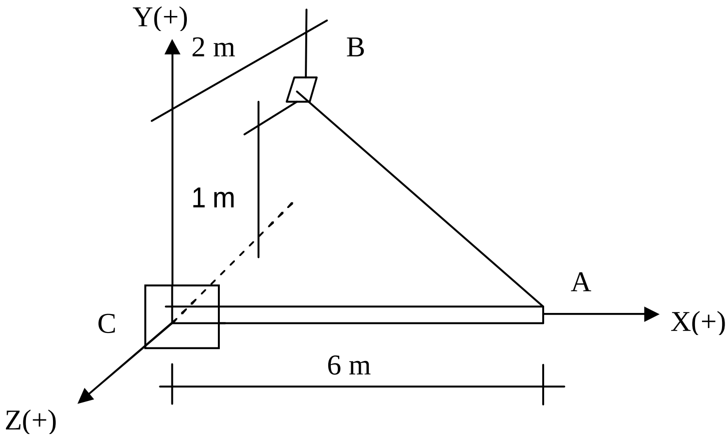
$$= (\overrightarrow{r_{OA}} - \overrightarrow{r_{OB}}) \times \vec{F}$$

Case 2: when coordinates of A and B are known

Then $\overrightarrow{m_B} = \begin{vmatrix} i & j & k \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ f_x & f_y & f_z \end{vmatrix}$

Problem 24: A pipe AC, 6m long is fixed at C , and strectched by a cable from A to a point B on the vertical wall as shown in fig.If the tension in the cable is 400N, determine

(i)The moment of the force exerted at A about C and
(ii)The moment of the force exerted at B about C



Soln:

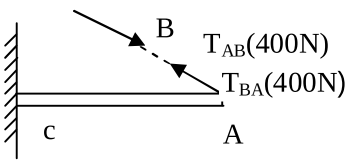
Tension T_{AB} of 400 N acts from A to B and Tension T_{BA} of same magnitude acts from B to A are the collinear from and the cable is in equilibrium.

T_{BA} produces clockwise moment about C , and

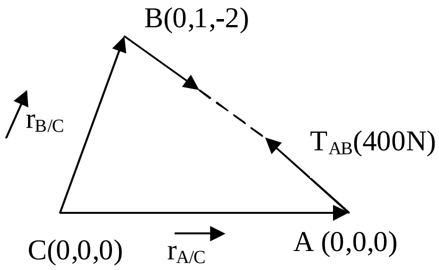
T_{AB} produces anticlockwise moment about C.

But magnitude of these two moments will to equal

Freebody diagram:



i)moment of force executed at A about C



In this case ,the force is directed from A to B

co-ordinates of A (6,0,0)

co-ordinates of B(0,1,-2)

co-ordinates of C(0,0,0)

writing in vector from (T_{AB})

$$\overrightarrow{T_{AB}} = T_{AB} \times \lambda_{AB}$$

Now

$$\lambda_{AB} = \frac{(0-6)i+(1-0)j+(-2-0)k}{\sqrt{(-6)^2+1^2+(-2)^2}}$$

$$\lambda_{AB} = \frac{-6i+1j-2k}{6.4}$$

$$\overrightarrow{T_{AB}} = T_{AB} \cdot \lambda_{AB} = 400\left[\frac{-6i+1j-2k}{6.4}\right]$$

$$\overrightarrow{T_{AB}} = -375\, i + 62.5\, j - 125\, k$$

lll[ly] $\overrightarrow{r_{AC}} = (6-0)i + 0\,j + 0\,k = 6i$

∴ Moment about C,

$$\overrightarrow{m_C} = \overrightarrow{r_{AC}} \times \overrightarrow{T_{AB}}$$

$$= 6\,i \times (-375\,i + 62.5\,j - 125\,k)$$

$$= \begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ -375 & 62.5 & -125 \end{vmatrix}$$

$$\overrightarrow{m_C} = 750\,j + 375\,k$$

<u>(ii) moment of force exerted at B about C</u>

$$\lambda_{BA} = \frac{(6-0)i+(0-1)j+(0+2)k}{\sqrt{6^2+1^2+2^2}} = \frac{6i-1j+2k}{6.4}$$

$$\therefore \overrightarrow{T_{BA}} = T_{BA} \cdot \lambda_{BA} = 400\left[\frac{6i - j + 2k}{6.4}\right]$$

$$= 375\,i - 62.5\,j + 125\,k$$

And

$$\overrightarrow{r_{\frac{B}{C}}} = (0-0)i + (1-0)j + (-2-0)k = 1i - 2\,K$$

$$\therefore \overrightarrow{m_C} = \overrightarrow{r_{\frac{B}{C}}} \times \overrightarrow{T_{BA}}$$

$$= (1j - 2k) \times (375\,i - 62.5\,j + 125\,k)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 1 & -2 \\ 375 & -62.5 & 125 \end{vmatrix}$$

$$= (125 - 125)i - (0 + 750)j + (0 - 375)k$$

$$\overrightarrow{m_C} = -750\,j - 375\,k$$