UNIT-II

EQUILIBRIUM OF RIGID BODIES

Statics of Rigid bodies in Two Dimensions

So far we have analyzed the force systems on a particle in which we assumed each body as a particle. This assumption is valid if the body has negligible size (or) subjected to external forces which are concurrent with its center of gravity.

But in many cases this assumption may not be possible. "When a body is not subjected to collinear /or concurrent force system then the body to be idealized as a rigid body".

In rigid body deformations are small due to the non-concurrent force system the body may translate or rotate with respect to a point or an axis.

Resultant force of parallel forces

In collinear force system the resultant force also lies on the same line of action of the given forces and in concurrent force system the resultant force also passes through the point of intersection of forces.

But in parallel force system location of resultant force is also required. To find the location of resultant force, "Principle of moments "is applied.

Moment of a force

Moment of a force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from that point.

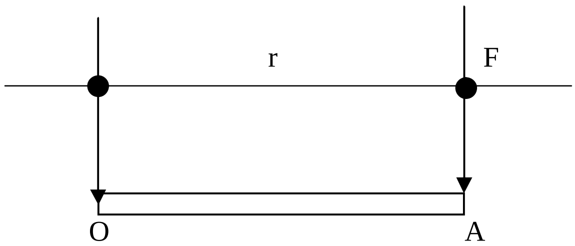


Figure 1

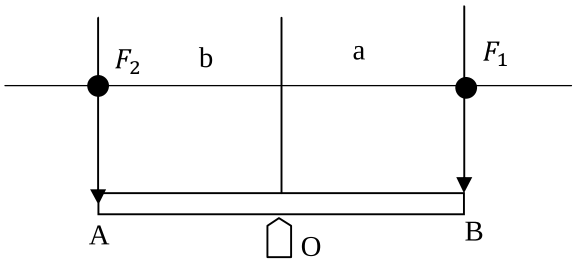


Figure 2

The moment (m) of the force F about 'o' is given b (fig1)

$$M_o = F \times r$$

Hence moment is also defined as turning effect produced by a force

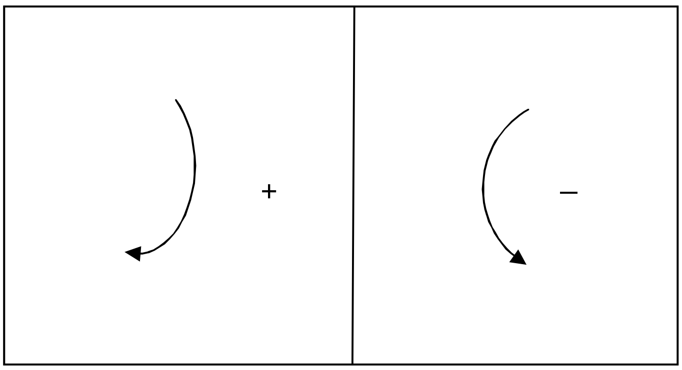
For fig 2

For $F_1, M_o = F_1 \times a$ ($clockwise$ ↓)

For F_2,, $M_o = F_2 \times b$ ($Anticlockwise$ ↓)

Sign convention

In the proceeding articles, we use positive sign for clockwise moment and negative sign for anticlockwise moment

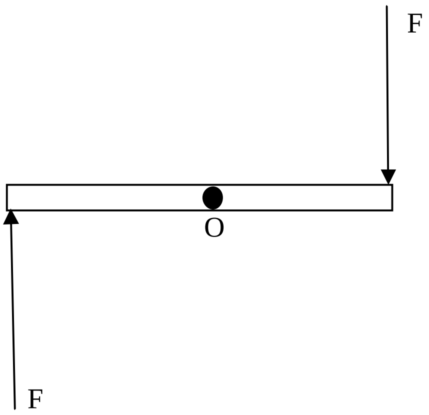


Unit of moment

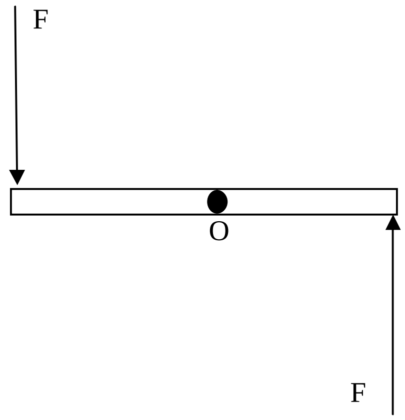
In S.I system unit of moment is Newton -meter (Nm)

Force is measured in Newton and the distance is measured in meter.

Moments of vertical forces

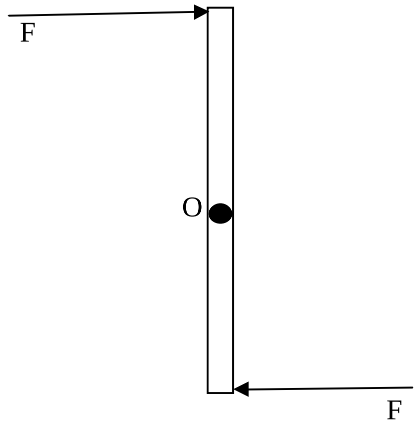


Clockwise moment

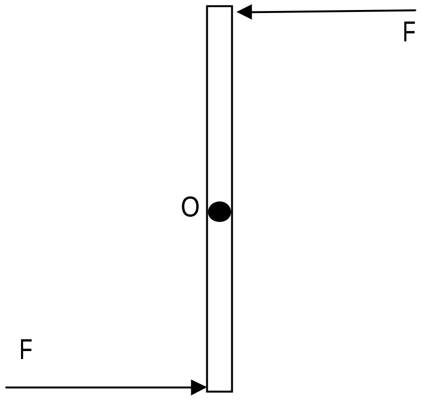


Anticlockwise moment

<u>Moment horizontal forces of</u>

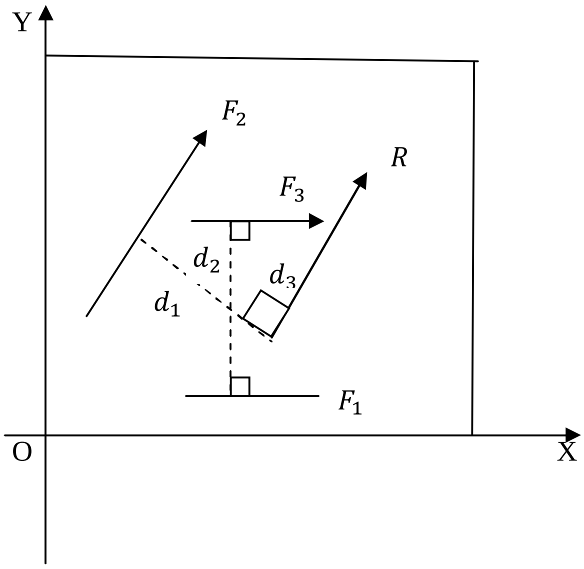


Clockwise moment



Anticlockwise moment

Varignon's theorem



"The algebric sum of the moments of any number of forces about any point in their plane is equal to the moment of their resultant about the same point". Varignon's theorem is also known as theorem of moments.

Consider a rigid body is subjected to three coplanar forces F_1, F_2 *and* F_3 as shown in figure at perpendicular distance d_1, d_2 *and* d_3 from a point$'o'$. Let the resultant force R is at a distance $'d'$ from $'o'$

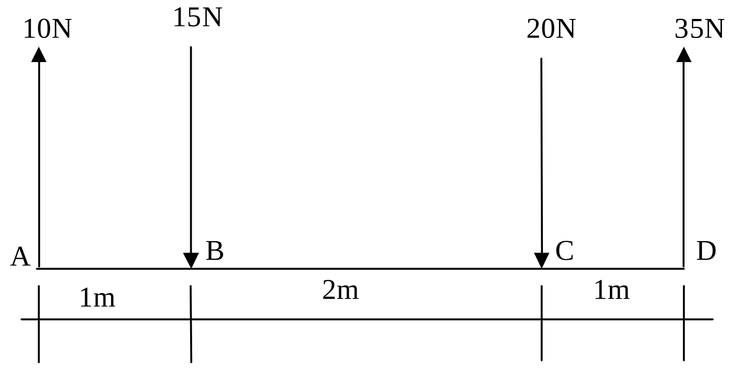
From Varignon's theorem

$$F_1 d_1 + F_2 d_2 + F_3 d_3 = R.d$$

Sum of the moment of all the forces about a point= moment of their resultant force about the same point.

Problem 1:

Four parallel forces of magnitudes 10N, 15N, 20N and 35N are shown in fig. determine the magnitude and direction of the resultant. Find also the distance of the resultant from point A



Solution

$$Magnitude\ of\ the\ resultant\ force\ R = 10 - 15 - 20 + 35 = 10N$$

Direction of the resultant; upwards (R is positive)

Location of the resultant force

From Varignon's theorem

$$\sum m_A = R \times x (moment\ of\ the\ resultant\ force)$$

Algebric sum of the moments about A

$$\therefore \sum m_A = (10 \times 0) + (15 \times 1) + (20 \times 3) - (35 \times 4)$$

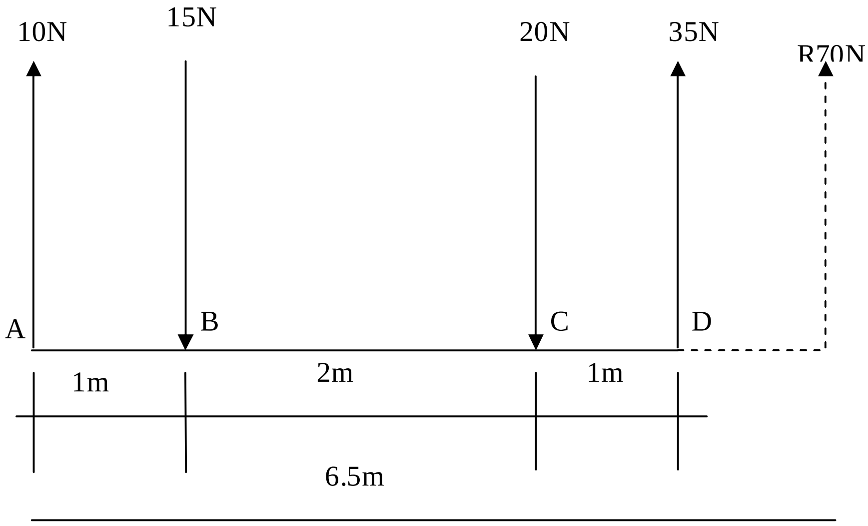
$$= 15 + 60 - 140 = -65 Nm (satisfied)$$

Since

$$\sum m_A = R \times x$$

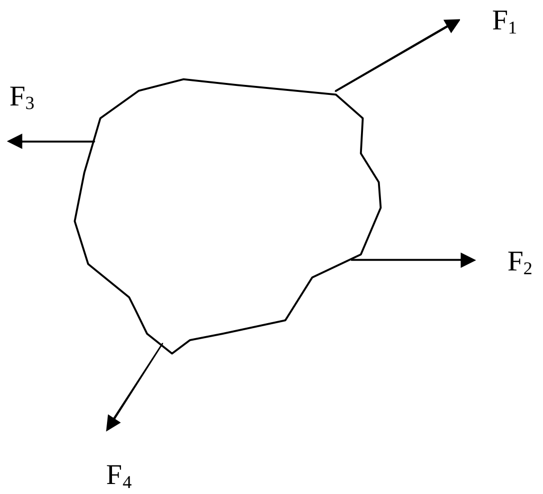
$$65 = 10 \times x$$

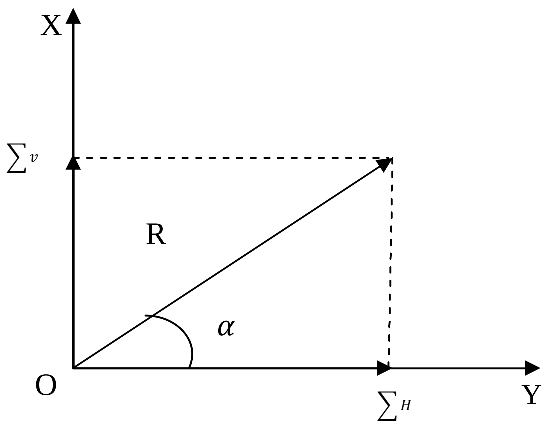
$$x = 6.5m$$



Resultant force of Nm –concurrent & non parallel forces

The magnitude and direction of resultant force can be determined by analytical method as same for concurrent force system. But, the location of resultant force of non concurrent and non parallel force system is determined by the concept of moment and Varignon’s principle.





Magnitude of resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum v\right)^2}$$

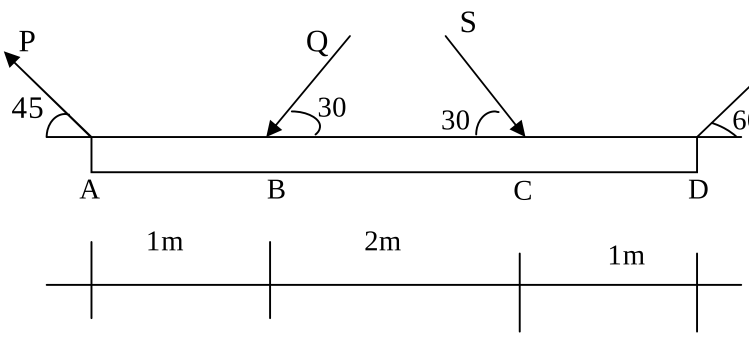
Direction of resultant force $\alpha = \tan^{-1}\left[\frac{\sum H}{\sum v}\right]$

Location of resultant

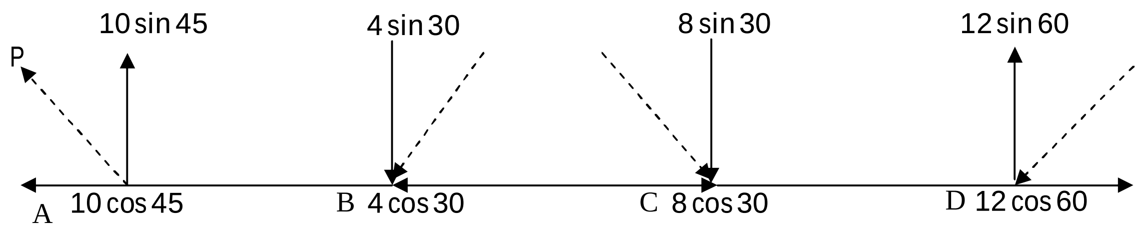
$$\sum m = R \times x$$

Problem 3:

ABCD is a weightless rod under the action of forces P, Q, S and T as shown in fig. if $P = 10N, Q = 4N, S = 8N$ and $T = 12N$, calculate the resultant and mark the same in direction with respect to the end A of the rod.



Solution:



Algebric sum of horizontal components

$$\sum H = -10\cos 45 - 4\cos 30 + 8\cos 30 + 12\cos 60$$

$$= -7.071 - 3.464 + 6.928 + 6 = 2.393N$$

Algebric sum of vertical components

$$\sum v = -10\sin 45 - 4\sin 30 - 8\sin 30 + 12\sin 60$$

$$= 11.463N$$

Magnitude of the resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum v\right)^2}$$

$$R = \sqrt{(2.393)^2 + (11.453)^2} = 11.71N$$

Direction of the resultant force

$$\alpha = \tan^{-1}\left(\frac{\sum H}{\sum v}\right) = \tan^{-1}\left(\frac{11.463}{2.393}\right) = 78.2$$

<u>Location of resultant force</u>

Let us locate the resultant from the point A. Z-Perpendicular distance of resultant force from A.

$$\sum m_A = R \times Z$$

Algebric sum of the moments about A

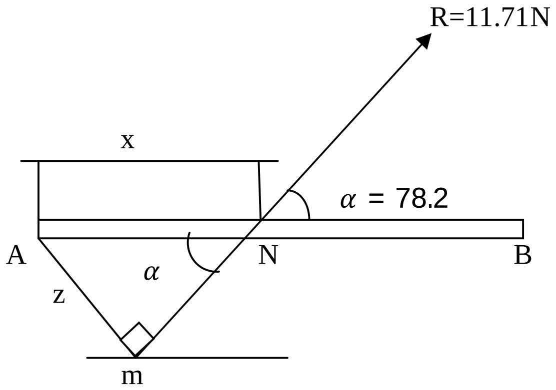
$$\sum m_A = (-10\sin 45 \times 0) + (4\sin 30 \times 1) + (8\sin 30 \times 2) - (12\sin 60 \times 3)$$

$$\sum m_A = -21.176Nm$$

(Moments of all other forces about the point A are zero, as they are passing through it).

(-) sign,

Hence, resultant force should also produce anticlockwise moment about A. Hence R should be taken on the right side A.



$$\sum m_A = R \times Z$$

$$21.176 = 11.71 \times Z$$

$$Z = 1.808m$$

To find x

In triangle AMN

$$AMN = 90$$

$$AMN = 78.2(\alpha)$$

$$\sin 78.2 = \frac{Am}{AN} = \frac{Z}{x}$$

$$x = \frac{Z}{\sin 78.2} = \frac{1.808}{\sin 78.2} = 1..847m$$

<u>Alternate method</u>

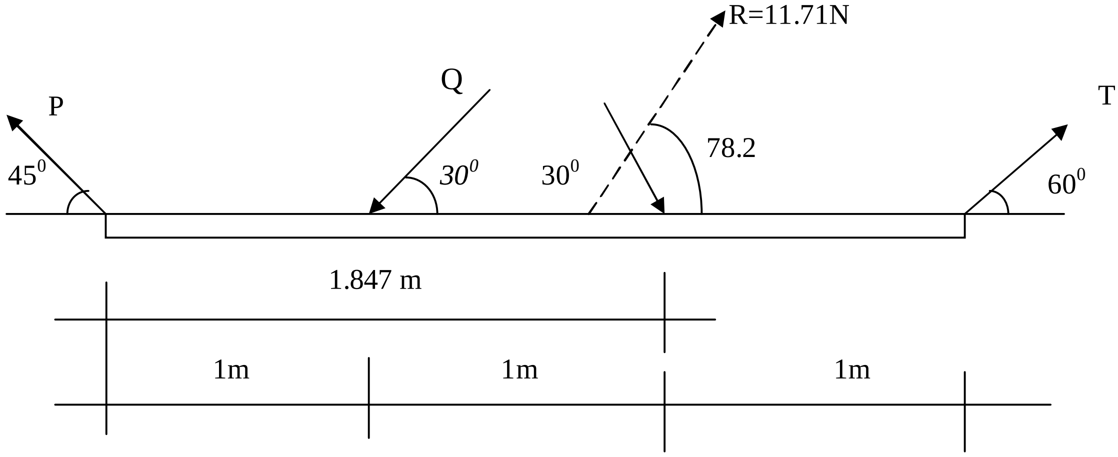
$$\sum M_A = R \times x$$

$$\sum M_A = \left(\sqrt{\left(\sum H\right)^2 + \left(\sum v\right)^2}\right) \times x$$

(moment of $\sum H$ about A is zero)

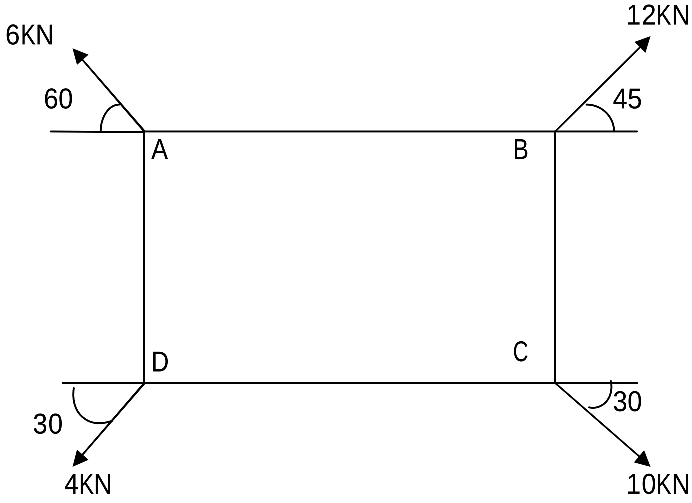
$$\sum M_A = \sum v \times x$$

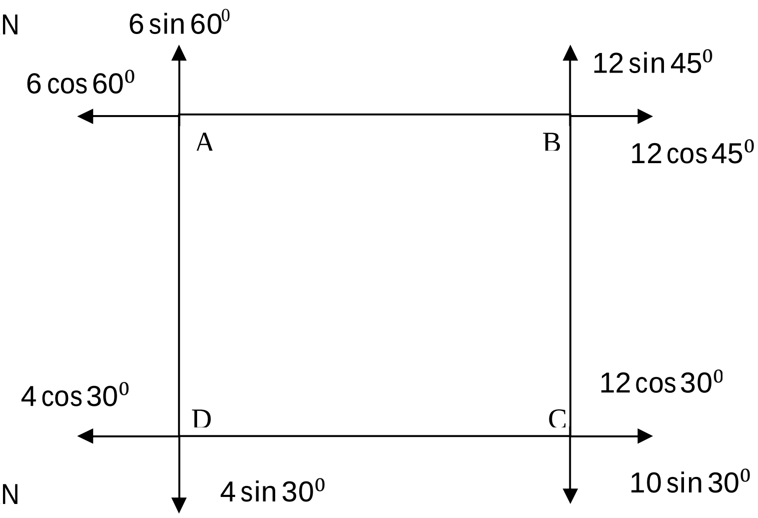
$$x = \frac{\sum M_A}{\sum v} = \frac{21.176}{11.463} = 1.847m$$



Problem 4:

Four forces of magnitude and direction acting on a square ABCD of side 2m one shown in fig. calculate the resultant in magnitude and direction and also locate its point of application with respect to the sides AB and AD





Solution:

Algebric sum of horizontal forces

$$\sum H = 12\cos 45 + 10\cos 30 - 4\cos 30 - 60\cos 60 = 10.681KN$$

Algebric sum of vertical forces

$$\sum v = 12\sin 45 - 10\sin 30 - 4\sin 30 + 60\sin 60$$

$$= 6.681KN$$

Magnitude of the resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum v\right)^2} = \sqrt{(10.681)^2 + (6.681)^2} = 12.598KN$$

Direction of the resultant force

$$\alpha = \tan^{-1}\left(\frac{\sum v}{\sum H}\right) = \tan^{-1}\left(\frac{6.681}{10.681}\right) = 32$$

Location of the resultant force

$$\therefore \sum m_A = (4\cos 30 \times 2) + (10\sin 30 \times 2) - (10\cos 30 \times 2) - (12\sin 45 \times 2)$$

$$= -17.36KNm\ (Anticlockwise)$$

Hence, to have anticlockwise moment by the resultant force, R is to be taken on the right hand side of A.

Location of resultant force w.r.t AB

Resolve the resultant force into two components $\sum H$ and $\sum V$ at m.

$$\sum m_A = R \times x$$

$$= \sqrt{(\sum H)^2 + (\sum V)^2} \times x$$

as $\sum H$ moment about A is zero.

So, $\sum m_A = \sum V \times x$

17.36 = 6.681 × x

x=2.598m

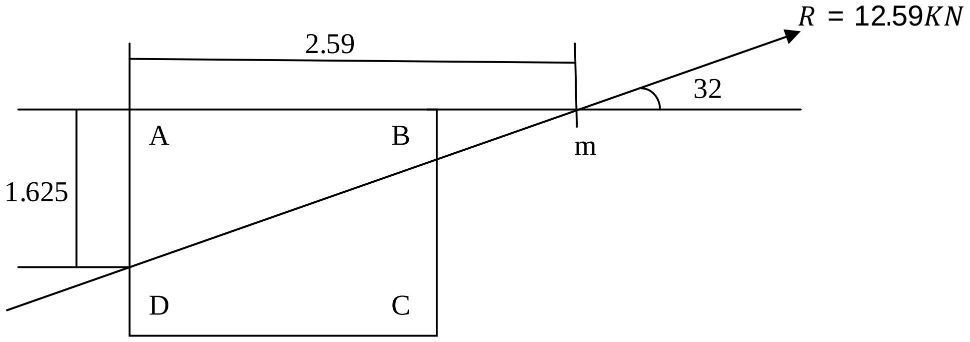
Location of Resultant force w.r.t AD

Resolve the force

$\sum m_A = \sum H \times y$ (As moment of $\sum V$ about A is zero)

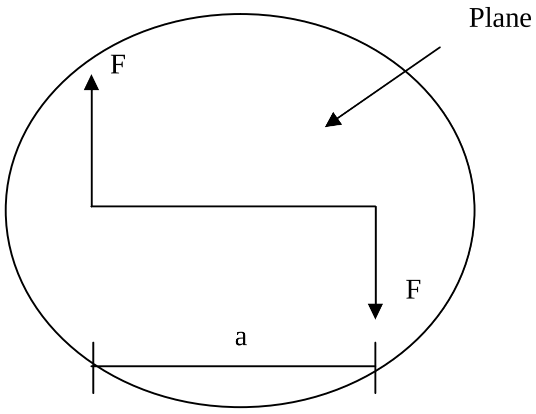
17.36 = 10.681× y

y = 1.625m



Statistics of Rigid bodies – Force couple system

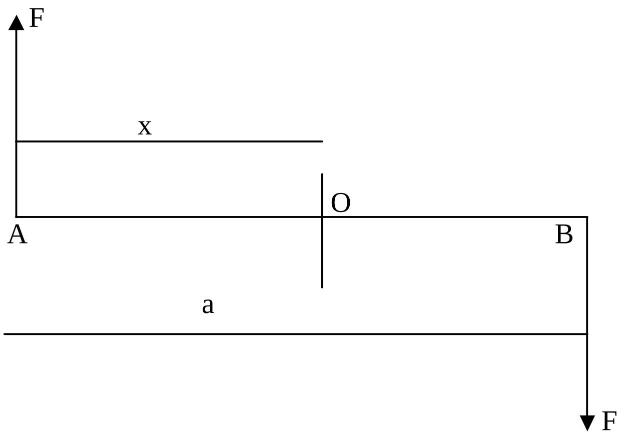
Couple:



Two forces F and –F having the same magnitude , parallel lines of action and opposite sense one said to form a couple.

-Couple has a tendency to rotate the body

-The perpendicular distance between the parallel forces is called arm of the couple.



Here OA = x; OB = (a-x)

Sum of moments at A

$\sum m_A$=F× $a()$

Sum of moments at B

$\sum m_B$=F× a

Sum of moments at o

$\sum m_o$= (F$\times$ x) + (F$\times$ $(a - x)$)

=F$\times$ $x + F \times a - F \times x$

$\sum m_o$= Fa ()

The sum of the moments of couple forces about any point is same magnitude and nature.

Moment of a couple = Force × Arm of the couple

m=F×a

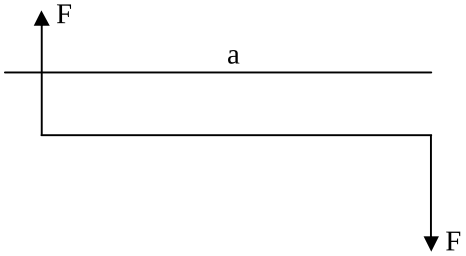
Difference between moment and couple

The couple is a pure turning effect which may be moved anywhere in its own plane or into a parallel plane without change of its effect of the body, but the moment of a force must include a description of the reference axis about which the moment is taken.

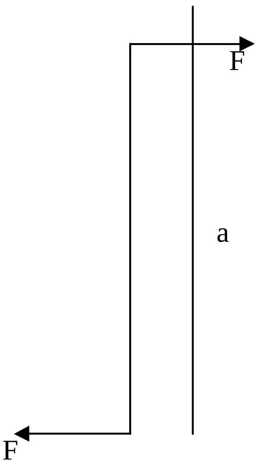
Types of Couple:

Based on its nature

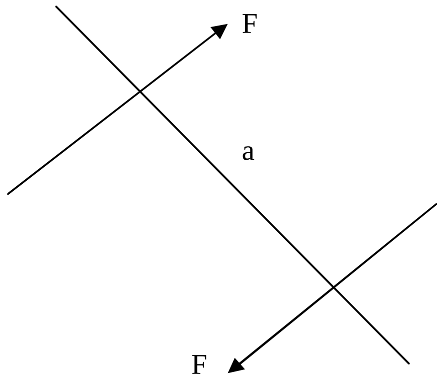
(i) Clockwise couple
(ii) Anticlockwise couple



Vertical

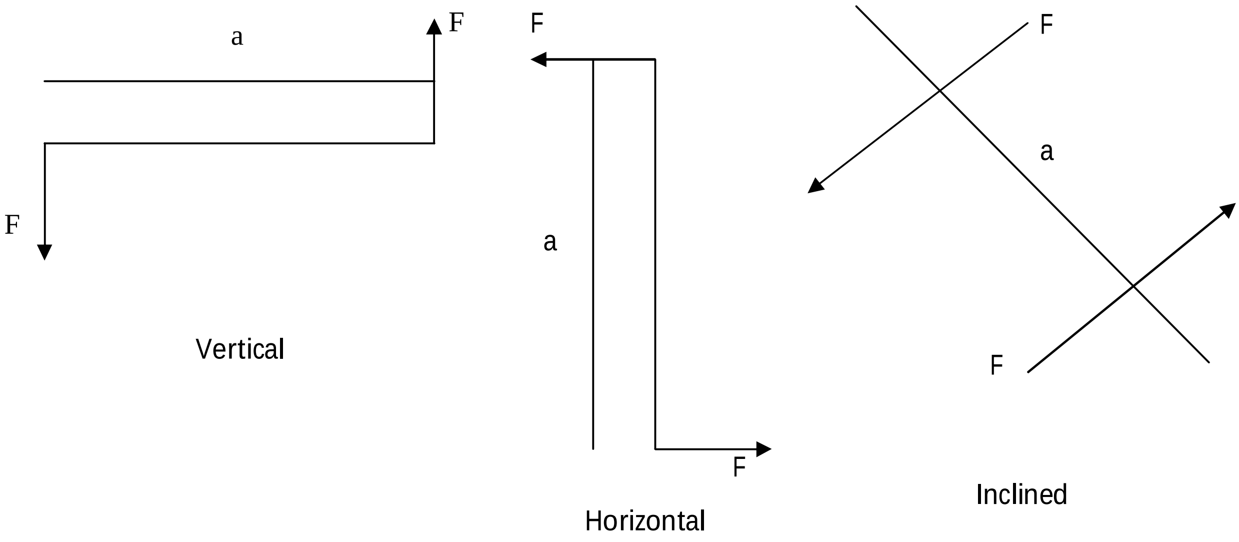


Horizontal



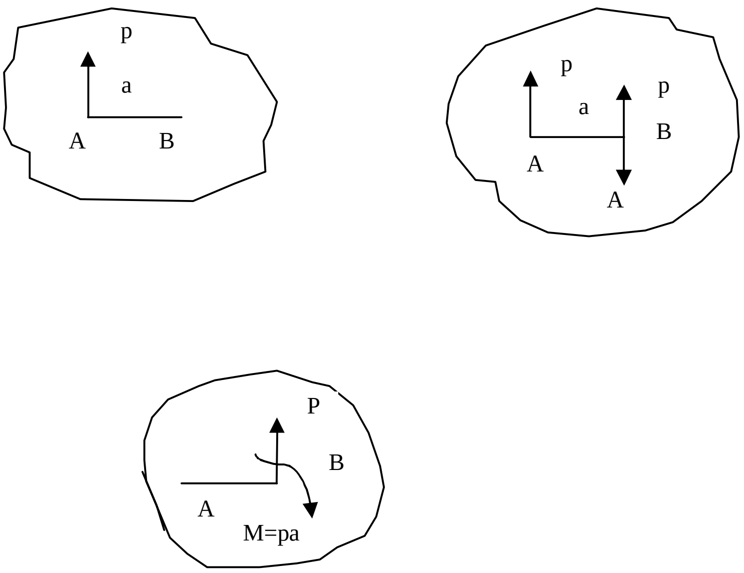
Inclined

Clockwise couple



Anticlockwise couple

Resolution of a force into a force and a couple at a point



Principle of transmissibility of forces

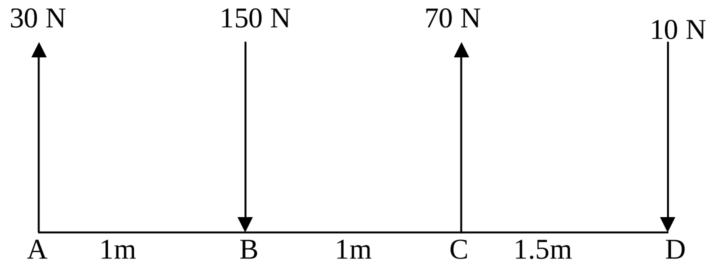
If a force acts at any point on a rigid body it may also be considered to act at any other point on its line of action.

Problem 6:

A system of parallel forces are acting on rigid bar as shown in fig. Reduce the system to

(i) A single force
(ii) A single force and a couple at A

(iii) A single force and a couple at B.



Solution:

(i) Single force system

The single force system will consist only resultant force.

Magnitude of resultant force R = 30-150+70-10

R=-60N

Direction of Resultant: Vertical & Downwards (as R in negative)

Location of Resultant force

Sum of all moments about A $\sum m_A = R \times x$

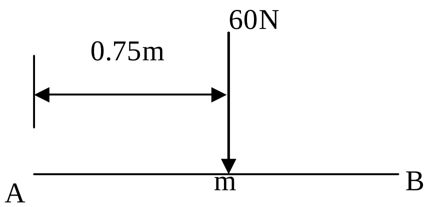
$$\sum m_A = (30 \times 0) + (150 \times 1) - (70 \times 2) + (10 \times 3.5)$$

$$\sum m_A = 45\ Nm\ (Clockwise)$$

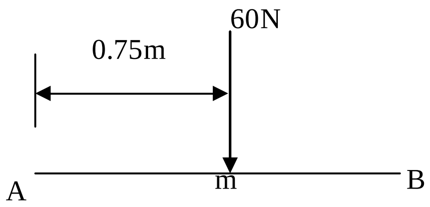
$$\sum m_A = R \times x$$

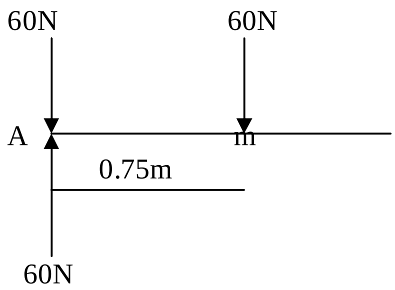
$$45 = 60 \times x$$

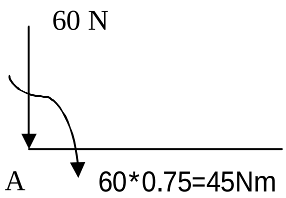
$$x = 0.75m$$



(ii) A Single force and a couple at A

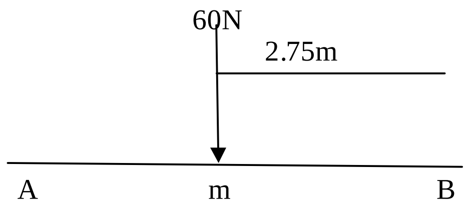


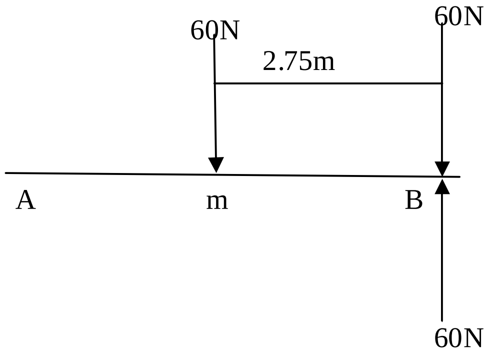


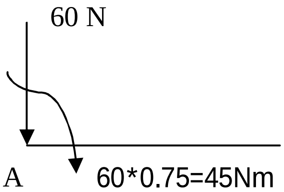


Clockwise couple

(iii) <u>A single force and a couple at B</u>





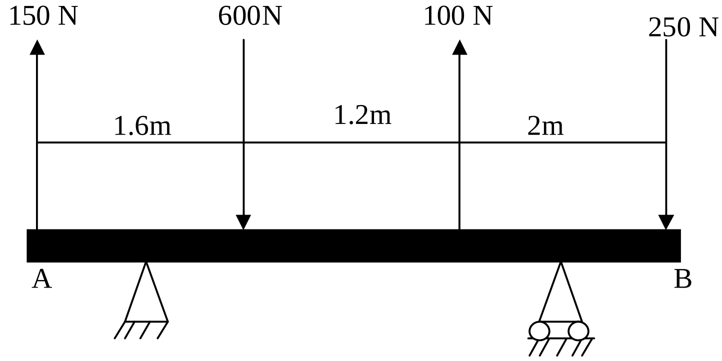


Anticlockwise couple

Problem 7:

A 4.8m beam is subjected to the forces shown in fig. Reduce the given system of forces to

(i) A single force
(ii) An equivalent force –couple system at A
(iii) Force couple system at B.



Solution:

(i) A single force (or Resultant force)

Magnitude of Resultant, R = 150-600+100-250

= -600N

Direction of Resultant force: Vertically downwards (R is (-))

Location of Resultant force

Algebric sum of moments of all forces $\sum m_A = (150 \times 0) + (600 \times 1.6) - (100 \times 2.8) + 250(1.6 + 1.2 + 2)$

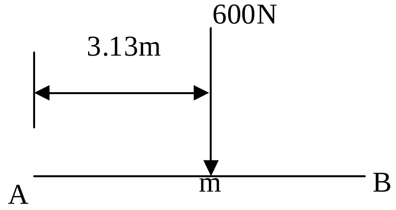
= 1880 Nm (Clockwise)

Sum of moments about A = moment of Resultant force about A

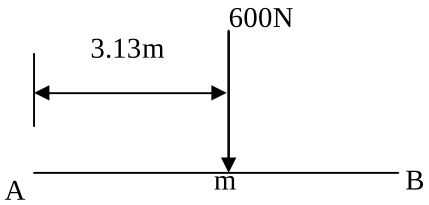
1880 = R× x

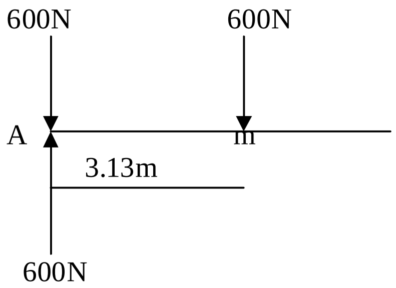
1880 = 600× x

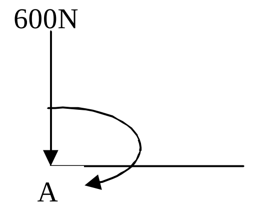
$x = 3.13m$



(ii) <u>Force -Couple system at A</u>

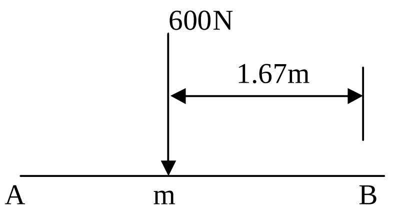


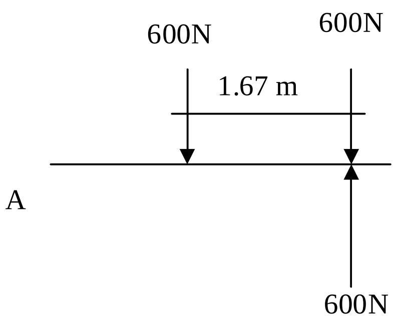


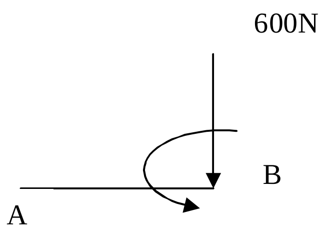


60X3.13=1878Nm

(iii) <u>Force – Couple system at B</u>



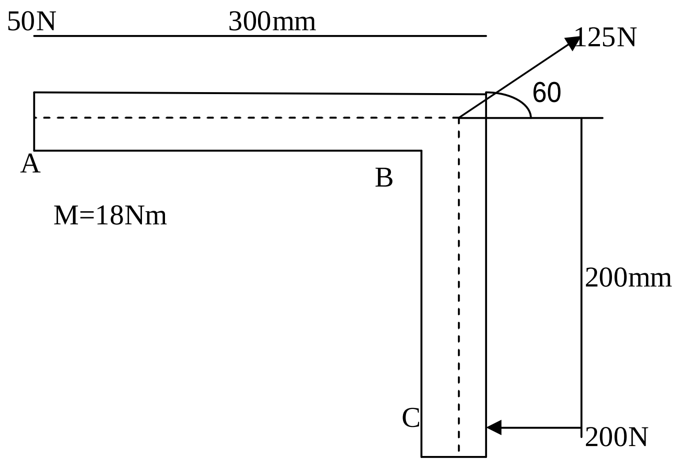


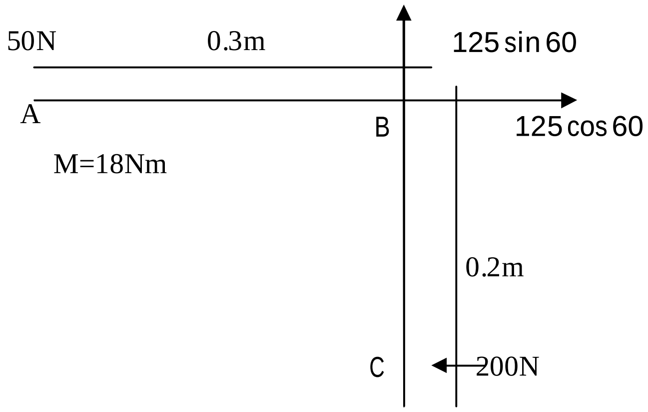


600X1.67=*1002Nm*

<u>Problem 10:</u>

The three forces and a couple of magnitude m = 18 Nm are applied to an angled bracket as shown in fig. (i) Find the resultant of this system of forces (ii) Locate the points where the line of action of the resultant intersects line AB and line BC.





Solution:

(i) Resultant force

Algebric sum of Horizontal forces,

$$\sum H = 125\cos 60 - 200 = -137.5\ N$$

Algebric sum of Vertical forces

$$\sum V = 125\sin 60 - 50 = 58.25\ N$$

Magnitude of resultant force

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(137.5)^2 + (58.25)^2} = 149.32\ \text{N}$$

Direction of the resultant force

$$\propto = tan^{-1}\left(\frac{\sum V}{\sum H}\right) = tan^{-1}\left(\frac{58.25}{137.5}\right) = 22.95$$

Location of the resultant force

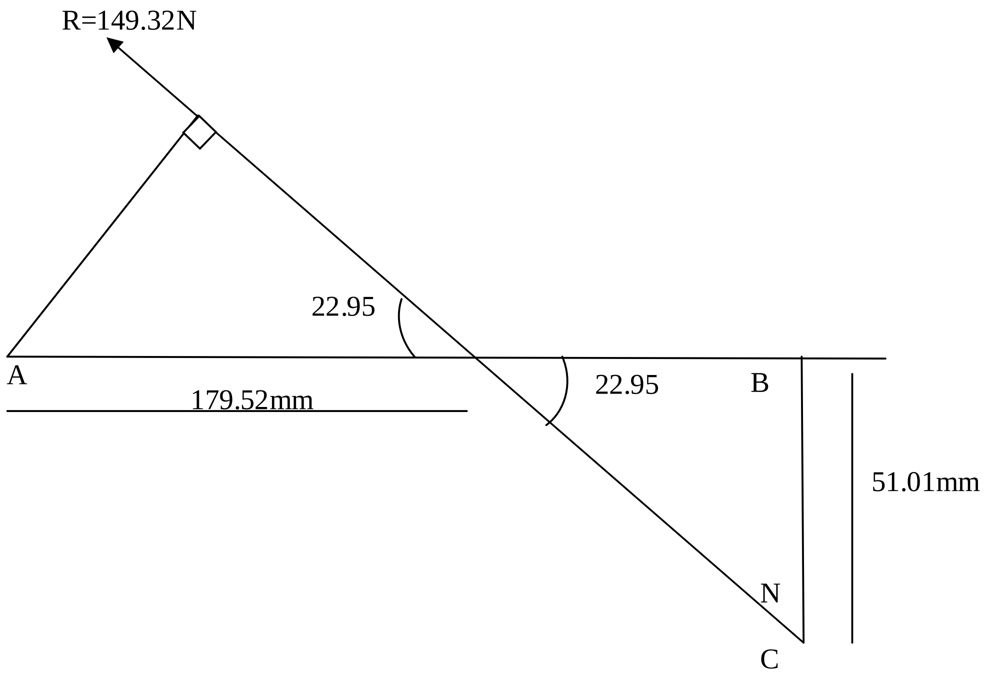
Sum of moments $\sum m_A = -(125 \sin 60 \times 0.3) - 18 + (200 \times 0.2)$

$$= -10.475\ Nm\ (Anticlockwise\ moment)$$

Let x be the perpendicular distance of the resultant from A.

$$\sum m_A = R \times x$$

$$x = \frac{10.475}{149.32} = 0.07\ m = 70\ mm$$



(ii) Intersection of resultant force with line AB and BC

To find AM

In right triangle APM $< APM = 90^\circ$; $< PMA = 22.95^\circ$; $AP = 70\ mm$

$$\sin 22.95 = \frac{AP}{AM} = \frac{70}{AM}$$

$$AM = \frac{70}{\sin 22.95} = 179.52\ mm$$

To find BN

In right angled triangle BMN

$$BM = AB - AM = 300 - 179.52 = 120.48\ mm$$

$$\tan 22.95 = \frac{BN}{BM} = \frac{BN}{120.48}$$

$$BN = 51.01\ mm$$

EQILIBRIUM OF RIGIDBODIES – SUPPORT REACTIONS

Beam:

A beam is a horizontal structural member which carries a load, transverse (perpendicular) to its axis and transfers the load through support reactions to supporting columns or walls.

Frame:

A structure made up of several members, riveted or welded together is known as frame.

Support reactions of Beam:

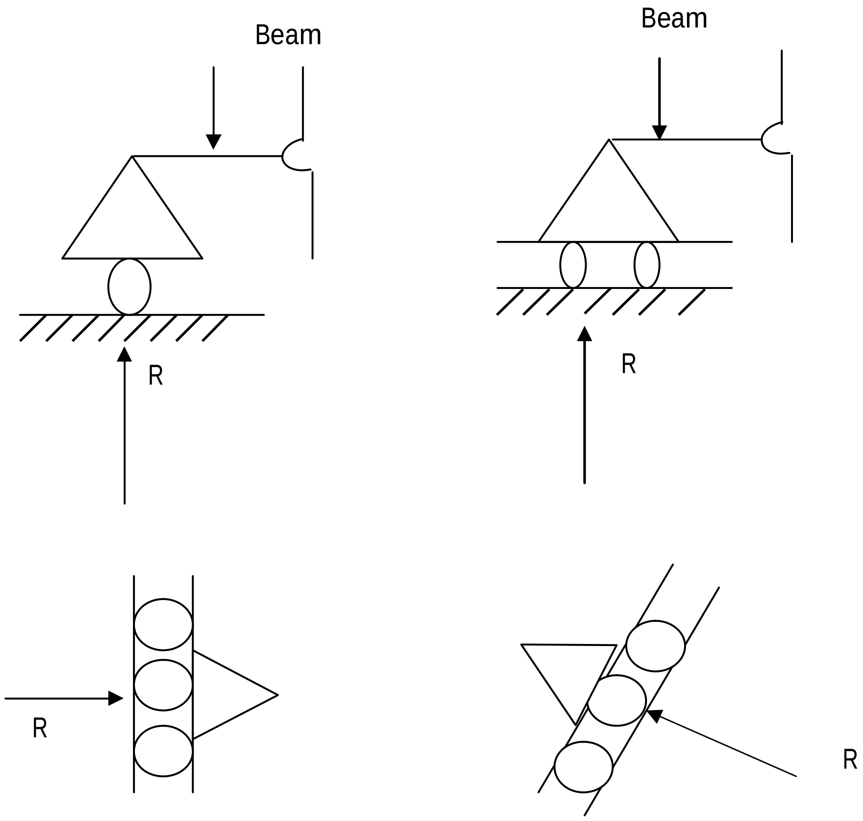
The force of resistance exerted by the support on the beam is called as support reactions.

- Support reaction of beam depends upon the type of loading and type of support.

Types of Supports

1. Roller support
2. Hinged support
3. Fixed support

1. Roller support:



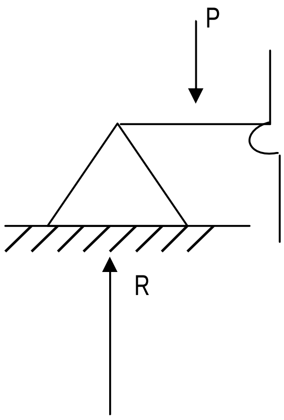
This type of support cannot withstand any force parallel to its own plane. This support will simply roll of if there is some parallel force to its plane.
Hence, the roller support has only one reaction.

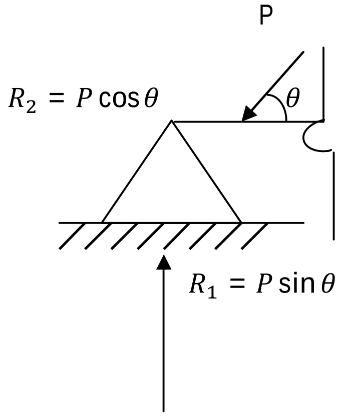
2. <u>Hinged support:</u>

This type of support can withstand any type of both horizontal and vertical. Hence it has two reaction components, vertical and horizontal.

It is to be noted that if the load is vertical, even though it can offer two reaction forces, in this particular case, the reaction will be vertical only. Its horizontal reaction is zero.

But if the load is inclined then the reaction will also be inclined i.e. resolving we get vertical and horizontal components.

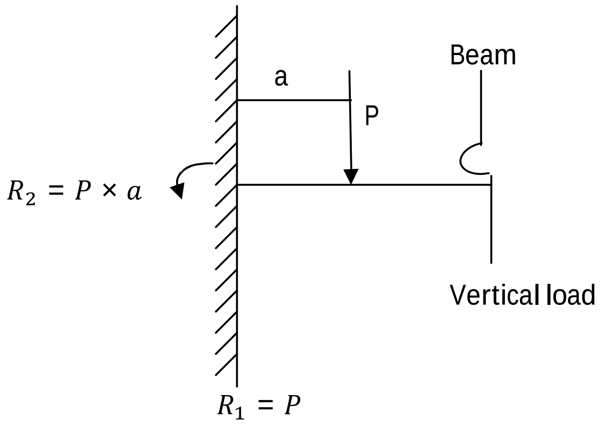


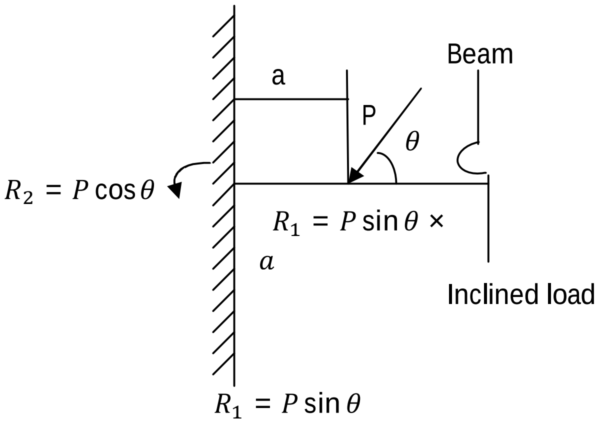


It is also called Pin – Joint support.

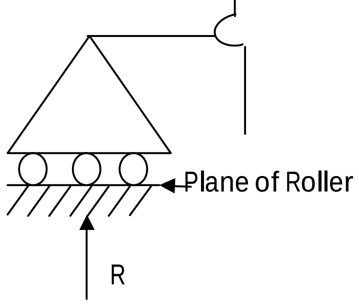
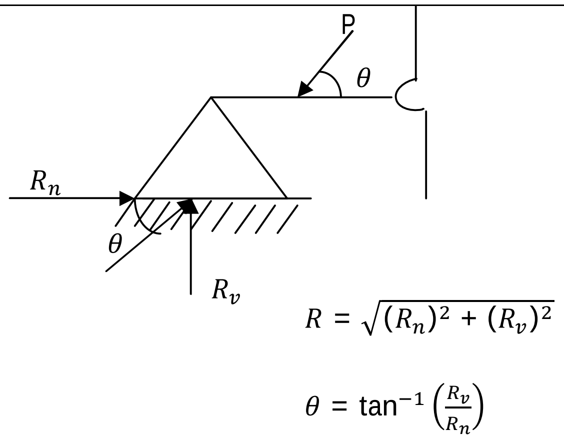
3. Fixed support

Both roller and hinged supports can resist only displacement (i.e. vertical and horizontal movement of beam at ends) but rotation of the beam is resisted by both the supports. This can be given by the fixed supports. Hence, fixed support has three reaction components, horizontal reaction, vertical reaction and rotational reaction. Fixed support is considered as the strongest support.





Comparison between Roller support and Hinged support

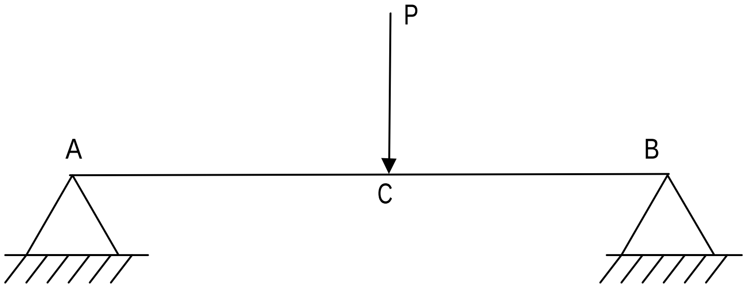
	
Roller support has the known line of action i.e. always normal to the plane of rollers.	Hinged support has an unknown line of action of reaction i.e. at any angle θ with horizontal.
	R can be resolved b its two components R_n , R_v ($\sum H = 0$, $\sum V = 0$ for solving problems)

Types of Loads

1. Point load
2. Uniformly distributed load (U.D.L)
3. Uniformly varying load (U.V.L)

1. Point Load

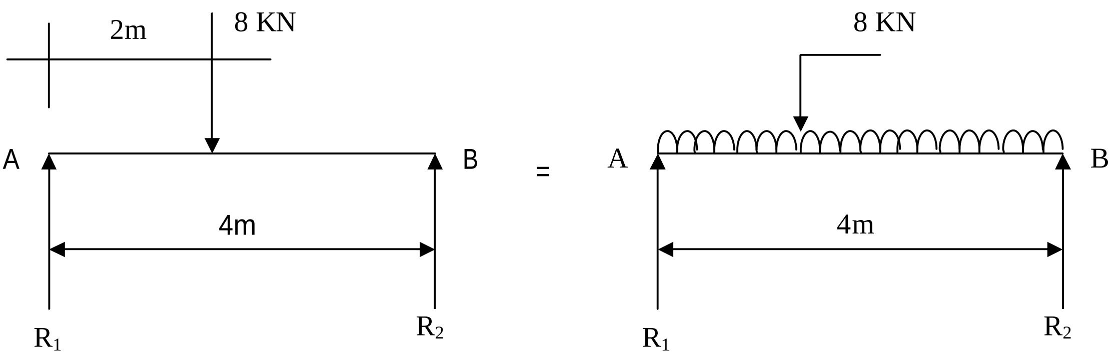
A load acting at a point on a beam is known as point load.

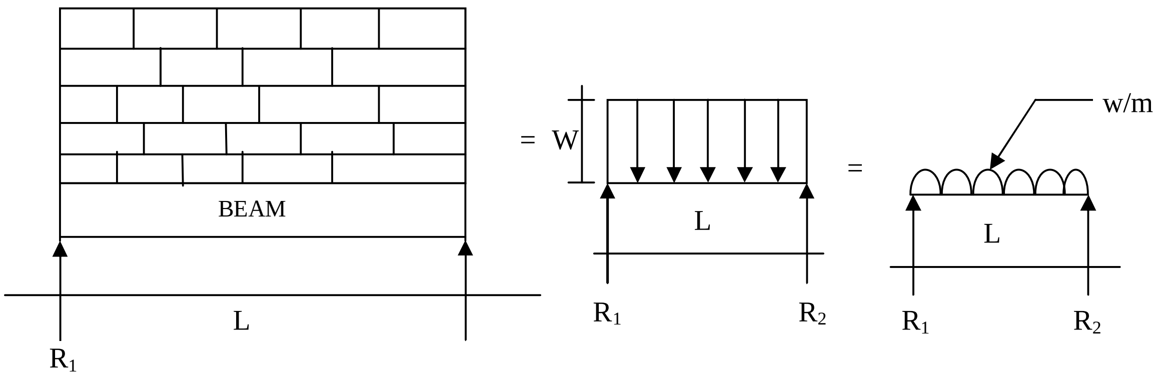


In actual practice it is not possible to apply a load at a particular point. Because any load will have some contact area.

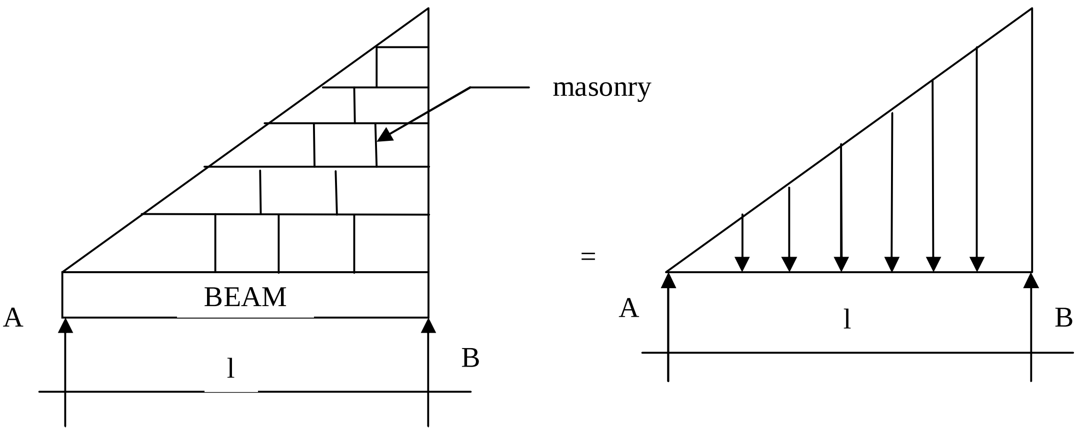
2. Uniformly distributed Load

A load which is spread over a bean in such a manner that each unit length of the beam carries same intensity of the load is called uniformly distributed load.





Uniformly varying Load



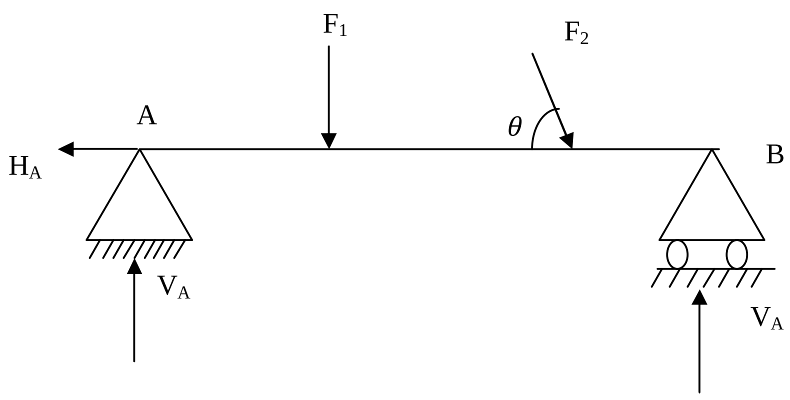
The total load on the beam is equal to the area of the load diagram and acts at the centre of gravity of the load diagram i.e. at one-third of length from B. Self weight of beam is neglected. If it is also considered then we get an additional udl due to weight of beam.

Statically determinate structure

A structure which can be analysed (in this initial stage say, finding the support reaction) by static conditions of equilibrium ($\sum H = 0$, $\sum V = 0$ and $\sum m = 0$) alone, is called statically determinate structure.

The structure which cannot be completely analysed by these equations and needs some additional equations to solve is called statically indeterminate structure.

Analytical method of solving support reaction of a beam



In a statically determinate beam as shown in fig. where,

$$V_A - \ Vertical\ reaction\ at\ A$$

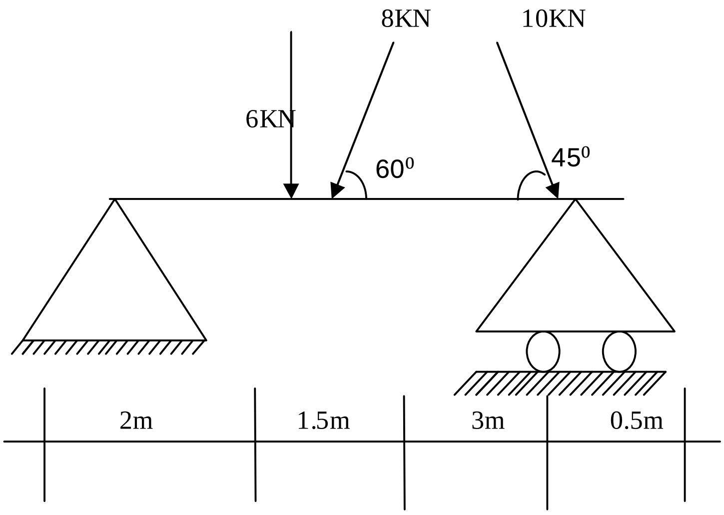
$$V_B - \ Vertical\ reaction\ at\ B$$

$$H_A - \ Horizontal\ reaction\ at\ B$$

These three unknowns can be determined b using the equilibrium equations.

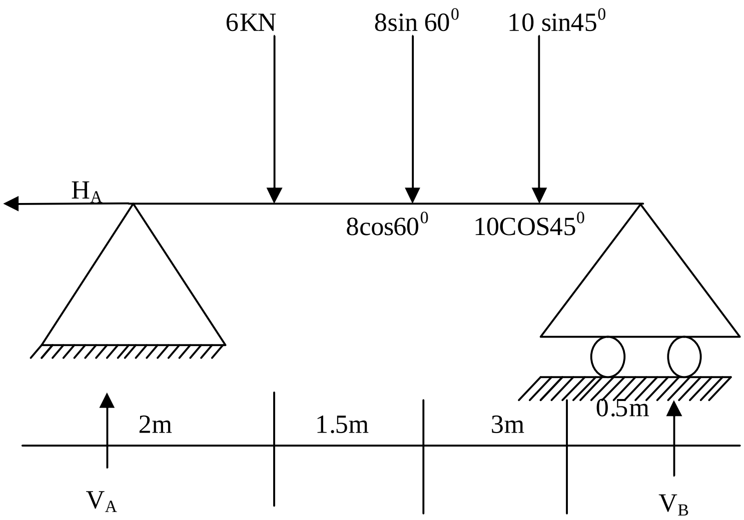
1. Apply $\sum H = 0$ to find H_A
2. Apply $\sum V = 0$ to find $(V_A + V_B)$
3. Apply $\sum m_A = 0$ to find V_B
   (Or)
   Apply $\sum m_B = 0$ to find V_A
4. Then substitute V_B in $\sum V = 0$ equation and find V_A. (or) substitute V_A in $\sum H = 0$ equation and find V_B.

Problem 13: Determine the support reactions of the beam show in figure.



Solution:

First of all the inclined forces are to be resolved into two components.



Applying $\sum H = 0(\rightarrow +)$

$$10\cos 45° - 8\cos 60° - H_A = 0$$

$$H_A = 3.07\ KN$$

H_A is positive, hence direction of H_A assumed is correct $(\leftarrow)$

Applying $\sum V = 0(\uparrow +)$

$$V_A + V_B - 6 - 8\sin 60 - 10\sin 45 = 0$$

$$V_A + V_B = 20\ KN \longrightarrow (1)$$

Applying $\sum m_A = 0(\curvearrowright +)$

$$(H_A \times 0) + (V_A \times 0) + (6 \times 2) + (8\sin 60 \times 3.5) + (10\sin 45 \times 6.5) - (V_B \times 7) = 0$$

$$V_B \times 7 = 8.22$$

$$V_B = 11.74\ KN$$

Sub $V_B = 11.74\ KN\ in\ (1)$

$$V_A + V_B = 20$$

$$V_A + 11.74 = 20\ KN$$

$$V_A = 8.26\ KN$$

Both V_A and V_B are positive, Hence assumed directions are correct. Both are acting upwards.

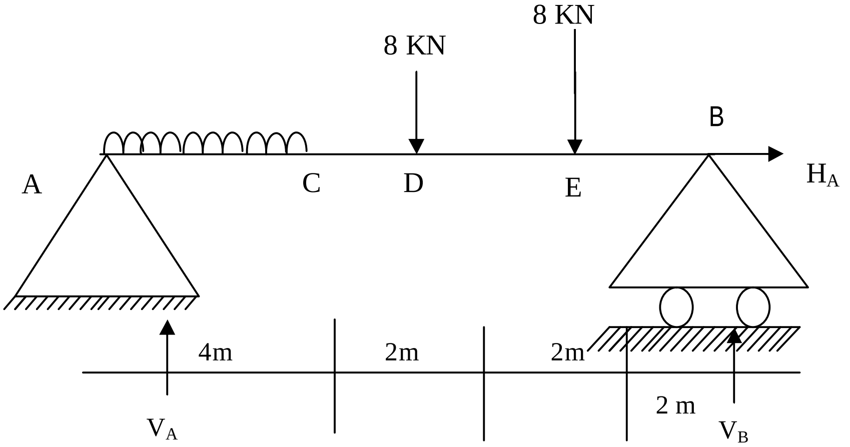
Results:

$H_A = 3.07\ KN(\leftarrow)$

$V_A = 8.26\ KN(\uparrow)$

$V_B = 11.74\ KN(\uparrow)$

Problem 14:A beam AB of spam 10m is loaded as shown in fig Determine the recations at A and B.



Solution:

For u.d.l total load is (3*4)=12 KN which acts at mid point of AC , i.e at $\frac{4}{2} = 2m$ from A

Applying $\sum H = 0(\rightarrow +)$

$H_B = 0$

Applying $\sum V = 0(\uparrow +)$

$$V_A + V_B - 8 - 8 - (3 \times 4) = 0$$

$$V_A + V_B = 28 \longrightarrow (1)$$

Applying $\sum m_A = 0(\curvearrowright +)$

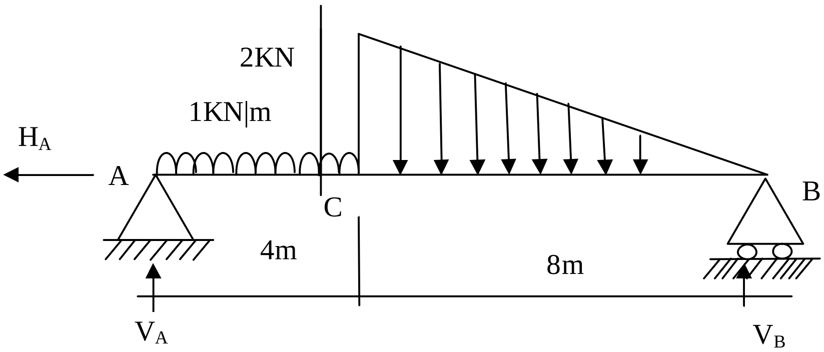
$$(8 \times 6) + (8 \times 6) + \left(3 \times 4 \times \frac{4}{2}\right) - (V_B \times 10) = 0$$

$$V_B = 13.6\,KN(\uparrow)$$

Substitute V_B in (1)

$V_A = 14.4\,KN(\uparrow)$

Problem 15: Calculated the support reaction of a simply supported beam shown in fig



Solution:

Total udl load is $1 \times 4 = 4KN$ which is located at mid point of AC.

Total load of triangular load is areaof the triangle i.e $\frac{1}{2} \times 8 \times 2 = 8KN$ acts at centroid of the triangle,at $\frac{2}{3} \times 8 = 5.33\ m$ from B.

Applying $\sum H = 0$; $H_A = 0$

Applying $\sum V = 0(\uparrow +)$

$$V_A + V_B - (1 \times 4) - (\frac{1}{2} \times 8 \times 2) = 0$$

$$V_A + V_B = 12 \longrightarrow \quad (1)$$

Applying $\sum m_A = 0$ (↷+)

$$\left(1 \times 4 \times \frac{4}{2}\right) - [8 \times (12 - 5.33)] - (V_B \times 12) = 0$$

$$12\,V_B = 8 + 53.36$$

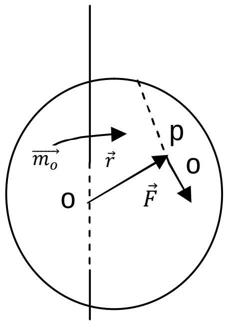
$$V_B = 5.11\,KN\,(\uparrow)$$

Substitute $V_B\ in\ (1)$

$$V_A = 6.89\,KN$$

EQUILIBRIUM OF RIGID BODIES IN THREE DIMENSIONS

Moment of a figure about a point



$$If\ \overrightarrow{r}\ and\ \overrightarrow{f}\ are\ giving\ by$$

$$\overrightarrow{r} = xi + yj + zk$$

and

$$\vec{f} = f_x i + f_y j + f_z k$$

Then moment

$$\vec{m} = \vec{r} \times \vec{f}$$

But $\vec{m} = m_x i + m_y j + m_z k$

Writing $\vec{m} = \vec{r} \times \vec{f}$

$$\vec{m} = \begin{vmatrix} i & j & k \\ x & y & z \\ f_x & f_y & f_z \end{vmatrix}$$

$$\vec{m} = i\left(f_z y - f_y z\right) + j(f_x z - f_z x) + k(f_y x + f_x y)$$

$$m_x = f_z y - f_y z;\ m_y = f_x z - f_z x;\ m_z = f_y x - f_x y$$

Magnitude of moment , $m = \sqrt{m_x^2 + m_y^2 + m_z^2}$

Direction of moment $\vec{m}$

Let the moment of $\vec{m}$,makes angles Φ_x, Φ_y, Φ_z about x,y and z axes

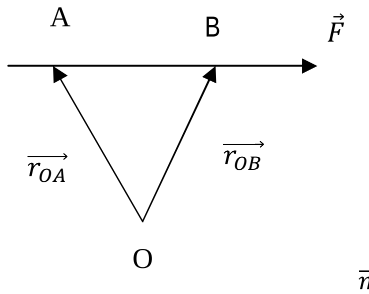
Then

$$\cos\Phi_x = \frac{m_x}{m} \quad \Longrightarrow \quad \Phi_x = \cos^{-1}(\frac{m_x}{m})$$

lll[ly] $\Phi_y = \cos^{-1}\frac{m_y}{m} \quad and\ \Phi_z = \cos^{-1}(\frac{m_z}{m})$

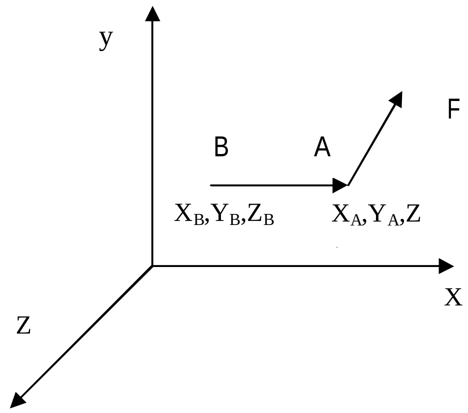
Note:

1.The point P may be taken any where on the line of action of $\vec{f}$



$$\overrightarrow{m_O} = \overrightarrow{r_{OA}} \times \vec{F} = \overrightarrow{r_{OB}} \times \vec{F}$$

2. In case , if moment about any arbitrary point B, of force $\vec{F}$ acting at A is required , the relative position vector of A , with respect to B should be used (write this as $r_{\frac{A}{B}}$)



Case 1:When position vector of A and B are known

$$\overrightarrow{m_B} = \overrightarrow{r_{\frac{A}{B}}} \times \vec{F}$$

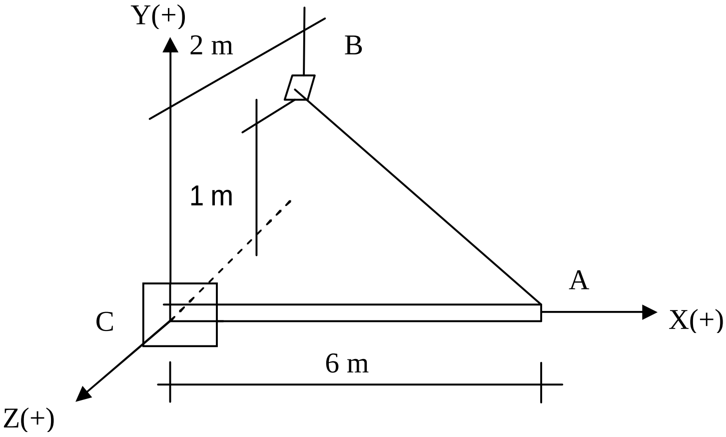
$$= (\overrightarrow{r_{OA}} - \overrightarrow{r_{OB}}) \times \vec{F}$$

Case 2: when coordinates of A and B are known

$$\text{Then} \quad \overrightarrow{m_B} = \begin{vmatrix} i & j & k \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ f_x & f_y & f_z \end{vmatrix}$$

Problem 24: A pipe AC, 6m long is fixed at C , and strectched by a cable from A to a point B on the vertical wall as shown in fig.If the tension in the cable is 400N, determine

(i)The moment of the force exerted at A about C and
(ii)The moment of the force exerted at B about C



Soln:

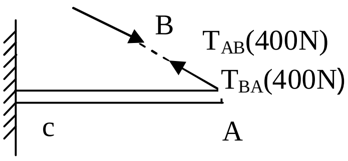
Tension T_{AB} of 400 N acts from A to B and Tension T_{BA} of same magnitude acts from B to A are the collinear from and the cable is in equilibrium.

T_{BA} produces clockwise moment about C , and

T_{AB} produces anticlockwise moment about C.

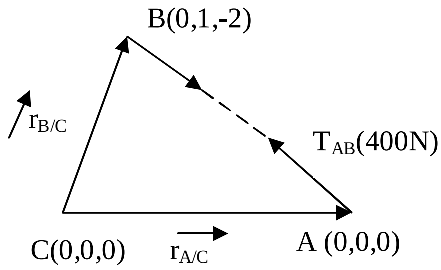
But magnitude of these two moments will to equal

Freebody diagram:



i)moment of force executed at A about C

In this case ,the force is directed from A to B



co-ordinates of A (6,0,0)

co-ordinates of B(0,1,-2)

co-ordinates of C(0,0,0)

writing in vector from (T_{AB})

$$\overrightarrow{T_{AB}} = T_{AB} \times \lambda_{AB}$$

Now

$$\lambda_{AB} = \frac{(0-6)i+(1-0)j+(-2-0)k}{\sqrt{(-6)^2+1^2+(-2)^2}}$$

$$\lambda_{AB} = \frac{-6i+1j-2k}{6.4}$$

$$\overrightarrow{T_{AB}} = T_{AB} \cdot \lambda_{AB} = 400\left[\frac{-6i+1j-2k}{6.4}\right]$$

$$\overrightarrow{T_{AB}} = -375\, i + 62.5\, j - 125\, k$$

lll[ly] $\overrightarrow{r_{AC}} = (6 - 0)i + 0\,j + 0\,k = 6i$

∴ Moment about C,

$$\overrightarrow{m_C} = \overrightarrow{r_{AC}} \times \overrightarrow{T_{AB}}$$

$$= 6\,i \times (-375\,i + 62.5\,j - 125\,k)$$

$$= \begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ -375 & 62.5 & -125 \end{vmatrix}$$

$$\overrightarrow{m_C} = 750\,j + 375\,k$$

<u>(ii) moment of force exerted at B about C</u>

$$\lambda_{BA} = \frac{(6-0)i+(0-1)j+(0+2)k}{\sqrt{6^2+1^2+2^2}} = \frac{6i-1j+2k}{6.4}$$

$$\therefore\ \overrightarrow{T_{BA}} = T_{BA} \cdot \lambda_{BA} = 400\left[\frac{6i - j + 2k}{6.4}\right]$$

$$= 375\,i - 62.5\,j + 125\,k$$

And

$$\overrightarrow{r_{\frac{B}{C}}} = (0 - 0)i + (1 - 0)j + (-2 - 0)k = 1i - 2\,K$$

$$\therefore\ \overrightarrow{m_C} = \overrightarrow{r_{\frac{B}{C}}} \times \overrightarrow{T_{BA}}$$

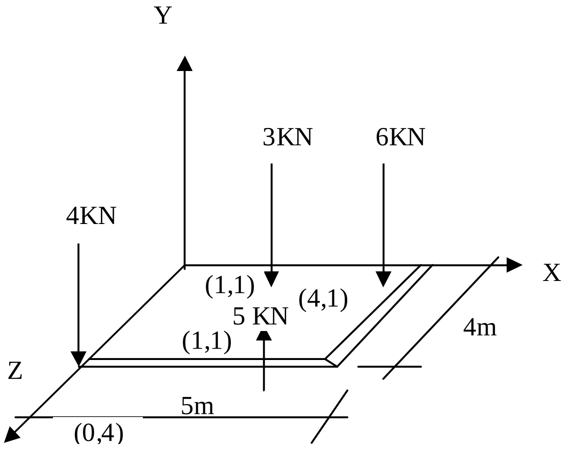
$$= (1j - 2k) \times (375\,i - 62.5\,j + 125\,k)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 1 & -2 \\ 375 & -62.5 & 125 \end{vmatrix}$$

$$= (125 - 125)i - (0 + 750)j + (0 - 375)k$$

$$\overrightarrow{m_C} = -750\,j - 375\,k$$

<u>**Problem 26**</u>: A slab of 4m * 5m carries from parallel forces as shown in fig. (locate the resultant force by scalar and vector approach.)



<u>Solution:</u>

Scalar approach

Magnititude of Resultant force $R = -4 + 5 - 3 - 6 = -8\,KN$

$$R = 8\,KN(\downarrow)$$

Talking moment of forces about x-axis

$$\sum m_x = -(4 \times 4) - (3 \times 1) + (5 \times 2)$$

$$= -15\,KNm$$

$$= 15\,KNm(Anticlockwise)$$

Talking moment about z-axis

$$\sum m_z = (4 \times 0) + (3 \times 1) + (6 \times 4) - (5 \times 3)$$

$$= 12\,KNm\ \ (clockwise)$$

Let 'R' be the resultant force passing through E of co-ordinates x,z as shown in fig.

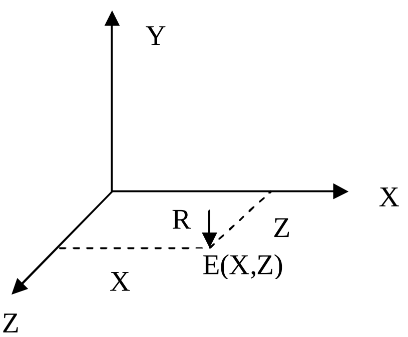
Moment of resultant force about

x-axis

$$\sum m_x = R \times Z$$

$$15 = 8\,Z$$

$$Z = 1.875\,m$$



Ill^{ly} moment of resultant force about Z axis

$$\sum m_z = R \times x$$

$12 = 8\,x$

$x = 1.5\,m$

Vector method:

Force vector of 4 KN is -4 j and co-ordinates A (0,0,4)

Force vector of 5 KN is 5 j and co-ordinates B (3,0,2)

Force vector of 3 KN is -3 j and co-ordinates C (1,0,1)

Force vector of 6 KN is -6 j and co-ordinates D (4,0,1)

The position vectors are

$\overrightarrow{r_{OA}} = 4\,K$

$\overrightarrow{r_{OB}} = 3\,i + 2k$

$\overrightarrow{r_{OC}} = i + k$

$\overrightarrow{r_{OB}} = 4i + k$

∴ Resultant vector, $\vec{R} = \overrightarrow{F_A} + \overrightarrow{F_B} + \overrightarrow{F_C} + \overrightarrow{F_C}$

$= -4j + 5j - 3j - 6j$

$= -8j$

Resultant moment, $\vec{m} = \overrightarrow{r_{OA}} \times F_A + \overrightarrow{r_{OB}} \times F_B + \overrightarrow{r_{OC}} \times F_C + \overrightarrow{r_{OD}} \times F_D$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 4 \\ 0 & -4 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 0 & 5 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & -3 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 0 & 1 \\ 0 & -6 & 0 \end{vmatrix}$$

$= [i[0-(-16)] + j(0) + k(0)] + [i(-10) - j(0) + k(15)] + [i(3) - j(0) + k(-3)] + [i(6) - i(0) + k(-24)]$

$= 16\,i + (10\,i + 15k) + (3i - 3k) + (6i - 24k)$

$\vec{m} = 15\,i + 12\,k \rightarrow (1)$

Let the resultant vector passing through a point P of co-ordinates (x,y,z) hence

$\overrightarrow{r_{OP}} = xi + yj + 2k$

From varignon's theorem

$$\overrightarrow{m_O} = \overrightarrow{r_{OP}} \times \vec{R} = \begin{vmatrix} i & j & k \\ x & y & z \\ 0 & -8 & 0 \end{vmatrix}$$

$15\,i - 12\,k = i(8z) = j(0) + k(-8x)$

$$15\,i - 12\,k = 8\,zi - 8\,kx$$

$$\therefore \quad 15 = 8\,z \qquad ; \qquad -12 = -8z$$

$$z = 1.875\,m \qquad ; \qquad x = 1.5\,m$$

Co-oridnates of resultant R is (1.5,0,1.875)m

Types of support in 3D force system:

Cable

Plane or smooth curved surface

Roller

Ball and socket

Plane or curved rough surface

Clamped or fixed

Single smooth pin