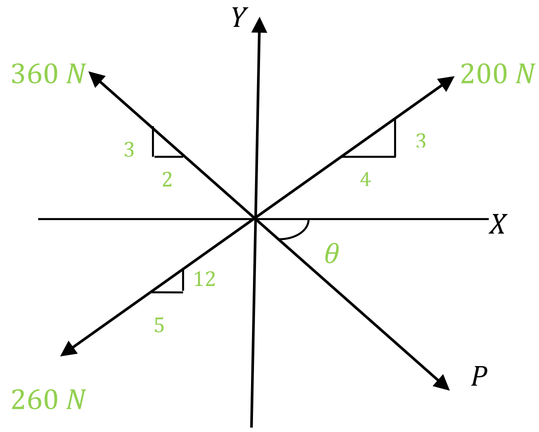
The resultant of the force system shown in fig. is 520N along the negative direction of y axis. Determine P and θ



Solution:

Resultant is 520 N acting along the negative (↓) direction of y axis.

As resultant force is a truly vertical downward force, $\sum H = 0$ and

$\sum V = R = -520N.$

Let,

$$F_1 = 200\text{ N}, \qquad \theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$F_2 = P, \text{angle is } \theta$$

$$F_3 = 260\text{ N}, \theta_3 = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$$

$$F_4 = 360\text{ N}, \theta_4 = \tan^{-1}\left(\frac{3}{2}\right) = 56.31^\circ$$

Algebraic sum of horizontal forces,

$$\sum H = 200\cos 36.87 + P\cos\theta - 260\cos 67.38 - 360\cos 56.31$$

$$0 = 160 + P\cos\theta - 100 - 199.69$$

$$0 = P\cos\theta - 139.69$$

$$\therefore P\cos\theta = 139.69N \longrightarrow (1)$$

Algebraic sum of vertical forces,

$\sum V = 200 \text{ Sin } 36.87^0 - P \sin\theta - 260 \sin 67.38^0 + 360 \sin 56.31^0$

$-520 = 120 - P \sin\theta - 240 + 299.53$

$-520 = 179.53 - P \sin\theta$

$P \sin\theta = 699.53 \rightarrow (2)$

$$\frac{(2)}{(1)} \rightarrow \frac{p \sin}{p \cos} = \frac{699.53}{139.69}$$

$\tan\theta = 5.007$

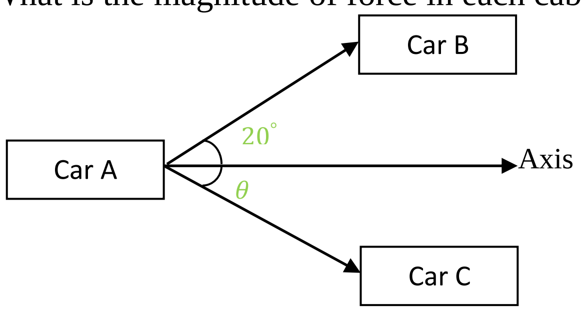
$\theta = \tan^{-1}(5.007) = 78.7^0$

Substitute (2) in (1)

$P \cos\theta = 136.69$

$$P = \frac{136.69}{\cos 78.7} = 712.9N$$

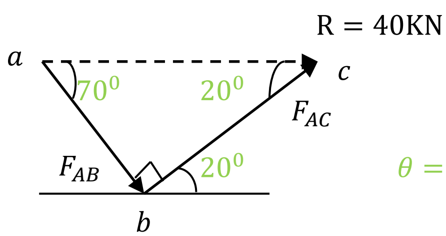
Problem:16: A car is pulled by means of two cars as shown in figure. If the resultant of the two forces acting on the car A is 40KN being directed along the positive direction of X axis, determine the angle Q of the cable attached to the car at B, such that the force in cable AB is minimum. What is the magnitude of force in each cable when this occurs?



To find θ for F_{AB} minimum:

Here F_{AB} is minimum. For this condition the angle between F_{AB} and F_{AC} should be equal to 90^0 (In the triangle length of the side will be minimum only when the sides are perpendicular to each other ($\angle$ABC, $ab \perp^r to\ bc$)

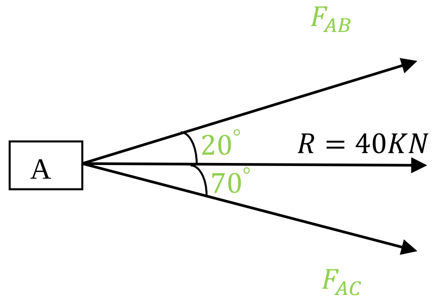
Triangle law



[F_{AB} will be least minimum when F_{AB} & F_{AC} $\perp^r$ to each other]

$\theta = 70^0$

Forces in the cables



$$\frac{40}{\sin 90^0} = \frac{F_{AB}}{\sin 20^0} = \frac{F_{AC}}{\sin 70^0}$$

$$\frac{F_{AB}}{\sin 20^0} = \frac{40}{\sin 90^0} => F_{AB} = 13.68 \text{ KN}$$

$$\frac{F_{AC}}{\sin 70^0} = \frac{40}{\sin 90^0} => F_{AC} = 37.587 \text{ KN}$$

Method of resolution (Alternate method)

Resolving forces horizontally

$$\sum H = F_{AC} \cos 20^0 + F_{AB} \cos 70^0 \rightarrow (1)$$

$$\sum H = 0.939\, F_{AC} + 0.342\, F_{AB}$$

Resolving forces vertically

$$\sum V = F_{AC} \sin 20^0 - F_{AB} \cos 70^0$$

$$= 0.342\, F_{AC} - 0.939\, F_{AB} \rightarrow (2)$$

Since the resultant force acting in positive direction of *x* axis, (given in problem)

$$R = \sum M = 40 \text{ KN} \;\&\; \sum V = 0$$

(1), (2) $\Rightarrow$

$$40 = 0.939\, F_{AC} + 0.342\, F_{AB}$$

$$0 = 0.342\, F_{AC} - 0.939\, F_{AB}$$

By solving we get

$$F_{AB} = 13.68 \text{ KN and } F_{AC} = 37.587 \text{ KN}$$