Unit-1
STATICS OF PARTICLES

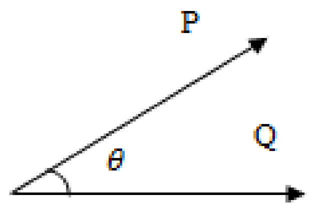
Topic-5

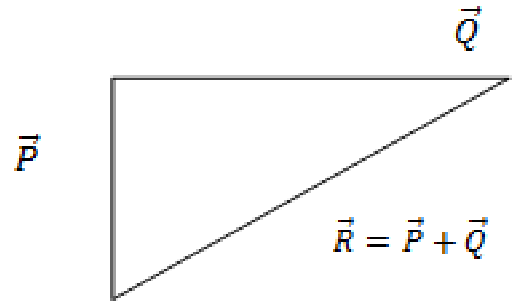
Vector Operations

Dr.R.Sudhakaran/professor/Mechanical

Vector addition

Triangle law – if two forces acting at a point are represented by the two sides of a triangle taken in order, then their resultant is represented by the third side taken in an opposite order.





$$\vec{P} = P x\, \dot{i} + Py\, \vec{j} + Pz\, \vec{K}$$

$$\vec{Q} = Q x\, \dot{i} + Qy\, \vec{j} + Qz\, \vec{K}$$

$$\vec{P} + \vec{Q} = (P_x + Q_x)\dot{i} + (P_y + Q_y)\vec{j} + (P_z + Q_z)\vec{K}$$

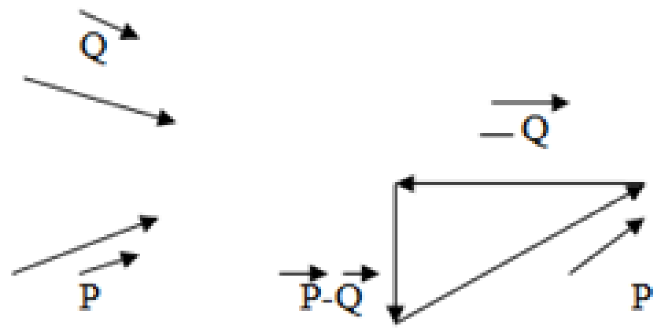
$$\boxed{\vec{P} + \vec{Q} = \vec{Q} + \vec{P}}$$ Commutative

$$\left(\vec{P} + \vec{Q} + \vec{S}\right) = \left(\vec{P} + \vec{Q}\right) + \vec{S} = \vec{P} + \left(\vec{Q} + \vec{S}\right) \pm \left(\vec{P} + \vec{S}\right) + \vec{Q}$$ Associative

$$(m + n)\vec{P} = m\vec{P} + n\vec{P}$$ Distributive

Vector subtraction

It is the addition of corresponding negative vectors.



Problem 4: Find the vector $(\bar{A} + 2\bar{B} + 3\bar{C})$ in terms of $\bar{i}$, $\bar{j}$, $\bar{k}$ and also find it's magnitude where $\bar{A} = 4\bar{i} - \bar{j} - 2\bar{k}$, $\bar{B} = 5\bar{i} + 2\bar{j} - 3\bar{k}$, $\bar{C} = 2\bar{i} - \bar{6}j + 4\bar{k}$

Soln:

$$\bar{A} + 2\bar{B} + 3\bar{C} = (4\bar{i} - \bar{j} - 2\bar{k}) + 2(5\bar{i} + 2\bar{j} - 3\bar{k}) + 3(2\bar{i} - 6\bar{j} + 4\bar{k})$$

$$= 4\bar{i} - \bar{j} - 2\bar{k} + 10\bar{i} + 4\bar{j} - 6\bar{k} + 6\bar{i} - 18\bar{j} + 12\bar{k}$$

$$= 20\,\bar{i} - 15\,\bar{j} + 4\bar{k}$$

Magnitude of $\bar{A} + 2\bar{B} + 3\bar{C} = \sqrt{(20)^2 + (-15)^2 + (4^2)}$

$$= 25.32$$

Multiplication of vectors by scalars

The product of scalar and vector gives vector quantity. The vector $\bar{p}$ is multiplied by a scalar gives a vector $m\bar{p}$

$$(m+n)\bar{p} = m\bar{p} + n\bar{p}$$

Dot or scalar product of vectors

The dot or scalar products of two vectors $\bar{A}$ &$\bar{B}$ written as $\bar{A} + \bar{B}$ is a scalar and is defined as the product of the magnitude of the two vectors and the coline of their included angel θ

$$\bar{A}.\bar{B} = \bar{B}.\bar{A}$$

$$\bar{A}.(\bar{B}+\bar{C}) = \bar{A}.\bar{B} + \bar{A}.\bar{C}$$

$$\bar{A} = A_x\bar{\imath} + A_y\bar{\jmath} + A_z\bar{k}$$

$$\bar{B} = B_x\bar{\imath} + B_y\bar{\jmath} + B_z\bar{k}$$

$$\bar{A}.\bar{B} = A_x.B_x + A_y.B_y + A_z.B_z$$

$$\bar{A}.\bar{A} = A_x^2 + A_y^2 + A_z^2$$

$$\bar{\imath}.\bar{\jmath} = \bar{\jmath}.\bar{k} = \bar{k}.\bar{\imath} = (1)(1)\cos 90^\circ$$

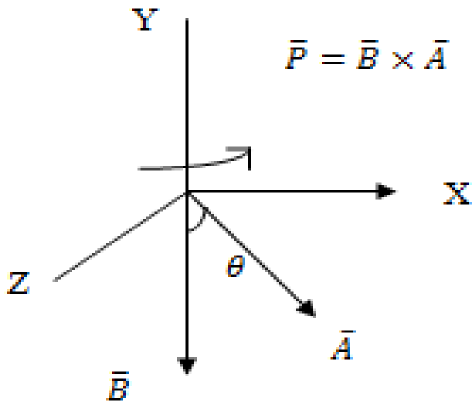
$$= 0$$

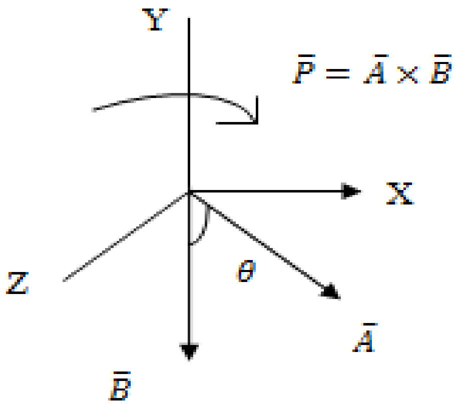
$$\bar{\imath}.\bar{\imath} = \bar{\jmath}.\bar{\jmath} = \bar{k}.\bar{k} = (1)(1)\cos 0$$

$$= 1$$

$$\bar{a}.\bar{b} = |\bar{a}|.|\bar{b}|\cos\theta$$

Cross (or) Vector product of vectors





$$\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A} \quad \bar{a} \times \bar{b} = |\bar{a}||\bar{b}| Sin\theta \ \hat{n}$$

$$\bar{A} \times \bar{B} = -(\bar{B} \times \bar{A})$$

$$\bar{i}, \bar{j}, \bar{k} \quad \bar{i}.\bar{i} = \bar{j}.\bar{j} = \bar{k}.\bar{k} = 0$$

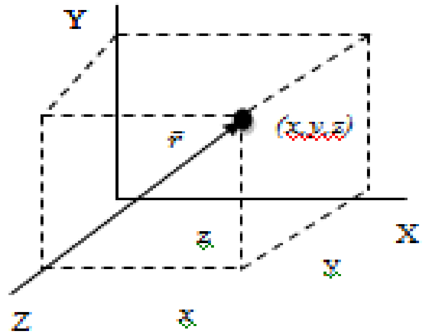
$$\bar{i} \times \bar{j} = \bar{k}, \ \bar{j} \times \bar{k} = \bar{i}, \bar{k} \times \bar{i} = \bar{j}$$

$$\bar{A} = A_x\bar{i} + A_y\bar{j} + A_z\bar{k}$$

$$\bar{B} = B_x\bar{i} + B_y\bar{j} + B_z\bar{k}$$

$$A = \begin{vmatrix} i & j & k \\ A_x & B_y & C_z \\ B_x & B_y & B_z \end{vmatrix}$$

Position vector



$$\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + 2\,\bar{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

Unit vector= $\hat{n} = \frac{\bar{r}}{|\bar{r}|}$

Problem 6 : Determine the unit vector along the line which originates at the point (4, 1, -2) and passes through the pint (2, 2, 6).

Solution:

$$O(0,0,0)$$

$$\bar{A}(4,1,-2)$$

$$\bar{B}(2,2,6)$$

$$\overline{OA} = 4i + \bar{j} - 2\bar{k} \qquad OB = 2\bar{i} + 2\bar{j} + 6\,\bar{k}$$

$$\overline{AB} = \overline{OB} - \overline{OA} \qquad = (2 - 4)\,\bar{i} + (2 - 1)\,\bar{j} + (6\text{-}(-2))\bar{k}$$

$$= -\,2i + j + 8\,\bar{k}$$

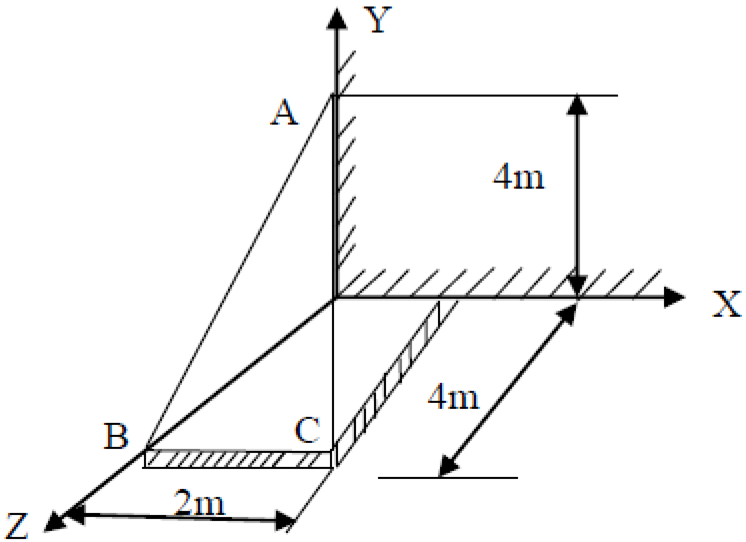
$$|\overline{AB}| = \sqrt{(-2)^2 + 1^2 + 8^2} = 8.3$$

$$\hat{n} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{-2\bar{\imath} + \bar{\jmath} + 8\bar{k}}{8.3}$$

Problem 31: The tension in cables AB and AC are 100N and 120N respectively in fig. Determine the magnitude of the resultant force acting at A.

Solution:

Considering the tension in cable AB



The force is directed from A to B.

A (0, 4, 0), B (0, 0, 4)

$x_1, y_1, z_1,$ x_2, y_2, z_2

A, B – Coordinates

Position vector $\overline{r_{AB}} = (x_2 - x_1)\,i + (y_2 - y_1)\,j + (z_2 - z_1)k$

$$= -4j + 4k$$

$\therefore$ Unit vector along $\overline{AB} = \frac{\overline{r_{AB}}}{r} = \frac{-4j+4k}{\sqrt{4^2+4^2}} = \frac{-4j+4k}{5.656}$

Tension in cable

$$\overline{T_{AB}} = T_{AB}.\lambda_{AB}$$

$$= 100\left[\frac{-4j+4k}{5.656}\right]$$

$$\overline{T_{AB}} = -70.72j + 70.72k$$

<u>Considering tension in cable AC</u>

Now the force is dissected from A to C

A (0, 4, 0) B (2, 0, 4)

x_1, y_1, z_1 x_2, y_2, z_2

A, B – Coordinates

$\therefore$ Position vector $\overline{r_{AC}} = (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1)k$

$$= 2i - 4j + 4k$$

Unit vector along $\overline{AC}, \lambda_{AC} = \frac{\overline{r_{AC}}}{r} = \frac{2i-4j+4k}{\sqrt{2^2+4^2+4^2}}$

$$= \frac{2i-4j+4k}{6}$$

$\therefore$ Tension in cable $\overline{AC}, \overline{T_{AC}} = T_{AC} . \lambda_{AC}$

$$= 120 \left[\frac{2i-4j+4k}{6}\right]$$

$$\overline{T_{AC}} = 40i - 80j + 80k$$

<u>Resultant force</u>

$$\bar{R} = \overline{T_{AB}} + \overline{T_{AC}}$$

$$= [-70.72j + 70.72k] + [40i + 80j + 80k]$$

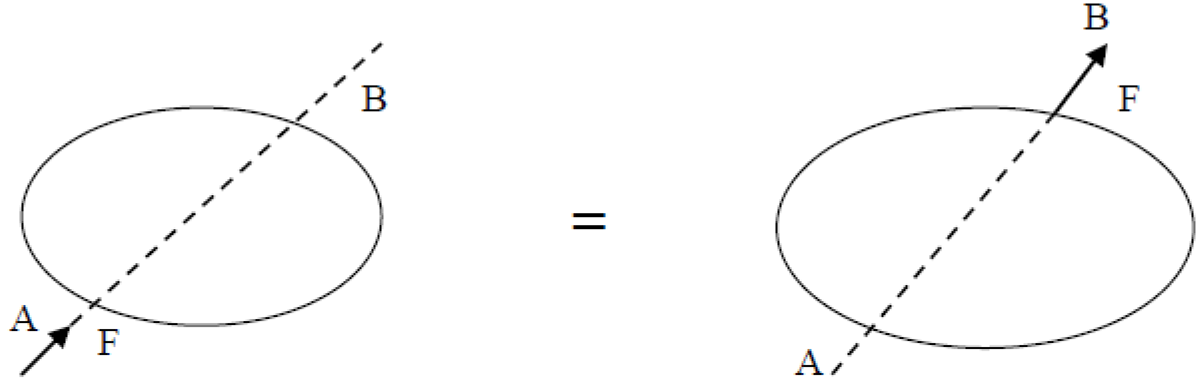
Now i, j and k components

$$\bar{R} = 40i - 150.72j + 150.72k$$

$\therefore$ Magnitude of Resultant force $R = \sqrt{40^2 + (-150.72)^2 + (150. - 72)^2}$

$$R = 216.87N$$

Principle of Transmissibility



The condition of equilibrium or motion of a rigid body remain, unchanged if a force acting at a given point of the rigid body is replaced by a force of same magnitude and direction, but acting at a different point provided that the two forces have the same line of action.