



# Unit-1

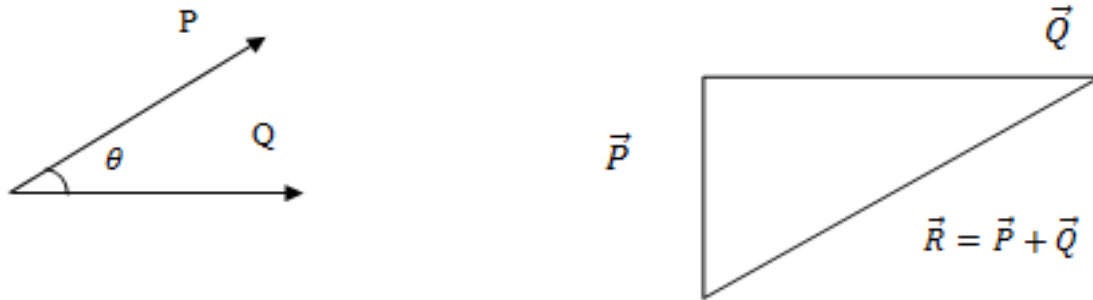
# STATICS OF PARTICLES

Topic-5

Vector Operations

## Vector addition

Triangle law – if two forces acting at a point are represented by the two sides of a triangle taken in order, then their resultant is represented by the third side taken in an opposite order.



$$\vec{P} = P_x \vec{i} + P_y \vec{j} + P_z \vec{k}$$

$$\vec{Q} = Q_x \vec{i} + Q_y \vec{j} + Q_z \vec{k}$$

$$\vec{P} + \vec{Q} = (P_x + Q_x)\vec{i} + (P_y + Q_y)\vec{j} + (P_z + Q_z)\vec{k}$$

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

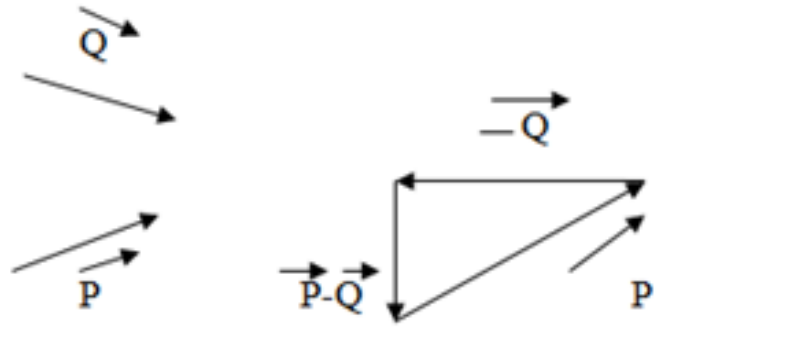
Commutative

$$(\vec{P} + \vec{Q} + \vec{S}) = (\vec{P} + \vec{Q}) + \vec{S} = \vec{P} + (\vec{Q} + \vec{S}) = (\vec{P} + \vec{S}) + \vec{Q} \quad \text{Associative}$$

$$(m + n)\vec{P} = m\vec{P} + n\vec{P} \quad \text{Distributive}$$

## Vector subtraction

It is the addition of corresponding negative vectors.



Problem 4: Find the vector  $(\vec{A} + 2\vec{B} + 3\vec{C})$  in terms of  $\vec{i}, \vec{j}, \vec{k}$  and also find its magnitude where  $\vec{A} = 4\vec{i} - \vec{j} - 2\vec{k}, \vec{B} = 5\vec{i} + 2\vec{j} - 3\vec{k}, \vec{C} = 2\vec{i} - 6\vec{j} + 4\vec{k}$

Soln:

$$\begin{aligned}\vec{A} + 2\vec{B} + 3\vec{C} &= (4\vec{i} - \vec{j} - 2\vec{k}) + 2(5\vec{i} + 2\vec{j} - 3\vec{k}) + 3(2\vec{i} - 6\vec{j} + 4\vec{k}) \\ &= 4\vec{i} - \vec{j} - 2\vec{k} + 10\vec{i} + 4\vec{j} - 6\vec{k} + 6\vec{i} - 18\vec{j} + 12\vec{k} \\ &= 20\vec{i} - 15\vec{j} + 4\vec{k}\end{aligned}$$

$$\begin{aligned}\text{Magnitude of } \vec{A} + 2\vec{B} + 3\vec{C} &= \sqrt{(20)^2 + (-15)^2 + (4)^2} \\ &= 25.32\end{aligned}$$



## Multiplication of vectors by scalars

The product of scalar and vector gives vector quantity. The vector  $\vec{p}$  is multiplied by a scalar gives a vector  $m\vec{p}$

$$(m + n)\vec{p} = m\vec{p} + n\vec{p}$$

## Dot or scalar product of vectors

The dot or scalar products of two vectors  $\vec{A}$  &  $\vec{B}$  written as  $\vec{A} \cdot \vec{B}$  is a scalar and is defined as the product of the magnitude of the two vectors and the cosine of their included angle  $\theta$



$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = (1)(1) \cos 90^\circ$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$= 0$$

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = (1)(1) \cos 0$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

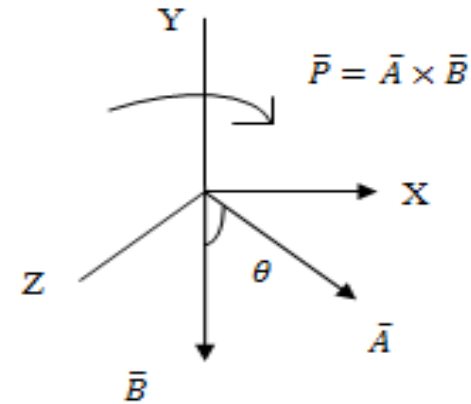
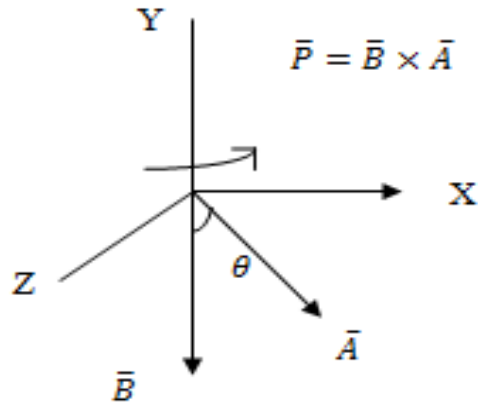
$$= 1$$

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos \theta$$

$$\vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2$$

## Cross (or) Vector product of vectors



$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \quad \vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta \hat{n}$$

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

$$\vec{i} \cdot \vec{j} \cdot \vec{k} \quad \vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 0$$

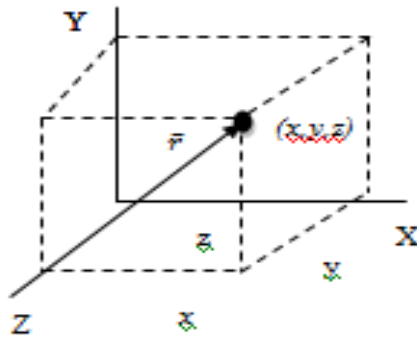
$$\vec{i} \times \vec{j} = \vec{k}, \quad \vec{j} \times \vec{k} = \vec{i}, \quad \vec{k} \times \vec{i} = \vec{j}$$

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

$$A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & B_y & C_z \\ B_x & B_y & B_z \end{vmatrix}$$

## Position vector



$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\text{Unit vector} = \hat{n} = \frac{\vec{r}}{r}$$

Problem 6: Determine the unit vector along the line which originates at the point (4, 1, -2) and passes through the point (2, 2, 6).

Solution:

$$O(0,0,0)$$

$$\vec{A}(4,1,-2)$$

$$\vec{B}(2,2,6)$$

$$\vec{OA} = 4\vec{i} + \vec{j} - 2\vec{k} \quad \vec{OB} = 2\vec{i} + 2\vec{j} + 6\vec{k}$$

$$\begin{aligned} \vec{AB} &= \vec{OB} - \vec{OA} = (2 - 4)\vec{i} + (2 - 1)\vec{j} + (6 - (-2))\vec{k} \\ &= -2\vec{i} + \vec{j} + 8\vec{k} \end{aligned}$$

$$|\vec{AB}| = \sqrt{(-2)^2 + 1^2 + 8^2} = 8.3$$

$$\hat{n} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{-2\vec{i} + \vec{j} + 8\vec{k}}{8.3}$$

Problem 31: The tension in cables AB and AC are 100N and 120N respectively in fig. Determine the magnitude of the resultant force acting at A.

Solution:

Considering the tension in cable AB

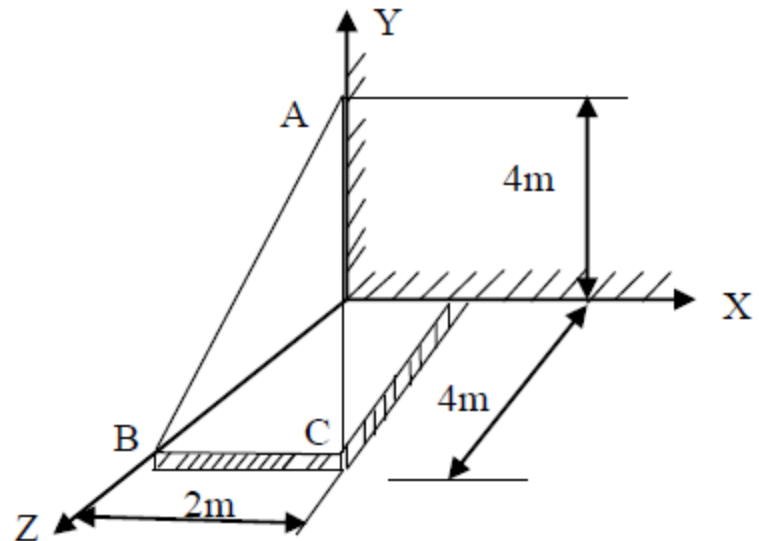
The force is directed from A to B.

A (0, 4, 0), B (0, 0, 4)

$x_1, y_1, z_1, \quad x_2, y_2, z_2$

A, B – Coordinates

$$\begin{aligned} \text{Position vector } \overline{r_{AB}} &= (x_2 - x_1) i + (y_2 - y_1) j + (z_2 - z_1) k \\ &= -4j + 4k \end{aligned}$$





$$\therefore \text{Unit vector along } \overline{AB} = \frac{\overline{r_{AB}}}{r} = \frac{-4j+4k}{\sqrt{4^2+4^2}} = \frac{-4j+4k}{5.656}$$

Tension in cable

$$\overline{T_{AB}} = T_{AB} \cdot \lambda_{AB}$$

$$= 100 \left[ \frac{-4j+4k}{5.656} \right]$$

$$\overline{T_{AB}} = -70.72j + 70.72k$$

Considering tension in cable AC

Now the force is dissected from A to C

$$A (0, 4, 0) \qquad B (2, 0, 4)$$

$$x_1, y_1, z_1 \qquad x_2, y_2, z_2$$

A, B – Coordinates

$$\begin{aligned} \therefore \text{Position vector } \overline{r_{AC}} &= (x_2 - x_1) i + (y_2 - y_1) j + (z_2 - z_1) k \\ &= 2i - 4j + 4k \end{aligned}$$





$$\begin{aligned}\text{Unit vector along } \overline{AC}, \lambda_{AC} &= \frac{\overline{r_{AC}}}{r} = \frac{2i-4j+4k}{\sqrt{2^2+4^2+4^2}} \\ &= \frac{2i-4j+4k}{6}\end{aligned}$$

$$\begin{aligned}\therefore \text{Tension in cable } \overline{AC}, \overline{T_{AC}} &= T_{AC} \cdot \lambda_{AC} \\ &= 120 \left[ \frac{2i-4j+4k}{6} \right]\end{aligned}$$

$$\overline{T_{AC}} = 40i - 80j + 80k$$

### Resultant force

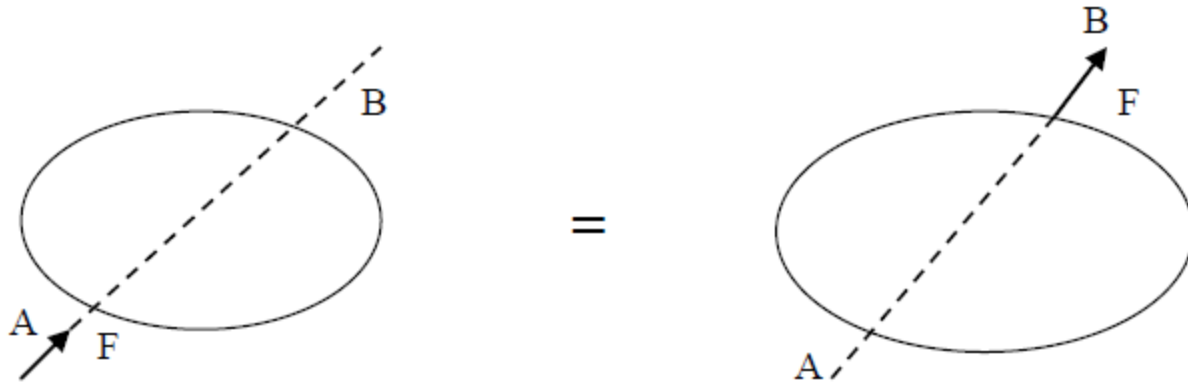
$$\begin{aligned}\overline{R} &= \overline{T_{AB}} + \overline{T_{AC}} \\ &= [-70.72j + 70.72k] + [40i + 80j + 80k]\end{aligned}$$

Now i, j and k components

$$\overline{R} = 40i - 150.72j + 150.72k$$

$$\begin{aligned}\therefore \text{Magnitude of Resultant force } R &= \sqrt{40^2 + (-150.72)^2 + (150.72)^2} \\ R &= 216.87\text{N}\end{aligned}$$

## Principle of Transmissibility



The condition of equilibrium or motion of a rigid body remain, unchanged if a force acting at a given point of the rigid body is replaced by a force of same magnitude and direction, but acting at a different point provided that the two forces have the same line of action.