Unit-1
STATICS OF PARTICLES

Topic-3

Parallellogram and Triangle Law of Forces

- The **law of parallelogram of forces** states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point .

MATHEMATICAL FORMULATION
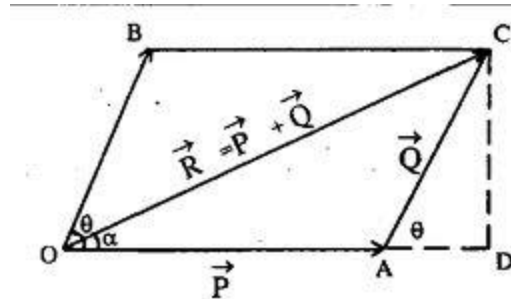
- Draw a perpendicular QN to OP produced.
- And let us assume that OP=A, OS= PQ= B, OQ=R and angle SOP= angle QPN = θ.
- Now considering this if we proceed further , in the case of triangle law of vector addition , the magnitude and direction of resultant vector will be given by
- R= sqrt of A^2 + B^2 + 2 AB cosθ
- tan B = B sinθ/ A+B cosθ

SPECIAL CASES

(1) When two vectors are acting in the same direction , then θ= 0 , cosθ=1 and sinθ= 0

- R= sqrt of A^2 + B^2 + 2 AB
  =sqrt of (A+B)^2 = A + B

- tan Beta = B X 0/ A+B = 0
  Beta = 0

- Thus for two vectors acting in the same direction the magnitude of the resultant vector is equal to the sum of the magnitudes of two vectors and act along the direction of A and B.

(2) When two vectors are acting in opposite directions , then θ= 180 , cos θ= -1 and sinθ= 0

- R= sqrt of A^2+ B^2+ 2 AB (-1)
  = sqrt of (A-B) or (B-A)

- tan beta = B X 0/ A+ B (-1)= 0
  Beta = 0 or 180.

- Thus for two vectors acting in opposite directions, the magnitude of the resultant vector is equal to the difference of the magnitudes of the two vectors and acts in the direction of bigger vector .

(3) When two vectors act at right angle to each other θ = 90 , sinθ = 1 and cosθ = 0

- R= sqrt of A^2+B^2 + 2 AB (0)
  = sqrt of A^2+B^2

- tan beta = B(1)/A+B(0)= B/A
  or, Beta = tan^-1 B/A

PROBLEMS

Problem 1: The resultant of the two forces, when they act at an angle of $60°$ is 14N. If the same forces are acting at right angles, their resultant is $\sqrt{136}\ N$. Determine the magnitude of the two forces.

PROBLEMS

Two forces 5 N and 20 N are acting at an angle of 120 degree between them . Find the resultant force in magnitude and direction.

- Solution : Here A = 5 N
B = 20 N ; θ= 120 degree ; R= ?; beta =?

- R= sqrt of A^2+ B^2+ 2 AB cosθ
= sqrt of 5^2 + 20^2+ 2 X 5 X 20 cos 120 degree
= sqrt of 325
= 18.03 N

- tan beta = 5 sin 120 /20 + 5 cos 120
= 5 sqrt 3 / 2 /20 +5 X (-0.5)
= 0.2475 = tan 13 degree 54 min

- hence, beta = 13 degree 5

TRIANGULAR LAW OF FORCES

- *The resultant of two forces acting at a point can also be found by using triangle law of forces.*

- *If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then the closing side of the triangle taken in the reversed order represents the resultant of the forces in magnitude and direction.*

- *Forces Vector P and Vector Q act at an angle ϑ. In order to find the resultant of Vector P and Vector Q, one can apply the head to tail method, to construct the triangle.*

- *In Fig., OA and AB represent Vector P and Vector Q in magnitude and direction. The closing side OB of the triangle taken in the reversed order represents the resultant Vector R of the forces Vector P and Vector Q. The magnitude and the direction of Vector R can be found by using sine and cosine laws of triangles.*

- *The triangle law of forces can also be stated as, if a body is in equilibrium under the action of three forces acting at a point, then the three forces can be completely represented by the three sides of a triangle taken in order.*

- *If Vector P , Vector Q and Vector R are the three forces acting at a point and they are represented by the three sides of a triangle then P/QA =Q/AB =R/ OB.*