$$125 = A\,(2 - j\,10)$$

$$A = \frac{125\,(2 + j10)}{(2 - j10)\,(2 + j10)}$$

$$A = 2.4 + j\,12.02$$

Put $s = -2$

$$250\,(-2) = C\,(-2 + j10)\,(-2 - j10)$$

$$-500 = C\,(4 + 100)$$

$$-500 = C\,104$$

$$C = \frac{-500}{104} = -4.81$$

$$I\,(s) = \frac{2.4 + j\,12.02}{s + j10} + \frac{2.4 - j\,12.02}{s - j10} - \frac{4.81}{s + 2}$$

Taking inverse Laplace transform

$$i\,(t) = (2.4 + j\,12.02)\,e^{-j10t} + (2.4 - j12.02)\,e^{j10t} - 4.81\,e^{-2t}$$

$$i\,(t) = 2.4\,e^{-j10t} + j12.02\,e^{-j10t} + 2.4\,e^{j10t} - j12.02e^{j10t} - 4.81\,e^{-2t}$$

$$i\,(t) = 2.4\,(e^{j10t} + e^{-j10t}) - j1.202\,(e^{j10t} - e^{-j10t}) - 4.81\,e^{-2t}$$

$$i\,(t) = 2.4\,(2\cos 10t) - j12.02\,(2j\sin 10t) - 4.81\,e^{-2t}$$

$$\boldsymbol{i\,(t) = 4.8\cos 10t + 24.04\sin 10t - 4.81\,e^{-2t}\ \text{A}}$$

3.4.3 Transient response for RLC circuit

3.4.3.1 Step input

Applying *KVL* to the series *RLC* circuit (Figure 3.15)

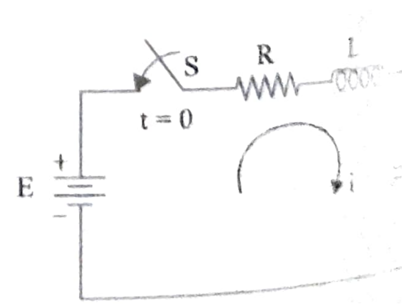


Fig. 3.15

$$iR + L\frac{di}{dt} + \frac{1}{C}\int idt = E$$

Assume no initial charge on the capacitor.

Taking Laplace transform on both sides,

$$RI\,(s) + L\,[sI\,(s) - i\,(0)] + \frac{1}{C}\frac{I\,(s)}{s} = \frac{E}{s}$$

Assuming $i(0) = 0$ we get

$$RI(s) + LsI(s) + \frac{I(s)}{Cs} = \frac{E}{s}$$

$$I(s)\left[R + sL + \frac{1}{Cs}\right] = \frac{E}{s}$$

$$I(s) = \frac{E}{s(R + sL + 1/Cs)}$$

$$I(s) = \frac{E}{\left(Rs + s^2 L + \frac{1}{C}\right)}$$

$$I(s) = \frac{E/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

The roots of the denominator are,

$$s = \frac{-R/L \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4/LC}}{2}$$

$$s = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s = \alpha \pm \beta$$

$$\alpha = \frac{-R}{2L}\,;\ \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case (i)

Discriminant positive

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

The two roots are real and distinct

$$s_1 = \alpha + \beta,\ s_2 = \alpha - \beta$$

EXAMPLE 35: *An RLC series circuit has $R = 10\ \Omega$, $L = 2\ H$. What value of capacitance will make the circuit critically damped.*

LOTS

Solution :

For critical damping

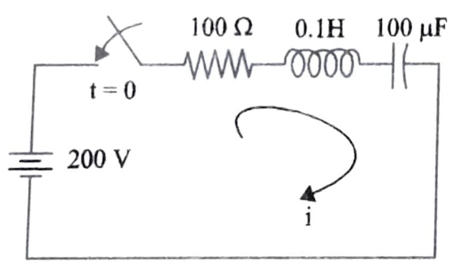
$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

$$C = \frac{4L}{R^2} = \frac{4 \times 2}{(10)^2} = 0.08\text{ F}$$

$$\boxed{C = 0.08\text{ F}}$$

EXAMPLE 36: *A series RLC circuit with $R = 100\ \Omega$, $L = 0.1\ H$ and $C = 100\ \mu F$ has a DC voltage of 200 volts applied to it at $t = 0$ through a switch. Find the expression for the transient current. Assume initially relaxed circuit conditions.*

HOTS



Solution :

When the switch is closed, the equation for the circuit is

$$100i + 0.1\frac{di}{dt} + \frac{1}{100 \times 10^{-6}} \int i\, dt = 200$$

Take Laplace transform on both sides and assuming zero initial conditions

$$100I(s) + 0.1\,sI(s) + \frac{1}{100 \times 10^{-6}}\frac{I(s)}{s} = \frac{200}{s}$$

$$I(s)\left[100 + 0.1s + \frac{1}{100 \times 10^{-6}s}\right] = \frac{200}{s}$$

$$I(s) = \frac{200}{s\left(100 + 0.1\,s + \dfrac{1}{100 \times 10^{-6}s}\right)}$$

$$I(s) = \frac{200}{100s + 0.1s^2 + \frac{1}{100 \times 10^{-6}}}$$

$$I(s) = \frac{2000}{s^2 + 1000s + 100 \times 10^3}$$

The roots of the denominator are

$$s^2 + 1000s + 100 \times 10^3 = 0$$

$$s_1, s_2 = \frac{-1000 \pm \sqrt{(1000)^2 - 4 \times 1 \times 100 \times 10^3}}{2}$$

$$s_1, s_2 = -500 \pm 387.2$$

$$s_1, s_2 = -112.8, -887.2$$

$$I(s) = \frac{K_1}{s + 112.8} + \frac{K_2}{s + 887.2}$$

$$2000 = K_1(s + 887.2) + K_2(s + 112.8)$$

When $s = -887.2$

$$K_2(-887.2 + 112.8) = 2000$$

$$K_2 = -2.58$$

When $s = -112.8$

$$K_1(-112.8 + 887.2) = 2000$$

$$K_1 = 2.58$$

$$I(s) = \frac{2.58}{s + 112.8} - \frac{2.58}{s + 887.2}$$

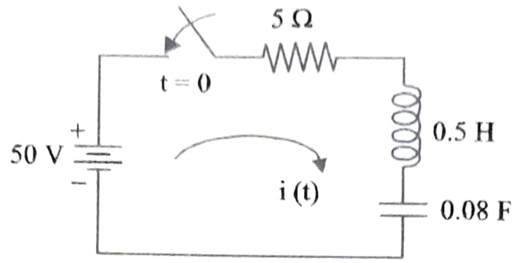
Taking inverse Laplace transform,

$$i(t) = 2.58\,e^{-112.8t} - 2.58\,e^{-887.2t}$$

$$\boxed{i(t) = 2.58\,[e^{-112.8t} - e^{-887.2t}]\ \text{A}}$$

EXAMPLE 37: *In the circuit shown in figure, find the transient current when the switch is closed at $t = 0$. Assume zero initial conditions.*

HOTS



Solution :

Applying *KVL*,

$$5i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{0.08}\int i(t)\,dt = 50$$

Taking Laplace transform on both sides,

$$5I(s) + 0.5\,sI(s) + \frac{1}{0.08}\frac{I(s)}{s} = \frac{50}{s}$$

$$I(s)\left[5 + 0.5s + \frac{1}{0.08s}\right] = \frac{50}{s}$$

$$I(s) = \frac{50}{s\left(5 + 0.5s + \dfrac{12.5}{s}\right)}$$

$$I(s) = \frac{50}{0.5s^2 + 5s + 12.5}$$

$$I(s) = \frac{100}{s^2 + 10s + 25}$$

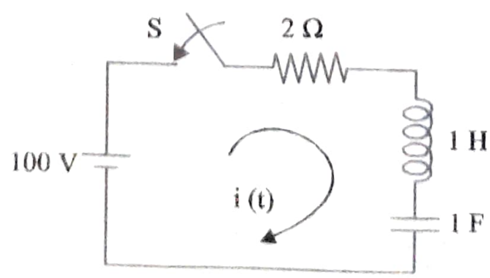
$$I(s) = \frac{100}{(s+5)^2}$$

Take inverse Laplace transform on both sides,

$$\boxed{\boldsymbol{i(t) = 100\,te^{-5t}\ \text{A}}}$$

EXAMPLE 38: *For the circuit shown in figure, determine the current in the circuit when the switch is closed at $t = 0$. Assume that there is no initial charge on the capacitor or current in the inductor.*

HOTS



Solution :

When the switch is closed, by applying *KVL*, we get

$$2\,i(t) + \frac{d\,i(t)}{dt} + 1 \int i\,dt = 100$$

Taking Laplace transform on both sides,

$$2I(s) + sI(s) + \frac{I(s)}{s} = \frac{100}{s}$$

$$I(s)\left[2 + s + \frac{1}{s}\right] = \frac{100}{s}$$

$$I(s) = \frac{100}{s^2 + 2s + 1}$$

$$I(s) = \frac{100}{(s+1)^2}$$

Taking inverse Laplace transform on both sides, we get

$$\boxed{i(t) = 100\,te^{-t}\ \text{A}}$$

3.4.3.3 Sinusoidal input

Figure 3.21 shows the series RLC circuit is excited by sinusoidal source under transient condition.

The switch in the series RLC circuit is closed at $t = 0$, we shall assume all initial conditions to be zero.

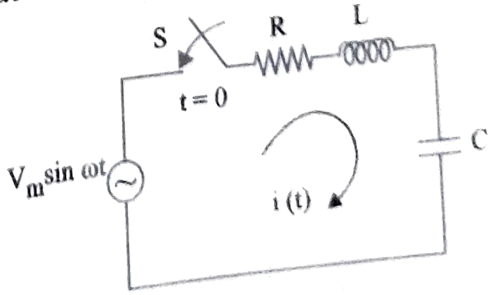


Fig. 3.21

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt = V_m \sin \omega t$$

Take Laplace transform,

$$RI(s) + LsI(s) + \frac{1}{C}\frac{I(s)}{s} = \frac{V_m\,\omega}{s^2 + \omega^2}$$

$$I(s)\left[R + Ls + \frac{1}{CS}\right] = \frac{V_m\,\omega}{s^2 + \omega^2}$$

$V_m\,\omega$

1

$$I(s) = \frac{V_m \, \omega \, Cs}{(RCs + s^2 LC + 1)(s^2 + \omega^2)}$$

$$I(s) = \frac{V_m \, \omega \, Cs}{LC\left(s^2 + \frac{Rs}{L} + \frac{1}{LC}\right)(s^2 + \omega^2)}$$

$$I(s) = \frac{V_m \, \omega \, s / L}{\left(s^2 + \frac{R}{L} s + \frac{1}{LC}\right)(s^2 + \omega^2)}$$

When $\quad s^2 + \omega^2 = 0$

$$s^2 = -\omega^2$$

$$s = \pm j\omega$$

When $\quad s^2 + \frac{R}{L} s + \frac{1}{C} = 0$

$$s = \frac{\frac{-R}{L} \pm \sqrt{\frac{R^2}{L^2} - 4 \times 1 \times \frac{1}{LC}}}{2}$$

$$s = \frac{-R}{2L} \pm \left[\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right]$$

The four roots of the denominator are $\pm j\omega, \alpha \pm \beta$

$$\alpha = \frac{-R}{2L} \; ; \; \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

The type of solution will depend on the value of $\left[\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}\right]$

Case (i)

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

In this case, the roots are real and distinct.

Case (ii)

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

Here, the two roots become identical and equal to α only.

Case (iii)

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

The roots are complex conjugate.

The expression for the steady state current will be

$$i_{SS} = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \sin(\omega t \pm \theta)$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

SHORT QUESTIONS AND ANSWERS

1. **R** **The transients are due to the presence of energy storing elements in the circuit - True or False**

Ans: True

(AU/ECE - Dec 2007)

2. **R** **What is transient?**

If a network containing energy storage elements, with change in excitation, the current and voltage change from one state to other state. The behaviour of the voltage or current, when it is changed from one state to another, is called the transient state.

3. **R** **Define transient time.**

The time taken for the circuit to change from one steady state to another steady state is called the transient time.

4. **R** **What do you mean by steady state?**

A circuit consisting of constant sources is said to be in steady state if the voltages and current do not change with time.