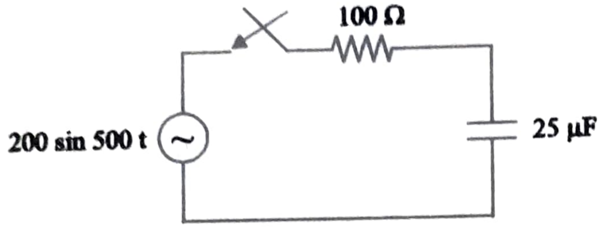
EXAMPLE 31: *In the circuit of figure, find the current i (t). Assume initial charge in the capacitor is zero.*

HOTS



(AU/EEE - Dec 2007)

Solution :

Applying KVL,

$$100\, i + \frac{1}{25 \times 10^{-6}} \int i\, dt = 200 \sin 500\, t$$

Taking Laplace transform on both sides,

$$100\, I(s) + 40000 \frac{I(s)}{s} = 200 \frac{500}{s^2 + 500^2}$$

$$I(s)\left[100 + \frac{40000}{s}\right] = \frac{100000}{s^2 + 500^2}$$

$$I(s) = \frac{100000}{\left(100 + \frac{40000}{s}\right)\left(s^2 + 500^2\right)}$$

$$I(s) = \frac{1000\, s}{(s + 400)(s^2 + 500^2)}$$

$$\text{or } I(s) + \frac{4oo\, I(s)}{s} = \frac{10^3}{s^2 + 5a^2}$$

$$(s + 400)\, I(s) = \frac{10^3\, s}{s^2 + 5a^2}$$

$$\text{or } I(s) = \frac{10^3\, s}{(s + 400)(s^2 + 500^2)}$$

$$\frac{1000\, s}{(s + 400)(s^2 + 500^2)} = \frac{K_1}{s + 400} + \frac{K_2}{s + j500} + \frac{K_3}{s - j500}$$

$$K_1 = \frac{-400}{41} = -9.76$$

$$K_2 = \frac{5(4 + j5)}{41}$$

$$K_3 = \frac{5(4 - j5)}{41}$$

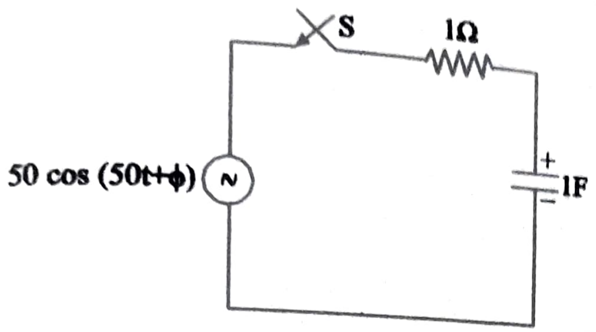
$$i(t) = -9.76\, e^{-400\, t} + \frac{5}{41}\left[(4 + j5)\, e^{-j500\, t} + (4 - j5)\, e^{j500\, t}\right]$$

$$i(t) = -9.76\, e^{-400\, t} + \frac{10}{\sqrt{41}}\left[\frac{4}{\sqrt{41}} \cos 500\, t - \frac{5}{\sqrt{41}} \sin 500\, t\right]$$

$$i(t) = -9.76\, e^{-400\, t} + 1.562 \sin(500\, t + 38.7^\circ)$$

EXAMPLE 32: ***Determine the current when the switch is closed at a time corresponding to $\phi = 0$ in the circuit shown in figure. The initial charge on the capacitor is 2 coulombs with polarity as shown.***

HOTS



(AU, Trichy/EEE - Dec 2008)

Solution :

Applying Kirchhoff's voltage law,

$$i(t) + \frac{1}{1}\int_{-\infty}^{t} i\,dt = 50\cos(50t)$$

$$i(t) + \int_{-\infty}^{0} \frac{dq}{dt}\,dt + \int_{0}^{t} i\,dt = 50\cos(50t)$$

Taking Laplace transforms on both sides, we have

$$I(s) + \frac{I(s)}{s} + \frac{q_0}{s} = \frac{50\,s}{s^2 + 50^2}$$

$$I(s)\left[1 + \frac{1}{s}\right] + \frac{2}{s} = \frac{50\,s}{s^2 + 50^2}$$

$$I(s) = \left[\frac{50\,s}{s^2 + 50^2} - \frac{2}{s}\right]\frac{s}{s+1}$$

$$I(s) = \frac{\left[50\,s^2 - 2s^2 - 2\,(50)^2\right]}{\left(s^2 + (50)^2\right)(s+1)}$$

$$I(s) = \frac{48\,s^2 - 2\,(50)^2}{\left(s^2 + (50)^2\right)(s+1)}$$

By partial fractions,

$$I(s) = \frac{K_1}{(s + j\,50)} + \frac{K_2}{(s - j\,50)} + \frac{K_3}{(s+1)}$$

$$K_1 = I(s)\,(s + j\,50)\Big|_{s=-j\,50}$$

$$K_1 = \frac{48\,s^2 - 2\,(50)^2}{(s - j\,50)\,(s+1)}\Bigg|_{s=-j\,50} \qquad K_1 = \frac{1250}{50 + j}$$

$$K_2 = I(s)\,(s - j\,50)\Big|_{s=j\,50}$$

$$K_2 = \frac{48\,s^2 - 2\,(50)^2}{(s + j\,50)\,(s+1)}\Bigg|_{s=j\,0} \qquad K_2 = \frac{1250}{50 - j}$$

$$K_3 = I(s)\,(s+1)\Big|_{s=-1}$$

$$= \frac{48\,s^2 - 2\,(50)^2}{s^2 + (50)^2}\Bigg|_{s=-1} \qquad K_3 = -\,1.98$$

Substituting the values of K_1, K_2 and K_3 we get

$$I(s) = \frac{1250}{(50+j)\,(s+j\,50)} + \frac{1250}{(50-j)\,(s-j\,50)} - \frac{1.98}{(s+1)}$$

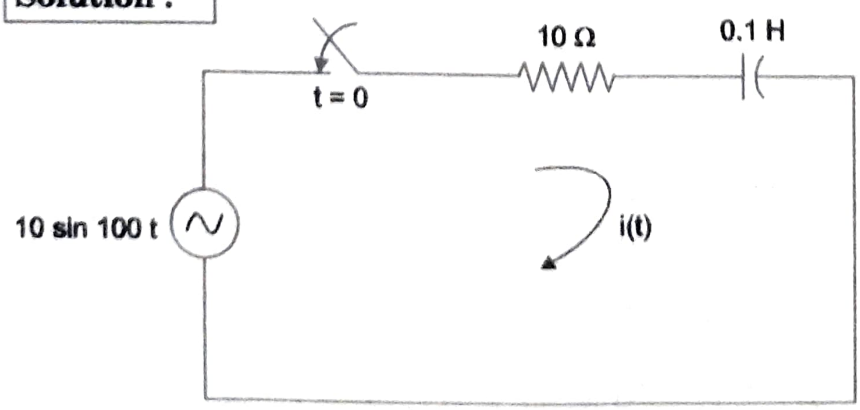
Taking inverse Laplace transform,

$$i(t) = \frac{1250}{(50+j)}\,e^{-j\,50t} = \frac{1250}{(50-j)}\,e^{j\,50t} - 1.98\,e^{-t}\ \text{A}$$

EXAMPLE 33: *A sinusoidal voltage of 10 sin 100 t is connected in series with a switch and R = 10 Ω and L = 0.1 H. If the switch is closed at t = 0, determine the transient current i (t).*

HOTS **(AU/EEE - Nov. 201**

Solution :



$$R = 10\,\Omega \quad L = 0.1\,H \quad v = 10 \sin 100\,t$$

$$10\,i\,(t) + 0.1 \frac{di\,(t)}{dt} = 10 \sin 100\,t$$

Taking Laplace transform on both sides, assuming zero initial conditions.

$$10\,I\,(s) + 0.1\,sI\,(s) = 10 \times \frac{100}{s^2 + 100^2}$$

$$I\,(s)\,[10 + 0.1\,s] = \frac{1000}{s^2 + 100^2}$$

$$I\,(s)\,0.1 \left[s + \frac{10}{0.1} \right] = \frac{1000}{s^2 + 100^2}$$

$$I\,(s)\,[s + 100] = \frac{1000}{0.1\,(s^2 + 1000^2)}$$

$$I\,(s)\,[s + 100] = \frac{10000}{s^2 + 100^2}$$

$$I\,(s) = \frac{10000}{(s + 100)\,(s^2 + 100^2)}$$

$$I\,(s) = \frac{10000}{(s + j\,100)\,(s - j\,100)\,(s + 100)}$$

$$\frac{10000}{(s + j\,100)\,(s - j\,100)\,(s + 100)} = \frac{A}{(s + j\,100)} + \frac{B}{(s - j\,100)} + \frac{C}{(s + 100)}$$

$$10000 = A\,(s - j100)\,(s + 100) + B\,(s + j100)\,(s + 100) + C\,(s + j100)\,(s - j100)$$

Put $s = -j\,100$

$$10000 = A\,(-j\,100 - j\,100)\,(-j\,100 + 100)$$

$$10000 = A\,(-j\,200)\,(-j\,100 + 100)$$

$$10000 = A\,(j^2\,20000 - j\,20000)$$

$$10000 = 10000\,A\,(-2 - j2)$$

$$A = \frac{1}{(-2 - j2)} \times \frac{(-2 + j2)}{(-2 + j2)} = -0.25 + j\,0.25$$

Put $S = j\,100$

$$10000 = B\,(j100 + j100)\,(j100 + 100)$$

$$10000 = B\,(j200)\,(j100 + 100)$$

$$10000 = B(-20000 + j20000)$$

$$10000 = 10000\, B(-2 + j2)$$

$$B = \frac{1}{-2 + j2} \times \frac{(-2 - j2)}{(-2 - j2)}$$

$$B = -0.25 - j0.25$$

Put $S = -100$

$$10000 = C(-100 + j\,100)(-100 - j\,100)$$

$$10000 = C(10000 + 10000)$$

$$C = \frac{10000}{20000} = \frac{1}{2} = 0.5$$

$$I(s) = \frac{-0.25 + j\,0.25}{s + j\,100} + \frac{-0.25 - j\,0.25}{s - j\,100} + \frac{0.5}{s + 100}$$

Taking inverse Laplace transform

$$i(t) = (-0.25 + j\,0.25)\, e^{-j\,100\,t} + (-0.25 - j\,0.25)\, e^{j\,100\,t} + 0.5\, e^{-100t}$$

$$i(t) = -0.25\, e^{-j\,100\,t} + j\,0.25\, e^{-j\,100\,t} - 0.25\, e^{j\,100\,t} - j\,0.25\, e^{j\,100\,t} + 0.5\, e^{-100\,t}$$

$$i(t) = -0.25\,(e^{j\,100\,t} + e^{-j\,100t}) + (-j\,0.25)\,(e^{j\,100\,t} - e^{-j\,100\,t}) + 0.5\, e^{-100\,t}$$

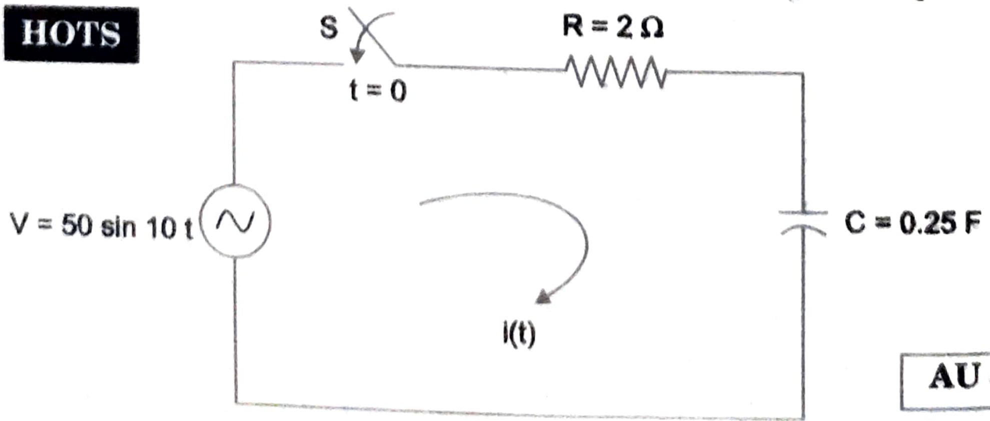
$$i(t) = (-0.25)\, 2\cos 100\,t + (-j\,0.25)\,(2j)\sin 100\,t + 0.5\, e^{-100t}$$

$$i(t) = -0.5\cos 100\,t + 0.5\sin 100\,t + 0.5\, e^{-100\,t}$$

$$i(t) = 0.5\,[-\cos 100t + \sin 100t + e^{-100\,t}]\ A$$

EXAMPLE 34: *A sinusoidally varying voltage of $V = 50 \sin 10\,t$ is applied to a series RC circuit shown in figure, is at time $t = 0$, $R = 2\ \Omega$ and $C = 0.25$ F. Find the equation for the current in the circuit assuming initial change on capacitor to be zero.*

HOTS



AU Chennai June 2011

Solution :

$$2\,i\,(t) + \frac{1}{0.25}\int i\,(t)\,dt = 50 \sin 10\,t$$

Taken Laplace transform on both sides, with zero initial condition

$$2\,I\,(s) + 4\,\frac{I\,(s)}{s} = 50\,\frac{10}{s^2+10^2}$$

$$I\,(s)\left[2 + \frac{4}{s}\right] = \frac{500}{s^2+10^2}$$

$$I\,(s)\left[\frac{2s+4}{s}\right] = \frac{500}{s^2+10^2}$$

$$I\,(s)\,[2s+4] = \frac{500\,s}{s^2+10^2}$$

$$I\,(s)\,2\,(s+2) = \frac{500\,s}{s^2+10^2}$$

$$I(s) = \frac{500\,s}{2\,(s^2+10^2)}$$

$$I\,(s) = \frac{250\,s}{(s+j\,10)\,(s-j\,10)\,(s+2)}$$

$$\frac{250\,s}{(s+j\,10)\,(s-j\,10)\,(s+2)} = \frac{A}{(s+j\,10)} + \frac{B}{(s-j\,10)} + \frac{C}{(s+2)}$$

$$250\,s = A\,(s-j\,10)\,(s+2) + B\,(s+j\,10)\,(s+2) + C\,(s+j\,10)\,(s-j\,10)$$

Put $s = j\,10$

$$250\,(j\,10) = B\,(j\,10 + j\,10)\,(j\,10+2)$$

$$j\,2500 = B\,(j\,20)\,(j\,10+2)$$

$$125 = B\,(j10+2)$$

$$B = \frac{125\,(2-j10)}{(2+j10)\,(2-j10)} = 2.4 - j\,12.02$$

Put $s = -j10$

$$250\,(-j\,10) = A\,(-j10-j\,10)\,(-j10+2)$$

$$-j2500 = A\,(-j20)\,(2-j10)$$

$$125 = A\,(2 - j\,10)$$

$$A = \frac{125\,(2 + j10)}{(2 - j10)\,(2 + j10)}$$

$$A = 2.4 + j\,12.02$$

Put $s = -2$

$$250\,(-2) = C\,(-2 + j10)\,(-2 - j10)$$

$$-500 = C\,(4 + 100)$$

$$-500 = C\,104$$

$$C = \frac{-500}{104} = -4.81$$

$$I\,(s) = \frac{2.4 + j\,12.02}{s + j10} + \frac{2.4 - j\,12.02}{s - j10} - \frac{4.81}{s + 2}$$

Taking inverse Laplace transform

$$i\,(t) = (2.4 + j\,12.02)\,e^{-j10t} + (2.4 - j12.02)\,e^{j10t} - 4.81 e^{-2t}$$

$$i\,(t) = 2.4\,e^{-j10t} + j12.02\,e^{-j10t} + 2.4\,e^{j10t} - j12.02 e^{j10t} - 4.81\,e^{-2t}$$

$$i\,(t) = 2.4\,(e^{j10t} + e^{-j10t}) - j1.202\,(e^{j10t} - e^{-j10t}) - 4.81\,e^{-2t}$$

$$i\,(t) = 2.4\,(2\cos 10t) - j12.02\,(2j\sin 10t) - 4.81 e^{-2t}$$

$$\mathbf{i\,(t) = 4.8\cos 10t + 24.04\sin 10t - 4.81\,e^{-2t}\,A}$$

3.4.3 Transient response for RLC circuit

3.4.3.1 Step input

Applying *KVL* to the series *RLC* circuit (Figure 3.15)

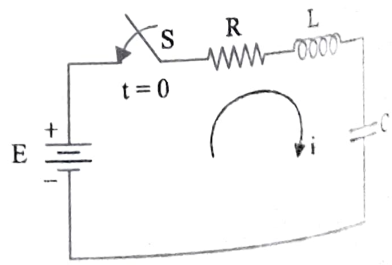


Fig. 3.15

$$iR + L\frac{di}{dt} + \frac{1}{C}\int idt = E$$

Assume no initial charge on the capacitor.

Taking Laplace transform on both sides,

$$RI\,(s) + L\,[sI\,(s) - i\,(0)] + \frac{1}{C}\frac{I\,(s)}{s} = \frac{E}{s}$$