3.4.2 Transient response for RC circuit

3.4.2.1 Step input

Consider the RC series circuit with a DC voltage applied through a switch as shown in figure 3.10. Let the capacitance have an initial charge of Q_0 coulombs.

The initial voltage on the capacitor $V_0 = \frac{Q_0}{C}$

Applying KVL, we get

$$iR + \frac{1}{C}\int i\,dt + V_0 = E$$

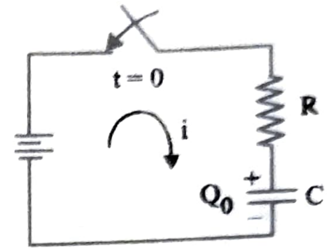


Fig. 3.10

If there is no initial charge $Q_0 = 0$ (i.e) $V_0 = 0$

$$\therefore\ iR + \frac{1}{C}\int i\,dt = E$$

The above integral equation may be solved by using Laplace transformation.

Taking Laplace transform on both sides,

$$RI(s) + \frac{1}{C}\frac{I(s)}{s} = \frac{E}{s}$$

$$\left(R + \frac{1}{sC}\right)I(s) = \frac{E}{s}$$

$$I(s) = \frac{E}{s\left(R + \frac{1}{sC}\right)}$$

$$I(s) = \frac{E}{sR + \frac{1}{C}}$$

$$I(s) = \frac{E}{R\left(s + \frac{1}{RC}\right)}$$

Taking inverse Laplace transform on both sides,

$$\boxed{i(t) = \frac{E}{R}e^{-t/RC}}$$

$$i(t)\big|_{t=0} = \frac{E}{R}$$

$$i(t)\big|_{t=\infty} = 0$$

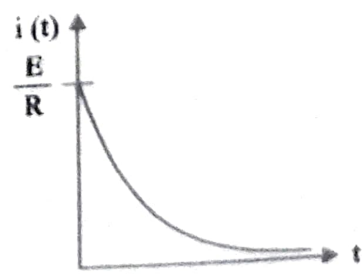


Fig. 3.11

The initial value of the current is $\frac{E}{R}$ amperes, and the final steady state value is zero ampere.

$$\boxed{\text{Voltage across the resistor } e_R = iR = E\, e^{-t/RC}}$$

Voltage across the capacitor $\quad e_c = \frac{1}{C}\int_0^t i(t)\, dt$

$$e_c = \frac{E}{CR}\int_0^t e^{-t/RC}\, dt$$

$$e_c = \frac{E}{CR}\left[\frac{-e^{-t/RC}}{1/RC}\right]_0^t$$

$$e_c = -E\left(e^{-t/RC}\right)_0^t$$

$$e_c = -E\,(e^{-t/RC} - 1)$$

$$\boxed{e_c = E\,(1 - e^{-t/RC})}$$

e_c increases with time when $t = \infty$ (steady state condition), e_c is equal to the applied voltage and opposing it. The current i then becomes zero.

Time constant $\tau = RC$ seconds

$$P_R = e_R\, i = \frac{E^2}{R} e^{-2t/RC}$$

$$P_C = e_C\, i = E\,(1 - e^{-t/RC})\frac{E}{R}(e^{-t/RC})$$

$$P_C = \frac{E^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$$

The energy stored in the capacitor is obtained by integrating P_C from 0 to ∞.

$$E_C = \frac{E^2}{R}\int_0^\infty (e^{-t/RC} - e^{-2t/RC})\, dt$$

$$E_C = \frac{E^2}{R}\left[\frac{e^{-t/RC}}{-1/RC} + \frac{e^{-2t/RC}}{2/RC}\right]_0^\infty$$

$$E_C = \frac{E^2}{R} \times \frac{RC}{2}$$

$$E_C = \frac{1}{2} CE^2$$

At $t = RC$ sec, $i = \frac{E}{R} e^{-1}$

$= 36.8\%$ of the initial current

Hence, time constant $\tau = RC$ seconds.

3.4.2.2 Source free (*RC* Decaying transient)

Consider the circuit of figure 3.12. The switch has been in position 1 for sufficient time to establish steady state conditions and at $t = 0$, the switch is moved to position 2.

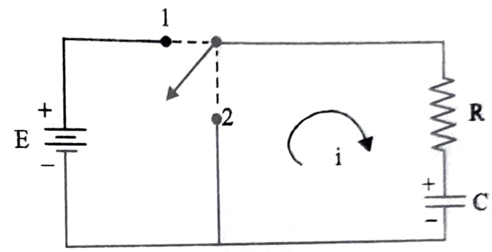


Fig. 3.12

Before the switch is moved to position 2, the capacitor gets charged to the voltage E with the polarity as shown.

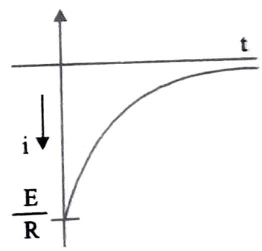


Fig. 3.13

The equation of the circuit is

$$iR + \frac{1}{C}\int i\,dt + E = 0$$

Taking Laplace transform on both sides

$$I(s)R + \frac{1}{C}\frac{I(s)}{s} = \frac{-E}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{-E}{s}$$

$$I(s) = \frac{-E}{s(R + 1/Cs)}$$

$$I(s) = \frac{-E}{Rs + \frac{1}{C}}$$

$$I(s) = \frac{-E}{R(s + 1/RC)}$$

Taking inverse Laplace transform, $i(t) = \frac{-E}{R} e^{-t/RC}$

EXAMPLE 24: *A resistor R and a 2 μF capacitor are in series across a 100 V DC supply. Across the capacitor is a cold cathode lamp which strikes at 60 V. Calculate R so that the lamp strikes 5 second after the switch is closed. If R = 5 M Ω after how much time would the lamp glow?*

LOTS **(AU/ Coimbatore/EEE - June 2009)**

Solution :

Case (i)

$$e_c = E\left[1 - e^{\frac{-t}{\tau}}\right] \qquad ...(1)$$

Here $e_c = 120$ Volts, E = 200 Volts, $t_1 = 5$ sec, $\tau_1 = R_1C$

Substituting the values, we get

$$120 = 200\left[1 - e^{\frac{-5}{\tau_1}}\right]$$

$$\therefore e^{\frac{-5}{\tau_1}} = 0.4$$

$$\frac{-5}{\tau_1} = \log_e 0.4 = \ln 0.4 = 0.916$$

$$\therefore \tau_1 = \frac{5}{0.916} = 5.4 \text{ sec}$$

$$R_1 = \frac{\tau_1}{C} = \frac{5.4}{2 \times 10^{-6}}$$

$$R_1 = 2.7 \times 10^6\ \Omega = 2.7\ M\Omega$$

Case (ii)

Here, $R_2 = 5\ M\Omega$

$$\tau_2 = R_2C = 5 \times 10^6 \times 2 \times 10^{-6} = 10 \text{ seconds}$$

Substituting in equation (1)

$$120 = 200\left[1 - e^{-\frac{t_2}{\tau_2}}\right]$$

$$e^{-\frac{t_2}{\tau_2}} = 0.4$$

$$\frac{-t_2}{\tau_2} = \ln 0.4 = -0.916$$

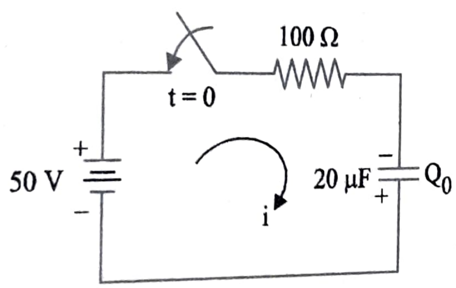
$$t_2 = 0.916 \times \tau_2$$

$$t_2 = 0.916 \times 10$$

$$t_2 = 9.16 \text{ seconds}$$

EXAMPLE 25: *The 20 μF capacitor in circuit of figure has an initial charge $Q_0 = 0.001$ coulomb as shown. The switch is closed at t = 0. Find the transient current.*

HOTS



Solution :

The differential equation of the circuit is given by

$$100\, i + \frac{1}{C} \int i\, dt - \frac{Q_0}{C} = 50$$

$$100\, i + \frac{1}{C} \int i\, dt = 50 + \frac{0.001}{20 \times 10^{-6}}$$

$$100\, i + \frac{1}{C} \int i\, dt = 100$$

Take Laplace transform on both sides

$$100\, I(s) + \frac{1}{C} \frac{I(s)}{s} = \frac{100}{s}$$

$$I(s)\left[100 + \frac{1}{Cs}\right] = \frac{100}{s}$$

$$I(s) = \frac{100}{s\,(100 + 1/Cs)}$$

$$I(s) = \frac{100}{(100s + 1/C)}$$

$$I(s) = \frac{100}{100\,(s + 1/100\,C)}$$

$$I(s) = \frac{1}{s + \frac{1}{100\,C}}$$

Taking inverse Laplace transform $i\ (t) = e^{(-1/100\ C)t}$

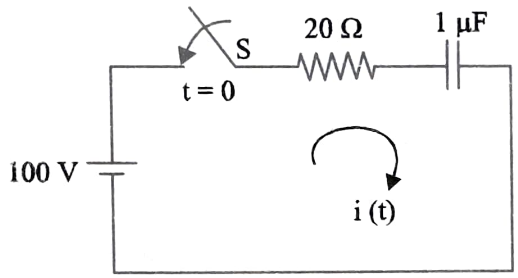
Substituting $c = 20\ \mu\ F$,

we get,

$$\boxed{i\ (t) = e^{-500t}}$$

EXAMPLE 26: *In the circuit shown in figure, find the time when the voltage a capacitor becomes 25 V, after the switch is closed at t = 0.*

HOTS



Solution :

Applying *KVL*,

$$20i\ (t) + \frac{1}{10^{-6}} \int_0^t i\ (t)\ dt = 100$$

Taking Laplace transform on both sides,

$$20I\ (s) + 10^6 \frac{I\ (s)}{s} = \frac{100}{s}$$

$$20s\ I\ (s) + 10^6\ I\ (s) = 100$$

$$I\ (s)\ [20s + 10^6] = 100$$

$$I\ (s) = \frac{100}{20s + 10^6}$$

$$I\ (s) = \frac{100}{10^6 \left(\frac{20}{10^6} s + 1\right)}$$

$$I\ (s) = \frac{100 \times 10^{-6}}{20 \times 10^{-6} \left(s + \frac{1}{20 \times 10^{-6}}\right)}$$

$$I\ (s) = \frac{5}{s + 50000}$$

Taking inverse Laplace transform

$$i(t) = 5e^{-50000t}$$

Voltage across the resistor $e_R = 20 \times 5e^{-50000t}$

$$e_R = 100\,e^{-50000t}$$

Voltage across the capacitor $e_C = E - e_R$

$$e_c = 100 - 100\,e^{-50000t}$$

$$e_c = 100\,(1 - e^{-50000t})$$

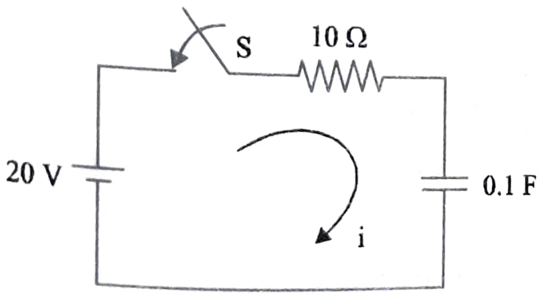
Given $\quad e_c = 25$ V

$$25 = 100\,(1 - e^{-50000t})$$

Hence, $\boxed{t = 5.75\ \mu\text{s}}$

EXAMPLE 27: *A series RC circuit consists of a resistor of 10 Ω and a capacitor of 0.1 F as shown in figure. A constant voltage of 20 V is applied to the circuit at t = 0. Obtain the current equation. Determine the voltage across the resistor and the capacitor.*

HOTS



(AU/EEE - Dec 2005 , May 2004)

Solution :

By applying *KVL*, we get

$$10i + \frac{1}{0.1}\int i\,dt = 20$$

Taking Laplace transform on both sides,

$$10I(s) + \frac{1}{0.1}\frac{I(s)}{s} = \frac{20}{s}$$

$$I(s)\left[10 + \frac{1}{0.1s}\right] = \frac{20}{s}$$

or $i + \int i dt = 2$

$$I(s) + \frac{1}{s}I(s) = \frac{2}{s}$$

$$(s+1)\,I(s) = 2$$

$$\therefore\ I(s) = \frac{2}{s+1}$$

$$I(s) = \frac{\frac{20}{(s+1)}}{0.1s}$$

$$I(s) = \frac{2}{(s+1)}$$

Taking inverse Laplace transform,

$$i(t) = 2e^{-t}$$

Voltage across the resistor $e_R = iR = 2e^{-t} \times 10$

$$\boxed{\mathbf{e_R = 20e^{-t}\ volts}}$$

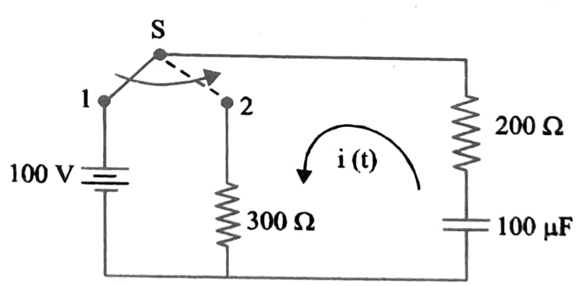
Voltage across the capacitor $e_c = E - e_R$

$$e_c = 20 - 20e^{-t}$$

$$\boxed{\mathbf{e_c = 20(1 - e^{-t})\ V}}$$

EXAMPLE 28: *In the circuit of figure, the switch s is in position 1 till steady state conditions are reached and then moved to 2. Find the energy dissipated in the two resistors. Show that this is equal to the energy stored in the capacitor before moving the switch.*

HOTS



Solution :

When steady state conditions have been reached in position 1, the capacitor has a voltage of 100 V across it. On moving the switch to 2, the equation for the circuit is

$$500i + \frac{1}{C}\int i\,dt = 100$$

Taking Laplace transform on both sides

$$500I(s) + \frac{I(s)}{Cs} = \frac{100}{s}$$

$$I(s)\left[500 + \frac{1}{Cs}\right] = \frac{100}{s}$$