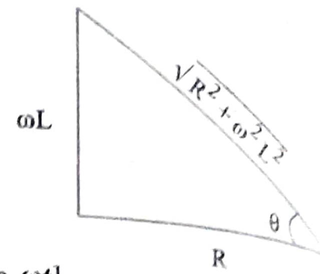
$$\sin\theta = \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}}\ ;\ \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\cos\theta = \frac{R}{\sqrt{R^2+\omega^2 L^2}}$$



$$i\,(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\,[e^{-\frac{R}{L}t}\sin\theta - \sin\theta\cos\omega t + \cos\theta\sin\omega t]$$

$$i\,(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\,[e^{-\frac{R}{L}t}\sin\theta + \sin(\omega t - \theta)]$$

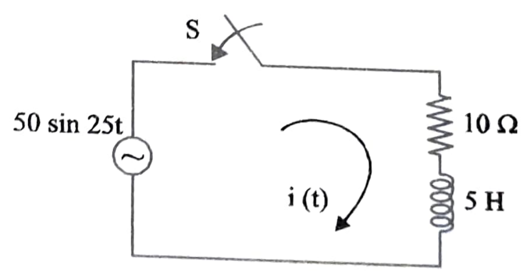
If the applied voltage is of the form $V_m \sin(\omega t + \phi)$ then in the solution, we substitute for ωt the value $(\omega t + \phi)$. The current is given by

$$i\,(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\,[e^{-\frac{R}{L}t}\sin\theta + \sin(\omega t + \phi - \theta)]$$

where θ is defined as before.

EXAMPLE 20: *The circuit shown in figure consists of series RL elements. The sinewave is applied to the circuit when the switch is closed at t = 0. Determine the current i (t).*

HOTS



Solution :

Applying *KVL*,

$$10i\,(t) + \frac{5\,di\,(t)}{dt} = 50\sin 25\,t$$

$$2i(t) + \frac{di(s)}{dt} = 10\sin 25\,t$$

Applying Laplace transform on both sides,

$$5\,[sI\,(s) - i\,(0)] + 10I\,(s) = 50 \times \frac{25}{s^2+(25)^2}$$

$$i\,(0) = 0$$

$$\therefore 5sI(s) + 10I(s) = \frac{50 \times 25}{s^2 + (25)^2}$$

$$I(s) = \frac{1250}{(s^2 + 625)(5s + 10)} = \frac{250}{(s^2 + 625)(s + 2)}$$

$$I(s) = \frac{250}{(s + 2)(s - j25)(s + j25)}$$

$$\frac{250}{(s + 2)(s - j25)(s + j25)} = \frac{A}{s + 2} + \frac{B}{s - j25} + \frac{C}{s + j25}$$

$$250 = A(s - j25)(s + j25) + B(s + 2)(s + j25) + C(s + 2)(s - j25)$$

Put $s = -2$

$$250 = A(-2 - j25)(-2 + j25)$$

$$250 = A(4 + 625)$$

$$\frac{250}{629} = A \Rightarrow A = 0.3974$$

Put $s = -j25$

$$250 = C(-j25 + 2)(-j25 - j25)$$

$$250 = C(-j25 + 2)(-j50)$$

$$\Rightarrow \quad 5 = C(-25 - 2j)$$

$$-5 = C(25 + j2)$$

$$C = \frac{-5}{25 + j2}$$

Put $s = j25$

$$250 = B(j25 + 2)(j25 + j25)$$

$$250 = B(j25 + 2)(j50)$$

$$250 = B(j^2 25 + j2)(50)$$

$$5 = B(-25 + j2)$$

$$B = \frac{-5}{(25 - j2)}$$

Substituting the values of A, B, C in $I(s)$,

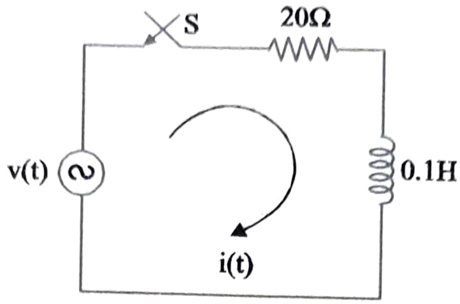
$$I(s) = \frac{0.3974}{s + 2} - \frac{5}{(s - j25)(25 - j2)} - \frac{5}{(s + j25)(25 + j2)}$$

Taking inverse Laplace transform on both sides

$$i(t) = 0.3974\, e^{-2t} - \frac{5}{(25 - j2)} e^{j25t} - \frac{5}{(25 + j2)} e^{-j25t} \text{ A}$$

EXAMPLE 21: *For the circuit shown in figure, determine the complete solution for the current, when switch S is closed at t = 0. Applied voltage is v (t) = 100 cos* $\left(1000t + \frac{\pi}{2}\right)$ *resistance R = 20 Ω and inductance L = 0.1 H.*

HOTS



Solution :

$$v(t) = 100\cos(1000t + \pi/2)$$

$$V(s) = \frac{-100000}{s^2 + 1000^2}$$

$$Ri(t) + L\frac{di(t)}{dt} = v(t)$$

$$I(s) = \frac{V(s)}{R + sL} = \frac{V(s)}{L(s + R/L)} = \frac{-100000}{(s^2 + 1000^2)\,L\left(s + \frac{R}{L}\right)}$$

$$I(s) = \frac{-100000}{(s^2 + 1000^2)\,0.1\left(s + \frac{20}{0.1}\right)} = \frac{-100000/0.1}{(s^2 + 1000^2)(s + 200)}$$

$$I(s) = \frac{-100000}{(s^2 + 1000^2)(s + 200)} = \frac{-1000000}{(s + 1000j)(s - 1000j)(s + 200)}$$

$$\frac{-1000000}{(s + 1000j)(s - 1000j)(s + 200)} = \frac{K_1}{s + 1000j} + \frac{K_2}{s - 1000j} + \frac{K_3}{s + 200}$$

$$K_1 = \left|\frac{-1000000}{(s - 1000j)(s + 200)}\right|_{s = -1000j}$$

$$K_1 = -(-0.48 + 0.096j) = 0.48 - 0.096j$$

$$K_2 = K_1^*$$

$$K_2 = -(-0.48 - 0.96j) = 0.48 + 0.096j$$

$$K_3 = \left. \frac{-1000000}{(s+1000j)(s-1000j)} \right|_{s=-200}$$

$$K_3 = -0.96$$

$$I(s) = \frac{0.48-0.096j}{s+1000j} + \frac{0.48+0.096j}{s-1000j} + \frac{-0.96}{s+200}$$

Taking inverse Laplace transform,

$$i(t) = -(-0.48 + j\,0.096j)\,e^{-1000jt} - (-0.48 - 0.096j)\,e^{1000jt} - 0.096e^{-200t}$$

$$i(t) = -\left[-0.48\,e^{-1000jt} + 0.096\,je^{-1000jt} - 0.48\,e^{1000jt} - 0.096\,je^{1000jt} + 0.096e^{-200t}\right]$$

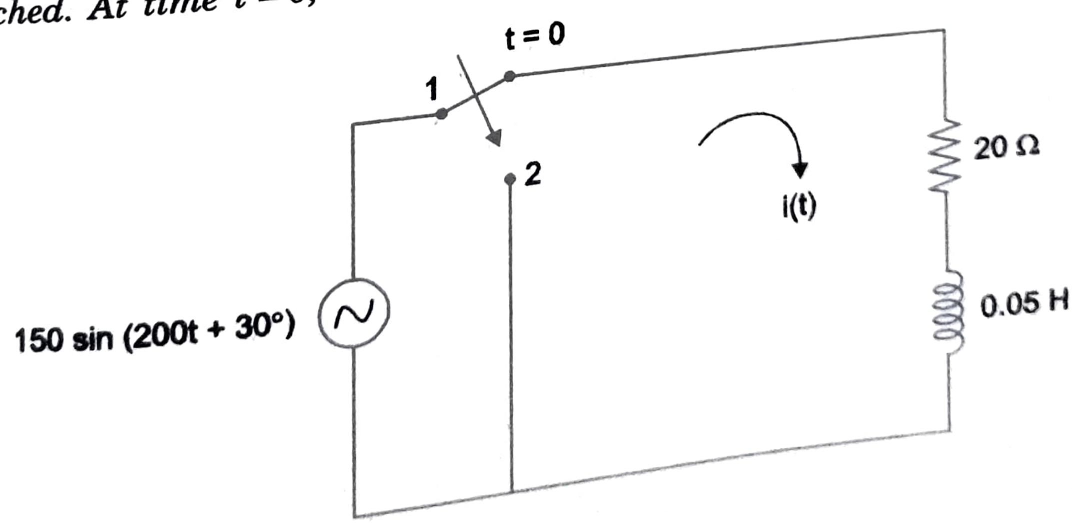
$$i(t) = 0.48\left[e^{j\,1000t} + e^{-j\,1000t}\right] - 0.096j\left[e^{j\,1000t} - e^{-j\,1000t}\right] - 0.96e^{-200t}$$

$$i(t) = 0.48\,[2\cos 1000t] - 0.096j\,[2j]\,\sin 1000t] - 0.96\,e^{-200t}$$

$$\boldsymbol{i(t) = 0.96\cos 1000t - 0.192\sin 1000t - 0.96\,e^{-200t}}$$

EXAMPLE 22: *In the circuit of figure, the switch remains in position - 1 until steady state is reached. At time t = 0, the switch is moved to position - 2. Find i (t)*

HOTS



Solution :

When switch is in position - 1

In position - 1, the circuit have attained steady state. The steady state of the RL circuit with switch in position - 1 is shown in figure. Let I_0 be the magnitude of rms value of current flowing in the circuit.

$$v_m = 150\text{ V},\ w = 200\text{ rad/s},\ \phi = 30^\circ$$

$$I_0 = \frac{\frac{150}{\sqrt{2}}}{\sqrt{20^2 + (200\times 0.05)^2}} = 4.7434\text{ A}$$