3.3.4 Impulse input signal

Figure 3.4 shows the impulse signal.

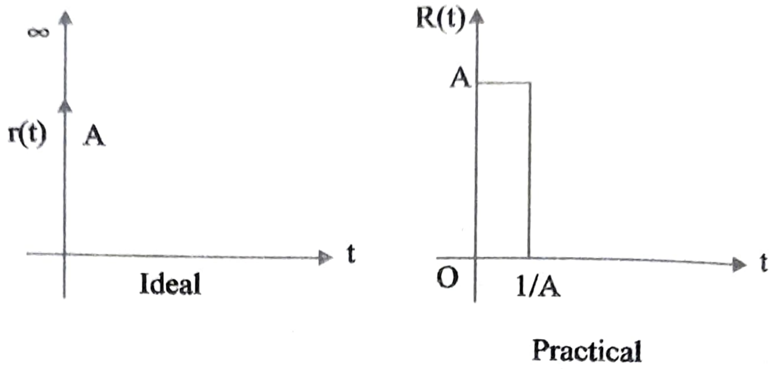


Fig. 3.4

The impulse function is zero for all t not equal to 0 and it is infinity at $t = 0$ it rises to infinity at $t = 0^-$ and comes back to zero at $t = 0^+$ enclosing a finite area. If the area is A, it is called as an impulse function of strength A. If $A = 1$. Then it is called unit impulse function.

Mathematically it is denoted as

$$r(t) = \begin{cases} \infty & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$

Laplace transform of impulse function is unity.

3.4 TRANSIENT RESPONSE

Learning Objective (LO 4)

- ***Students will be able to analyze the transient response of RL, RC and RLC circuit.***

Steady state

A circuit having constant sources is said to be in steady state if the currents and voltages do not change with time. Thus, circuits with currents and voltages having constant amplitude and constant frequency sinusoidal functions are also considered to be in a steady state. That means, the amplitude and frequency of a sinusoidal signal never change in a steady state circuit.

Transient state

In a network containing energy storage elements, with change in excitation, the currents and voltages change from one state to another state. The behaviour of the voltage and current when it is changed from one state to other state is called the transient state.

Transient time

The time taken for the circuit to change from one steady state to another steady state is called the transient time.

Natural and Forced responses

When we consider a circuit containing storage elements, which are independent of the source, the response depends upon the nature of the circuit and is called the natural response. It depends upon the type of elements, their size and the their connection. This response is independent of the source. A natural response dies out gradually. That is, it approaches zero as time becomes infinite. The natural response is also known as transient response.

When we consider sources acting on a circuit, the response depends on the nature of the source or sources. This response is called forced response.

The complete response of a circuit consists of two parts. The forced response and the transient response. When we consider a differential equation, the complete solution consists of two parts; the complementary function and the particular integral. The complementary function dies out after short interval, and is referred to as the transient response or source free response. The particular integral is the steady state response (or) the forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. After forming the differential equation, several methods can be used to find out the complete solution.

3.4.1 Transient response of RL circuit

3.4.1.1 Step input

An RL circuit shown in figure 3.5 is connected to a battery through a switch S. Assume that the switch is closed at time $t = 0$ and also assume that at the time of switching, the current is zero.

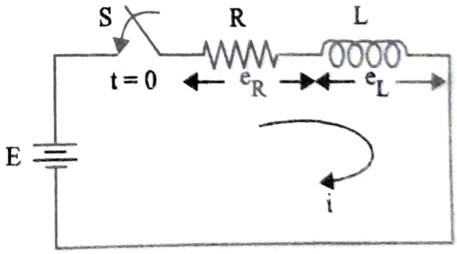


Fig. 3.5

Applying KVL for the loop $E = e_R + e_L$

Voltage across the resistor $e_R = iR$

Voltage across the inductor $e_L = L\dfrac{di}{dt}$

$$E = iR + L\frac{di}{dt} \qquad ...(1)$$

This differential equation may be solved by using Laplace transformation technique. Taking Laplace transforms on both sides of the equation

$$RI(s) + L\,[sI(s) - i(0)] = \frac{E}{s} \qquad ...(2)$$

We know that just before closing the switch, the current $i(0) = 0$

$\therefore$ Equation (2) reduces to

$$RI(s) + LsI(s) = \frac{E}{s}$$

or
$$I(s)\,[R + sL] = \frac{E}{s}$$

i.e.,
$$I(s) = \frac{E}{s(R + sL)}$$

Yet
$$\frac{E}{s(R + sL)} = \frac{A}{s} + \frac{B}{R + sL} \text{ (say)}$$

$$\therefore \quad E = A(R + sL) + Bs$$

Put $s = 0$

$$E = AR \Rightarrow A = \frac{E}{R}$$

Put $s = \frac{-R}{L}$

$$E = \frac{-BR}{L} \quad ; \quad B = \frac{-EL}{R}$$

$$I(s) = \frac{E/R}{s} - \frac{EL/R}{R + sL}$$

$$I(s) = \frac{E}{Rs} - \frac{EL}{R(R + sL)}$$

$$I(s) = \frac{E}{Rs} - \frac{EL}{RL\,[s + R/L]} = \frac{E}{Rs} - \frac{E}{R(s + R/L)}$$

Taking inverse Laplace transform on both sides

$$L^{-1}[I(s)] = \frac{E}{R} L^{-1}(1/s) - \frac{E}{R} L^{-1}\left(\frac{1}{s + R/L}\right)$$

$\therefore$
$$i(t) = \frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}t}$$

or
$$i(t) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad \text{...(3)}$$

The current is said to rise exponentially. The final value of the current may be obtained by substituting $t = \infty$ in the equation for $i(t)$.

$$i(t)\Big|_{t=\infty} = \frac{E}{R}$$

Figure 3.6 shows the transition period during which the current varies from zero to the steady state value. To determine the current at any time t_1, after the switch has been closed, we have to substitute $t = t_1$ in the equation.

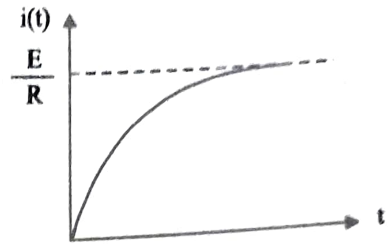


Fig. 3.6

Consider $t = \tau = L/R$

$$i(t)\Big|_{t=\frac{L}{R}} = \frac{E}{R}\left(1 - e^{-\frac{R}{L}\times\frac{L}{R}}\right)$$

$$= \frac{E}{R}(1 - e^{-1}) \qquad (e^{-1} = 0.3678)$$

$$= 0.632\frac{E}{R}$$

= 63.2% of the steady state (or) final value

Therefore, the time constant of an RL series circuit is defined as the time during which the current increases to 63.2% of its steady state value.

Note: Time constant $= \frac{L}{R}$

The transient voltage across the resistance

$$e_R = i\,R$$

$$e_R = \frac{E}{R}\left(1 - e^{\frac{-R}{L}t}\right)R$$

$$e_R = E\left(1 - e^{-\frac{R}{L}t}\right)$$

The transient voltage across the inductor

$$e_L = L\frac{di}{dt}$$

$$e_L = L\frac{d}{dt}\left(\frac{E}{R}\left(1 - e^{\frac{-R}{L}t}\right)\right)$$

$$e_L = \frac{E}{R}L\frac{d}{dt}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$e_L = \frac{EL}{R}\left[-e^{\frac{-R}{L}t}\left(\frac{-R}{L}\right)\right]$$

$$e_L = E\,e^{-\frac{R}{L}t}$$

EXAMPLE 7: *A D.C voltage of 100 volts is applied to a series RL circuit with $R = 25\ \Omega$. What will be the current in the circuit at twice the time constant?*

LOTS

Solution :

$$E = 100\text{ V}, R = 25\ \Omega$$

$$i(t) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\text{Time constant } \tau = \frac{L}{R}$$

$$i(t) = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Given $t = 2\tau$

$$i(t) = \frac{100}{25}\left(1 - e^{-\frac{2\tau}{\tau}}\right)$$

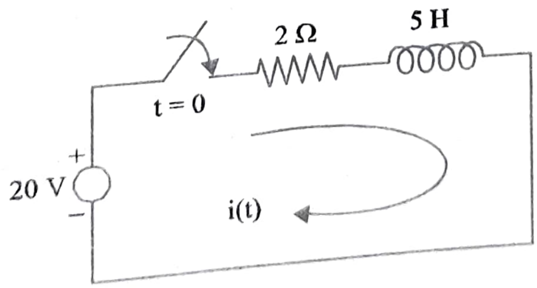
$$i(t) = 4\left(1 - e^{-2}\right)$$

$$i(t) = 4(1 - 0.1353)$$

$$\boldsymbol{i(t) = 3.45\text{ A}}$$

EXAMPLE 8: *In the circuit shown in figure, find the expression for transient current after the switch is closed at $t = 0$, assuming zero initial conditions.*

HOTS



Solution :

Applying Kirchhoff's voltage law for the loop,

$$2\,i(t) + 5\frac{d\,i(t)}{dt} = 20$$

Taking Laplace transform on both sides,

$$2I(s) + 5\,(sI(s) - i(0)) = \frac{20}{s}$$

Given $i(0) = 0$

$$\Rightarrow 2I(s) + 5s\,I(s) = \frac{20}{s}$$

$$I(s)\,[2 + 5s] = \frac{20}{s}$$

$$I(s) = \frac{20}{s(5s + 2)}$$

$$I(s) = \frac{4}{s(s + 0.4)}$$

Taking partial fractions

$$\frac{4}{s(s + 0.4)} = \frac{A}{s} + \frac{B}{s + 0.4} = \frac{10}{s} - \frac{10}{s + 0.4}$$

$$I(s) = \frac{10}{s} - \frac{10}{s + 0.4}$$

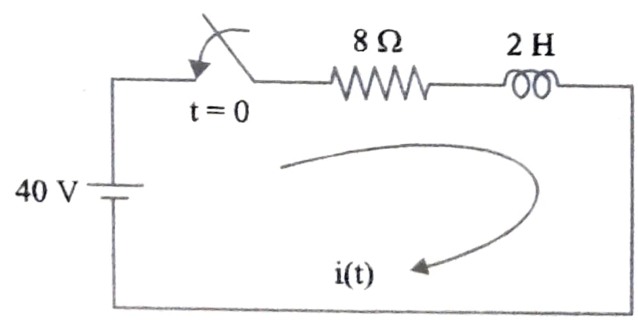
Taking inverse Laplace transform

$$L^{-1}[I(s)] = 10\,L^{-1}\left(\frac{1}{s}\right) - 10\,L^{-1}\left(\frac{1}{s + 0.4}\right)$$

$$\boxed{\boldsymbol{i(t) = 10 - 10\,e^{-0.4t}}}$$

EXAMPLE 9: *In the circuit shown in figure, find the transient current and the initial rate of growth of current when the switch is closed at* $t = 0$.

HOTS



Solution :

Applying Kirchhoff's voltage law for the loop,

$$8\,i(t) + 2\frac{d\,i(t)}{dt} = 40 \quad \text{or} \quad 4\,i(t) + \frac{d\,i(t)}{dt} = 20$$

Taking Laplace transform on both sides,

$$8\,I(s) + 2\,[sI(s) - i(0)] = \frac{40}{s}$$

Assuming zero initial conditions,

$$8\,I(s) + 2\,sI(s) = \frac{40}{s}$$

$$I(s)\,[8 + 2s] = \frac{40}{s}$$

$$I(s) = \frac{40}{s\,(8 + 2s)} = \frac{20}{s\,(s + 4)}$$

Taking partial fractions,

$$\frac{20}{s\,(s + 4)} = \frac{A}{s} + \frac{B}{s + 4}$$

$$20 = A\,(s + 4) + Bs$$

Put $s = 0$

$$20 = 4\,A \Rightarrow A = 5$$

Put $s = -4$; $20 = -4\,B \Rightarrow B = -5$

$$I(s) = \frac{5}{s} - \frac{5}{s + 4}$$

Taking inverse Laplace transform,

$$\boxed{\boldsymbol{i(t) = 5 - 5e^{-4t}\ \mathrm{A}}}$$

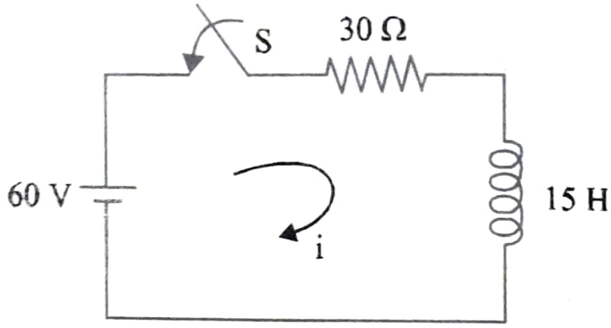
Rate of growth of current $\dfrac{di(t)}{dt} = 20\,e^{-4t}$

Initial rate of growth of current $= \left.\dfrac{di(t)}{dt}\right|_{t=0}$

$$\boxed{\left.\frac{di(t)}{dt}\right|_{t=0} = 20\ \text{A/second}}$$

EXAMPLE 10: *A series RL circuit with R = 30 Ω and L = 15 H has a constant voltage E = 60 V applied at t = 0 as shown in figure. Determine the current i, the voltage across resistor and the voltage across the inductor.*

HOTS



(AU, Tirunelveli/EEE - June 2011) (AU, Chennai/EEE - Dec. 2010) (AU/ECE - May 2005)

Solution:

By applying *KVL*, we get

$$30i + 15\frac{di}{dt} = 60$$

$$2i + \frac{di}{dt} = 4$$

Taking Laplace transform on both sides,

$$2I(s) + [sI(s) - i(0)] = \frac{4}{s}$$

Assuming zero initial conditions

$$2I(s) + sI(s) = \frac{4}{s}$$

$$I(s)[s + 2] = \frac{4}{s}$$

$$I(s) = \frac{4}{s(s+2)}$$

Taking partial fractions,

$$\frac{4}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} \Rightarrow A(s+2) + Bs = 4$$

Put $s = -2$

$4 = -2\,B \Rightarrow B = -2$

Put $s = 0$

$$4 = 2A \Rightarrow A = 2$$

$$I(s) = \frac{2}{s} - \frac{2}{s+2}$$

Taking inverse Laplace transform, we get

$$L^{-1}[I(s)] = 2L^{-1}\left(\frac{1}{s}\right) - 2L^{-1}\left(\frac{1}{s+2}\right)$$

$$i(t) = 2 - 2e^{-2t}$$

$$\boxed{i(t) = 2(1 - e^{-2t})\text{ A}}$$

Voltage across resistor $= iR$

$$e_R = 2(1 - e^{-2t}) \times 30$$

$$\boxed{e_R = 60(1 - e^{-2t})\text{ volts}}$$

Voltage across inductor $e_L = L\frac{di}{dt}$

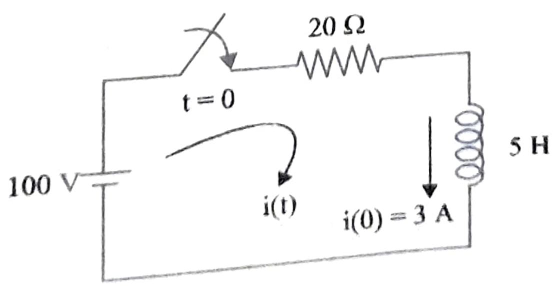
$$e_L = 15\frac{d}{dt}2(1 - e^{-2t})$$

$$e_L = 30 \times 2e^{-2t}$$

$$\boxed{e_L = 60e^{-2t}\text{ volts}}$$

EXAMPLE 11: *In the circuit shown in figure, find the transient voltage across R and L after the switch is closed at time t = 0. Assume the initial current through the inductor before the switch is closed is 3 A.*

HOTS



Solution :

Applying *KVL*, $$20\,i(t) + 5\frac{d\,i(t)}{dt} = 100$$

i.e., $$4i(t) + \frac{di(t)}{dt} = 20$$

Taking Laplace transform,

$$20I\ (s) + 5\ (sI\ (s) - i\ (0)) = \frac{100}{s}$$

$$20I\ (s) + 5\ sI\ (s) - 5 \times 3 = \frac{100}{s}$$

$$I\ (s) = \frac{\frac{100}{s} + 15}{5s + 20} = \frac{100 + 15s}{5s\ (s+4)}$$

$$I\ (s) = \frac{20 + 3s}{s\ (s+4)}$$

Taking partial fractions,

$$\frac{20 + 3s}{s\ (s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$20 + 3s = A\ (s + 4) + Bs$$

Put $s = 0$

$$20 = 4\ A$$

$$A = \frac{20}{4} = 5$$

Put $s = -4$

$$8 = -4B$$

$$B = \frac{8}{-4} = -2$$

$$I\ (s) = \frac{5}{s} - \frac{2}{s+4}$$

Taking inverse Laplace transform,

$$i\ (t) = 5 - 2e^{-4t}\ A$$

Voltage across the resistor $e_R = iR$

$$e_R = 20 \times (5 - 2e^{-4t})$$

$$\boxed{\mathbf{e_R = 100 - 40\ e^{-4t}\ volts}}$$

Voltage across the inductor $e_L = L\frac{di}{dt}$

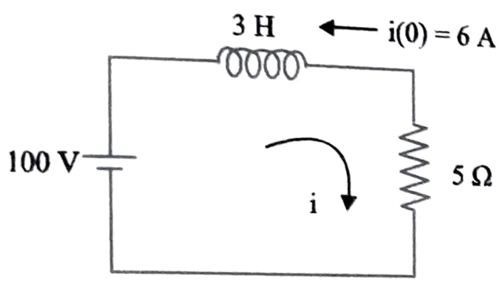
$$e_L = 5\frac{d}{dt}(5 - 2e^{-4t})$$

$$e_L = 5 \times 8e^{-4t}$$

$$\boxed{e_L = 40e^{-4t} \text{ volts}}$$

EXAMPLE 12: *In the circuit of figure, the current in the inductance is – 6 amp at $t = 0$. Find the expression for the transient current. Find also the initial rate of growth of current.*

HOTS



(AU/ECE - Dec 2005)

Solution :

$R = 5\ \Omega, L = 3H, i\ (0) = -6$ A

$V = 100$ volts

The differential equation for the circuit for $t > 0$ is

$$Ri + L\frac{di}{dt} = 100;\ i\ (0) = -6$$

$$5i + 3\frac{di}{dt} = 100$$

Taking Laplace transforms on both sides of the equation,

$$5I\ (s) + 3\ [sI\ (s) - i\ (0)] = \frac{100}{s}$$

$$5I\ (s) + 3\ [sI\ (s) + 6] = \frac{100}{s}$$

$$I\ (s)\ [5 + 3s] = \frac{100}{s} - 18$$

$$I\ (s) = \frac{100 - 18s}{s\ (5 + 3s)} = \frac{100 - 18\ s}{3s\ (s + 5/3)}$$

$$I(s) = \frac{\frac{100}{3} - 6s}{s(s+5/3)} = \frac{A}{s} + \frac{B}{(s+5/3)}$$

$$\frac{100}{3} - 6s = A(s+5/3) + Bs$$

Put $s = 0$

$$\frac{100}{3} = A\frac{5}{3} \quad ; \quad A = \frac{300}{15} = 20$$

Put $s = \frac{-5}{3}$

$$\frac{100}{3} - 6\left(\frac{-5}{3}\right) = B\left(\frac{-5}{3}\right)$$

$$\frac{100}{3} + \frac{30}{3} = \frac{-5B}{3}$$

$$\frac{130}{3} = \frac{-5}{3}B$$

$$-5B = 130$$

$$B = -26$$

$$I(s) = \frac{20}{s} - \frac{26}{(s+5/3)}$$

Taking inverse Laplace transform on both sides

$$\boxed{i(t) = 20 - 26\, e^{-(5/3)t}}$$

Rate of growth $= \frac{di}{dt} = (-26)\left(\frac{-5}{3}\right)e^{-\frac{5}{3}t}$

$$= \frac{130}{3}e^{-\frac{5}{3}t}$$

Initial rate of growth $= \left.\frac{di}{dt}\right|_{t=0} = \frac{130}{3}$

$$\boxed{= 43.333 \text{ A/sec}}$$