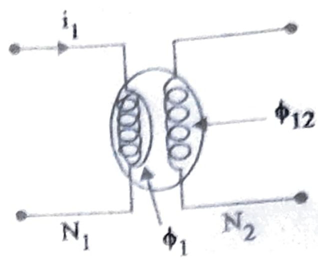
EXAMPLE 58: *Following data refer to the coupled coils 1 and 2 shown in figure.* $\phi_{11} = 0.5$ mwb ; $\phi_{12} = 0.3$ mwb; $N_1 = 100$ *turns;* $N_2 = 500$ *turns;* $i_1 = 1$ A.

Find K, the coefficient of coupling, the inductances L_1 *and* L_2 *and M, the mutual inductance.*



HOTS

Solution :

Total flux of coil-1

$$\phi_1 = \phi_{11} + \phi_{12}$$

$$\phi_1 = 0.5 \text{ mwb} + 0.3 \text{ mwb}$$

$$\phi_1 = 0.8 \text{ mwb}$$

$$L_1 = \frac{N_1 \phi_1}{i_1} = \frac{100 \times 0.8 \times 10^{-3}}{1}$$

$$\boxed{L_1 = 0.08 \text{ H}}$$

Coefficient of coupling $K = \frac{\phi_{12}}{\phi_1} = \frac{0.3}{0.8}$

$$\boxed{K = 0.375}$$

Mutual inductance

$$M = \frac{N_2 \phi_{12}}{i_1} = \frac{500 \times 0.3 \times 10^{-3}}{1}$$

$$\boxed{M = 0.15 \text{ H}}$$

We know $M = K\sqrt{L_1 L_2}$

$\therefore$ $$M^2 = K^2 L_1 L_2$$

or $$L_2 = \frac{M^2}{K^2 L_1} = \frac{(0.15)^2}{(0.375)^2 \times 0.08}$$

$$\boxed{L_2 = 2 \text{ H}}$$

4.2 Single Tuned Circuit

A single tuned circuit contains an adjustable capacitor in the secondary which can be tuned to resonance. It is shown in figure 4.23.

Let
R_S = Source resistance
R_1, R_2 = Resistance of coils 1 and 2 respectively.
L_1, L_2 = Self inductance of coils 1 and 2 respectively.

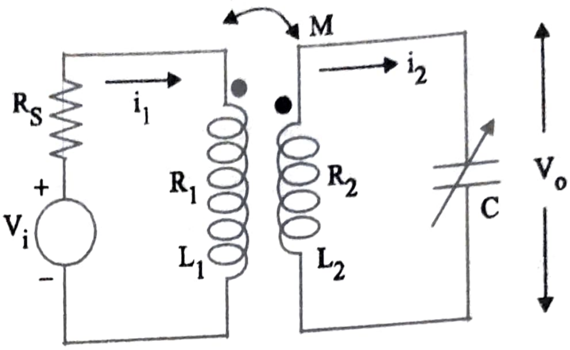


Fig. 4.23

Let $R_S + R_1 + j\omega L_1 = R_S$

with the assumption that $R_S >> R_1$ and $j\omega L_1$

The mesh equations for the circuit are

$$i_1 R_S - j\omega M i_2 = V_i$$

$$-j\omega M i_1 + \left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) i_2 = 0$$

Solving for i_2 we get

$$i_2 = \frac{jV_i \omega M}{R_S\left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) + \omega^2 M^2}$$

The output voltage $V_0 = i_2 \cdot \frac{1}{j\omega C}$

$$V_0 = \frac{jV_1 \omega M}{j\omega C\left\{R_S\left[R_2 + \left(j\omega L_2 - \frac{1}{\omega C}\right)\right] + \omega^2 M^2\right\}}$$

The voltage amplification or voltage transfer function is given by

$$\frac{V_0}{V_i} = A = \frac{M}{C\left\{R_S\left[R_2 + j\left(\omega L_2 - \frac{1}{\omega C}\right)\right] + \omega^2 M^2\right\}}$$

The circuit can be tuned by varying the capacitor C such that at a frequency ω_r, the circuit resonates. At resonant frequency ω_r,

$$\omega_r L_2 = \frac{1}{\omega_r C}$$

The amplification at resonance is given by

$$A = \frac{V_0}{V_i} = \frac{M}{C\,[R_S R_2 + \omega_r^2 M^2]}$$

The current i_2 at resonance is given by

$$i_2 = \frac{jV_i\omega_r M}{R_S R_2 + \omega_r^2 M^2}$$

Thus, it can be observed that the output voltage, current and amplification de on the mutual inductance M at resonant frequency, where $M = K\sqrt{L_1 L_2}$. The max value of V_0 depends on M. To get the condition for maximum output voltage

$$\frac{dV_0}{dM} = 0$$

$$\frac{dV_0}{dM} = \frac{d}{dM}\left[\frac{V_i M}{C\,(R_S R_2 + \omega_r^2 M^2)}\right]$$

$$= 1 - 2M^2\omega_r^2\,[R_S R_2 + \omega_r^2 M^2]^{-1} = 0$$

We get $R_S R_2 = \omega_r^2 M^2$ $\qquad M^2 = \frac{R_s R_2}{\omega_r^2}$ $\qquad M = \frac{\sqrt{R_s R_2}}{\omega_r}$

$$M = \frac{\sqrt{R_S R_2}}{\omega_r}$$

Using the above value of M, we can find the maximum output voltage.

$$V_{0M} = \frac{V_i}{2\omega_r C\sqrt{R_S R_2}}$$

The maximum amplification is given by

$$A_m = \frac{1}{2\omega_r C\sqrt{R_S R_2}} \quad \text{and} \quad i_2 = \frac{jV_i}{2\sqrt{R_S R_2}}$$

The frequency response is shown in figure 4.24.

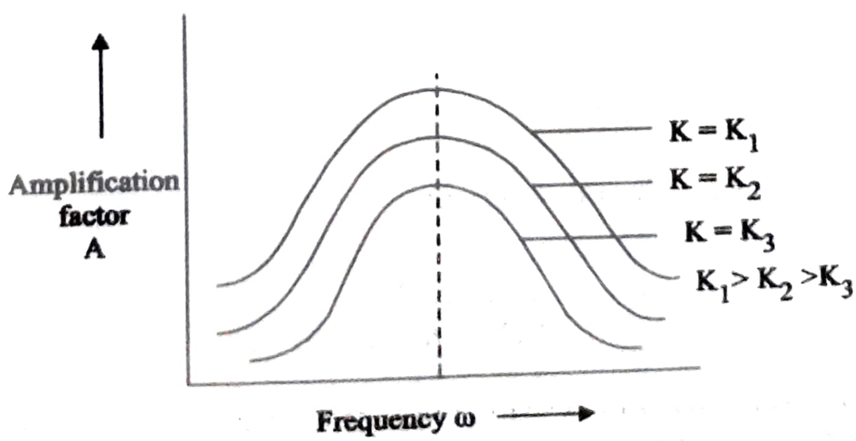


Fig. 4.24