EXAMPLE 46: *Two coils connected in series have an equivalent inductance of 0.4 H, when connected in aiding, and an equivalent inductance of 0.2 H, when the connection is opposing. Calculate the mutual inductance of the coils.*

LOTS **(AU, Trichy/EEE - June 2009) (AU, Chennai/EEE-May 2007)**

Solution :

$$L_1 + L_2 + 2M = 0.4H \text{ ... series aiding}$$

$$L_1 + L_2 - 2M = 0.2\,H \text{ ... series opposing}$$

Solving we get

$$4M = 0.2\text{ H}$$

$$M = 0.05H$$

$\therefore$ Mutual inductance $\boxed{M = 0.05\text{ H}}$

EXAMPLE 47: *Two coupled coils of self inductances $L_1 = 2$ H and $L_2 = 4$ H are coupled in (i) series aiding ; (ii) series opposing; (iii) parallel aiding; (iv) parallel opposing. If the mutual inductance is 0.5 H, find the equivalent inductance in each case.*

HOTS

Solution :

$$L_1 = 2\text{ H}, \quad L_2 = 4\text{ H}, \quad M = 0.5\text{ H}$$

(i) **Series aiding**

$$L_{eq} = L_1 + L_2 + 2M = 2 + 4 + (2 \times 0.5)$$

$$\boxed{L_{eq} = 7\text{ H}}$$

(ii) **Series opposing**

$$L_{eq} = L_1 + L_2 - 2M = 2 + 4 - (2 \times 0.5)$$

$$\boxed{L_{eq} = 5\text{ H}}$$

(iii) **Parallel aiding**

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{2 \times 4 - (0.5)^2}{2 + 4 - (2 \times 0.5)}$$

$$\boxed{L_{eq} = 1.55\text{ H}}$$

(iv) Parallel opposing

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{2 \times 4 - (0.5)^2}{2 + 4 + (2 \times 0.5)}$$

$$\boxed{L_{eq} = 1.1071 \text{ H}}$$

EXAMPLE 48: *Two coils connected in series have an equivalent inductance of 10 H. When the connections of one coil are reversed, the effective inductance is 6 H. If the co-efficient of coupling is 0.6, calculate the self inductance of each coil and the mutual inductance.*

HOTS

(AU, Coimbatore/EEE-May 2010)

Solution :

Co-efficient of coupling $K = 0.6$

$L_1 + L_2 + 2M = 10$ H ... Series aiding

$L_1 + L_2 - 2M = 6$ H ... Series opposing

Solving we get $4M = 4$ H

$$M = 1 \text{ H}$$

Substituting the value of M in the above equations we get

$$L_1 + L_2 + 2M = 10 \text{ H}$$

$$L_1 + L_2 - 2M = 6 \text{ H}$$

$$2\,(L_1 + L_2) = 16 \text{ H}$$

$$L_1 + L_2 = 8 \text{ H} \Rightarrow L_1 = 8 - L_2$$

We know that $K = \frac{M}{\sqrt{L_1L_2}}$

$$M = K\sqrt{L_1L_2}$$

$$M^2 = K^2 L_1L_2$$

$$1^2 = (0.6)^2 (L_1L_2)$$

$$1 = 0.36\,(8 - L_2)\,L_2$$

$$(8 - L_2)\,L_2 = 2.778$$

$$\therefore \quad L_2^2 - 8L_2 + 2.778 = 0$$

$$L_2 = \frac{8 \pm \sqrt{(8)^2 - 4 \times 1 \times (2.778)}}{2}$$

$$L_2 = \frac{8 \pm 7.27}{2}$$

$$L_2 = 7.635 \text{ H (or) } 0.365 \text{ H}$$

When $L_2 = 7.635$ H ; $L_1 = 0.365$ H

When $L_2 = 0.365$ H ; $L_1 = 7.635$ H

EXAMPLE 49: *Two identical coupled coils in series has an equivalent inductance of 0.080 H and 0.0354 H when connected in series aiding and series opposing. Find the values of the inductance, mutual inductance and the co-efficient of coupling.*

LOTS **(AU, Coimbatore/EEE - Dec. 2010)(AU/EEE - May 2008)**

Solution :

The equivalent inductance of two coupled coils in series is given by

$$L_{eq} = L_1 + L_2 \pm 2M$$

$$\therefore \quad L_1 + L_2 + 2M = 0.080$$

$$L_1 + L_2 - 2M = 0.0354$$

$$\therefore \quad 4M = 0.0446$$

or

$$M = \frac{0.0446}{4}$$

Mutual Inductance $M = 0.01115$ H

Then $L_1 + L_2 = 0.080 - 2M = 0.080 - 2 \times 0.0115 = 0.0577$

Since the two coils are identical

$$\therefore \quad L_1 = L_2 = \frac{0.0577}{2} = 0.02885$$

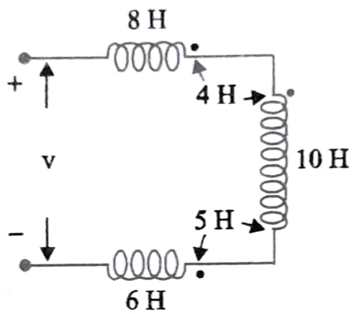
$L_1 = L_2 = 0.02885$ H

Coefficient of coupling $K = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L}$; as $L_1 = L_2 = \frac{0.01115}{0.02885}$

$$K = 0.3865$$

EXAMPLE 50: *Calculate the effective inductance of the circuit shown in figure.*

(AU/ECE - May 2006)



LOTS

Solution :

Let i = Current through the circuit

$$\therefore \quad v = 8\frac{di}{dt} - 4\frac{di}{dt} + 10\frac{di}{dt} - 4\frac{di}{dt} + 5\frac{di}{dt} + 6\frac{di}{dt} + 5\frac{di}{dt} = (34-8)\frac{di}{dt} = 26\frac{di}{dt}$$

Let L = effective inductance of the circuit

Then, the voltage across the circuit

$$v = L\frac{di}{dt} = 26\frac{di}{dt}$$

Hence, the equivalent inductance of the circuit $\boxed{L = 26 \text{ H}}$

EXAMPLE 51: *Two coupled coils with $L_1 = 0.02$ H $L_2 = 0.01$ H and $K = 0.5$ are connected in series aiding arrangement. Obtain the equivalent inductance.*

LOTS

(AU Coimbatore/EEE - Dec 2010)

Solution :

$L_1 = 0.02$ H, $L_2 = 0.01$ H, $K = 0.5$

Series aiding:

Effective inductance $= L_1 + L_2 + 2M$

$$M = K\sqrt{L_1 L_2} = 0.5\sqrt{0.02 \times 0.01} = 0.00707 \text{ H}$$

$$L_{eq} = 0.02 + 0.01 + (2 \times 0.00707)$$

$$\boxed{L_{eq} = 0.044 \text{ H}}$$

EXAMPLE 52: *A coil of 800 μH is magnetically coupled with another coil of 200 μH. The coefficient of coupling between the two coils is 0.05. Calculate inductance if the two coils are connected in (1) Parallel aiding (2) Parallel opposing.*

LOTS

(AU/EEE - Dec 2005)

Solution :

$$L_1 = 800\ \mu\ \text{H},\ L_2 = 200\ \mu\ \text{H},\ K = 0.05$$

$$M = K\sqrt{L_1 L_2} = 0.05\sqrt{800 \times 10^{-6} \times 200 \times 10^{-6}} = 20\ \mu\ \text{H}$$

(i) **Parallel aiding**

$$\text{Effective inductance } L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{800 \times 10^{-6} \times 200 \times 10^{-6} - (20 \times 10^{-6})^2}{800 \times 10^{-6} + 200 \times 10^{-6} - (2 \times 20 \times 10^{-6})}$$

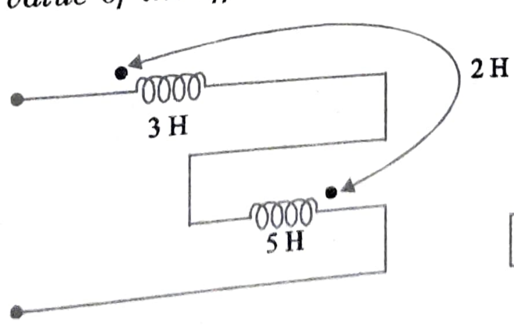
$$\boxed{L_{eq} = 166.25\ \mu\ \text{H}}$$

(ii) **Parallel opposing**

$$\text{Effective inductance } L_{eq} = \frac{L_1 L_2 - M^2}{L_1 L_2 + 2M} = \frac{800 \times 10^{-6} \times 200 \times 10^{-6} - (20 \times 10^{-6})^2}{800 \times 10^{-6} + 200 \times 10^{-6} + (2 \times 20 \times 10^{-6})}$$

$$\boxed{L_{eq} = 153.46\ \mu\ \text{H}}$$

EXAMPLE 53: *Find the value of the effective inductance of the combination.*



(AU/ECE - Dec 2005)

LOTS

Solution :

The current is entering one coil at the dot and leaving the other coil at the dot. So, the coupling is series opposing.

$$\text{Effective inductance } L_{eq} = L_1 + L_2 - 2M = 3 + 5 - 2 \times 2$$

$$\boxed{L_{eq} = 4\ \text{H}}$$

EXAMPLE 54: *Two identical coils with $L = 0.03\,H$ have a coupling coefficient of $K = 0.8$. Find mutual inductance and the equivalent inductance with the coils connected in series opposing mode.*

LOTS

(AU/EEE - May 2004)

Solution :

$$L = L_1 = L_2 = 0.03\text{ H},\ K = 0.8$$

Mutual inductance $M = K\sqrt{L_1 L_2} = 0.8 \times 0.03$

$$M = 0.024\text{ H}$$

Equivalent inductance with the coils connected in series opposing.

$$L_{eq} = L_1 + L_2 - 2M = 0.03 + 0.03 - 2 \times 0.024$$

$$\boxed{L_{eq} = 0.012\text{ H}}$$

EXAMPLE 55: *A coil having an inductance of 100 mH is magnetically coupled to another coil having an inductance of 900 mH. The coefficient of coupling between the coils is 0.45. Calculate the equivalent inductance if the two coils are connected in 1. Series aiding 2. Series opposing 3. Parallel aiding 4. Parallel opposing.*

HOTS

(AU/EEE - Dec 2007)

Solution :

$$L_1 = 100\text{ mH},\ L_2 = 900\text{ mH},\ K = 0.45$$

$$M = K\sqrt{L_1 L_2} = 0.45\sqrt{100 \times 10^{-3} \times 900 \times 10^{-3}} = 0.135\text{ H}$$

1. Series aiding

Effective inductance $L_{eq} = L_1 + L_2 + 2M = 100 \times 10^{-3} + 900 \times 10^{-3} + 2 \times 0.135$

$$\boxed{L_{eq} = 1.27\text{ H}}$$

2. Series opposing

$$L_{eq} = L_1 + L_2 - 2M = 100 \times 10^{-3} + 900 \times 10^{-3} - 2 \times 0.135$$

$$\boxed{L_{eq} = 0.73\text{ H}}$$

3. **Parallel aiding**

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{100 \times 10^{-3} \times 900 \times 10^{-3} - (0.135)^2}{100 \times 10^{-3} + 900 \times 10^{-3} - 2 \times 0.135}$$

$$\boxed{L_{eq} = 0.0983\,\text{H}}$$

4. **Parallel opposing**

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{100 \times 10^{-3} \times 900 \times 10^{-3} - (0.135)^2}{100 \times 10^{-3} + 900 \times 10^{-3} + 2 \times 0.135}$$

$$\boxed{L_{eq} = 0.0565\,\text{H}}$$

4.4 ANALYSIS OF COUPLED CIRCUITS

Learning Objective (LO 4)

- ***Students will be able to analyze single tuned and double tuned circuit.***

Consider the coupled circuits shown in figure 4.20. Each circuit contains a voltage source. As both currents I_1 and I_2 enter the coils through the dotted ends, M is taken as positive. By applying KVL, the two loop equations may be written as below.

$$i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = e_1$$

$$i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = e_2$$

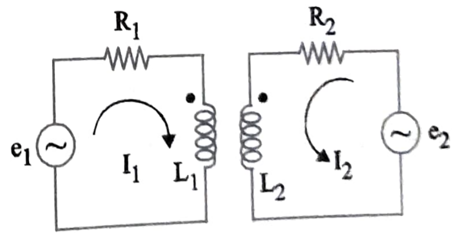


Fig. 4.20

In the sinusoidal steady state, the above equations become

$$(R_1 + j\omega L_1) I_1 + j\omega M I_2 = E_1$$

$$j\omega M I_1 + (R_2 + j\omega L_2) I_2 = E_2$$

$$\begin{bmatrix} R_1 + j\omega L_1 & j\omega M \\ j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

EXAMPLE 56: *In the coupled circuit shown, find V_2 if $I_1 = 0$*

LOTS