$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{L - CR_L^2}{L}}$$

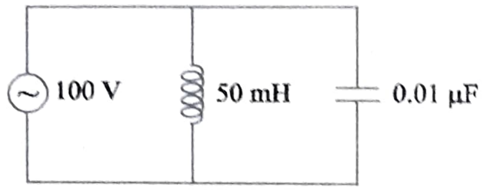
If R_L is also negligible, then

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{L}{L}} = \frac{1}{2\pi\sqrt{LC}}$$

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ which is same as for series resonance.

EXAMPLE 31: *Find the resonant frequency in the ideal parallel LC circuit shown in figure.*

LOTS



Solution :

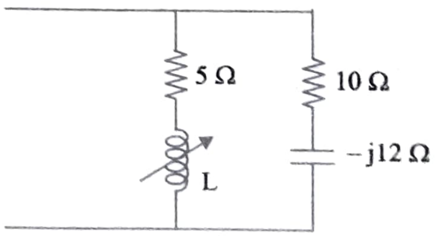
$$V = 100\text{ V}, L = 50\text{ mH}, C = 0.01\ \mu\text{F}$$

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{50\times10^{-3}\times0.01\times10^{-6}}}$

$$\boxed{f_r = 7117.625\text{ Hz}}$$

EXAMPLE 32: *Find the value of L at which the circuit resonates at a frequency of 1000 rad/second in the circuit shown in figure.*

HOTS



(AU/EEE - 2007)

Solution :

Net Admittance $Y = \dfrac{1}{10 - j12} + \dfrac{1}{5 + jX_L}$

$$Y = \frac{10 + j12}{10^2 + 12^2} + \frac{5 - jX_L}{5^2 + X_L^2}$$

$$Y = \frac{10}{244} + \frac{5}{25 + X_L^2} + j\left[\frac{12}{244} - \frac{X_L}{25 + X_L^2}\right]$$

At resonance condition, the susceptance becomes zero.

$$\therefore \quad \frac{X_L}{25 + X_L^2} = \frac{12}{244}$$

$$12X_L^2 - 244X_L + 300 = 0$$

Divide throughout by 12

$$X_L^2 - 20.33X_L + 25 = 0$$

$$X_L = \frac{20.33 \pm \sqrt{(20.33)^2 - 4 \times 25}}{2}$$

$$X_L = \frac{20.33 \pm \sqrt{413.3089 - 100}}{2}$$

$$X_L = \frac{20.33 \pm 17.7}{2} = 19.015\Omega \text{ (or) } 1.314\Omega$$

$$X_L = \omega L$$

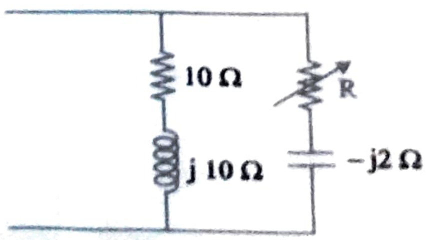
$$L = \frac{19.015}{1000} \text{ (or) } \frac{1.314}{1000}$$

$$\boxed{L = 19.01 \text{ mH (or) } 1.314 \text{ mH}}$$

EXAMPLE 33: *For the parallel network shown in figure, determine the value of R for resonance.*

HOTS

Solution :



The total admittance is given by

$$Y = \frac{1}{10 + j10} + \frac{1}{R - j2}$$

$$Y = \frac{10 - j10}{10^2 + 10^2} + \frac{R + j2}{R^2 + 4}$$

$$Y = \frac{10 - j10}{200} + \frac{R + j2}{R^2 + 4}$$

$$Y = \left[\frac{10}{200} + \frac{R}{R^2 + 4}\right] + j\left[\frac{2}{R^2 + 4} - \frac{10}{200}\right]$$

$$Y = \left[0.05 + \frac{R}{R^2 + 4}\right] + j\left[\frac{2}{R^2 + 4} - 0.05\right]$$

At resonance, the susceptance becomes zero.

$$\therefore \quad \frac{2}{R^2 + 4} - 0.05 = 0$$

$$\frac{2}{R^2 + 4} = 0.05 \; ; \; R^2 + 4 = \frac{2}{0.05}$$

$$R^2 + 4 = 40$$

$$R^2 = 36$$

$$\boxed{R = 6\Omega}$$

4.2.1 *RLC* parallel circuit

Consider the parallel *RLC* circuit shown in figure 4.9. In the circuit shown, the condition for resonance occurs when the susceptance part is zero.

Admittance $Y = G + jB$

$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

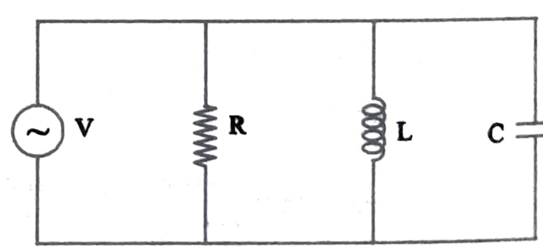


Fig 4.9

The frequency at which resonance occurs is

$$\left(\omega_r C - \frac{1}{\omega_r L}\right) = 0$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

The voltage and current variations with frequency are shown in figure 4.10. At $f = f_r$, the current is minimum.

$$Q = 2\pi \times \frac{\frac{1}{2} L \left(\frac{V}{\omega L} \right)^2 R}{\frac{V^2}{2} \times \frac{1}{f}}$$

$$Q = \frac{2\pi f L R}{\omega^2 L^2} = \frac{R}{\omega L}$$

In the case of a capacitor, the maximum energy stored $= \frac{1}{2} CV^2$

Energy dissipated per cycle $= \frac{V^2}{2R} \times \frac{1}{f}$

$$Q = 2\pi \times \frac{\frac{1}{2}(CV^2)}{\frac{V^2}{2R} \times \frac{1}{f}} = 2\pi f CR = \omega CR$$

4.2.2 Magnification

Current magnification occurs in a parallel resonant circuit. The voltage applied to the parallel circuit $V = IR$

$$I_L = \frac{V}{\omega_r L} = \frac{IR}{\omega_r L} = IQ_r$$

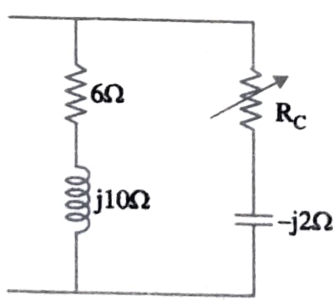
$$I_C = \frac{V}{\frac{1}{\omega_r C}} = IR\omega_r C = IQ_r$$

Therefore, the quality factor

$$Q_r = \frac{I_L}{I} \text{ (or) } \frac{I_C}{I}$$

EXAMPLE 34: *Determine the value of R_c in the network shown in figure to yield resonance.*

(AU/EEE - May 2008)



LOTS

Solution :

$$R_L = 6\ \Omega,\ X_L = 10\ \Omega,\ X_C = 2\ \Omega,\ R_C = ?$$

Condition for resonance, $\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$

$$\frac{10}{6^2 + 10^2} = \frac{2}{R_C^2 + 2^2}$$

$$10\,(R_C^2 + 4) = 2\,(6^2 + 10^2)$$

$$10\,R_C^2 + 40 = 272$$

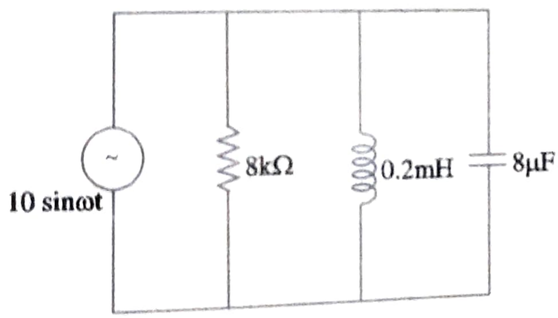
$$10\,R_C^2 = 272 - 40 = 232$$

$$R_C^2 = \frac{232}{10} = 23.2$$

$$\boxed{R_C = 4.816\ \Omega}$$

EXAMPLE 35: *In the parallel RLC circuit in figure, calculate resonant frequency, bandwidth, Q - factor.*

LOTS



(AU/EEE - May 2006)

Solution :

Angular resonant frequency $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = 25000$ rad/sec

Resonant frequency $= \frac{\omega_r}{2\pi} = \frac{25000}{2\pi} = 3978.8$ Hz

Bandwidth $BW = \frac{1}{RC} = \frac{1}{8 \times 10^3 \times 8 \times 10^{-6}} = 15.625$

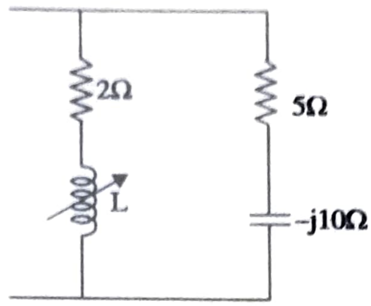
$$\boxed{BW = 15.625 \text{ rad/second}}$$

Q - factor $= \frac{R}{\omega_r L} = \frac{8 \times 10^3}{25000 \times 0.2 \times 10^{-3}}$

$$\boxed{Q = 1600}$$

EXAMPLE 36: *The circuit shown in figure is resonant at a frequency of $\omega = 5000\ rad/s$. Find the value of L.*

(AU/ECE - May 2006)



HOTS

Solution :

$$R_L = 2\ \Omega, R_C = 5\ \Omega, X_C = 10\ \Omega, L = ?$$

Condition for parallel resonance

$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\frac{X_L}{2^2 + X_L^2} = \frac{10}{5^2 + 10^2}$$

$$\frac{X_L}{4 + X_L^2} = 0.08$$

$$X_L = 0.08\,(4 + X_L^2)$$

$$X_L = 0.32 + 0.08\,X_L^2$$

$$0.08\,X_L^2 - X_L + 0.32 = 0$$

$$X_L^2 - 12.5\,X_L + 4 = 0$$

$$X_L = \frac{12.5 \pm \sqrt{156.25 - 10}}{2} = \frac{12.5 \pm 11.84}{2}$$

$$\therefore X_L = 12.17\ \Omega \text{ or } 0.33\ \Omega$$

Case I:

$$X_L = \omega L = 12.17\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{12.17}{5000}$$

$$\boxed{L = 2.434\ \text{mH}}$$

Case II:

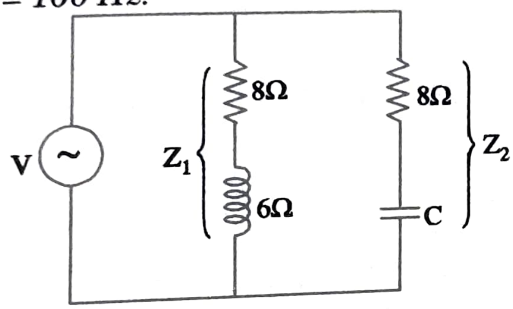
$$X_L = 0.33\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{0.33}{5000} = 66\ \mu\text{H}$$

$$\boxed{L = 66\ \mu\text{H}}$$

EXAMPLE 37: *For the circuit shown in figure, determine the value of C at which it resonates when f = 100 Hz.*

(AU/ECE - May 2006)



HOTS

Solution :

$$R_L = 8\ \Omega, X_L = 6\ \Omega, R_C = 8\ \Omega, f = 100\text{ Hz}, C = ?$$

Condition for parallel resonance

$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\frac{6}{8^2 + 6^2} = \frac{X_C}{8^2 + X_C^2}$$

$$0.06 = \frac{X_C}{64 + X_C}$$

$$0.06\,(64 + X_C^2) = X_C$$

$$3.84 + 0.06\,X_C^2 - X_C = 0$$

$$0.06\,X_C^2 - X_C + 3.84 = 0$$

$$X_C^2 - 16.66\,X_C + 64 = 0$$

$$X_C = \frac{16.66 \pm \sqrt{277.55 - 256}}{2} = \frac{16.66 \pm 4.64}{2}$$

$$= 10.65\ \Omega \text{ or } 6\ \Omega$$

Case I:

When $X_C = \frac{1}{2\pi fC} = 10.65\Omega$,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 100 \times 10.65}$$

$$\boxed{C = 149.44\ \mu\text{F}}$$

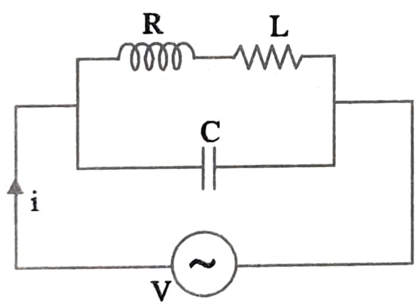
Case II:

When $X_C = 6\ \Omega$,

$$C = \frac{1}{2\pi \times 100 \times 6} = 265.25\ \mu\text{F}$$

$$\boxed{C = 265.25\ \mu\text{F}}$$

EXAMPLE 38: *Obtain an expression for the resonant frequency for the circuit shown in figure below.*



(AU/EEE - Dec. 2010, May 2005)

HOTS

Solution :

Impedance $Z_1 = R + jX_L$

Impedance $Z_2 = -jX_C$

Admittance $Y_1 = \frac{1}{R + jX_L} \times \frac{R - jX_L}{R - jX_L} = \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}$

$$Y_1 = G_L - jB_L$$

where G_L - Inductive conductance

B_L - Inductive susceptance

$$Y_2 = \frac{-1}{jX_C} = \frac{j1}{X_C}$$

$$Y_2 = j B_C$$

$$Y_{eq} = Y_1 + Y_2 = G_L + j B_C - j B_L$$

Condition for resonance is $\quad B_C = B_L$

$$\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$$

$$\omega_r C = \frac{\omega_r L}{R^2 + (\omega_r L)^2}$$

$$C = \frac{L}{R^2 + (\omega_r L)^2}$$

$$R^2 + (\omega_r L)^2 = \frac{L}{C}$$

$$(\omega_r L)^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

Resonant frequency $f_r = \dfrac{\omega_r}{2\pi}$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}}$$

XAMPLE 39: *Show that in a series RLC circuit, $\omega_1 \omega_2 = \omega_r^2$ where ω_r is the resonant equency and ω_1, ω_2 are the half power frequencies.*

LOTS

olution :

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$