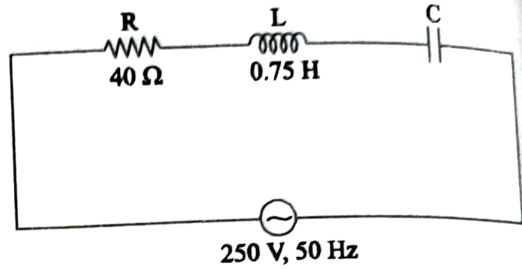
EXAMPLE 25: *A coil of resistance 40 Ω and inductance 0.75 H forms part of a series circuit for which the resonant frequency is 55 Hz. If the supply voltage is 250 V, 50 Hz find, (i) the line current (ii) power factor (iii) voltage across the coil.*

(AU, Coimbatore/EEE - June 2009)

HOTS

Solution :



First find the capacitor value

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f_r)^2 L} = \frac{1}{(2\pi \times 55)^2 \times 0.75}$$

$$\boxed{C = 11.16\ \mu F}$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.75 = 235.62\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 11.16 \times 10^{-6}} = 285.22\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (235.62 - 285.22)^2} = 63.71\ \Omega$$

(i) Current (I)

$$I = \frac{V}{Z} = \frac{250}{63.71} = 3.92\ A$$

$$\boxed{I = 3.92\ A}$$

(ii) Power factor ($\cos\phi$)

$$\cos\phi = \frac{R}{Z} = \frac{40}{63.71} = 0.627$$

$$\boxed{\cos\phi = 0.627 \text{ (leading)}}$$

(iii) Voltage across the coil (V_{coil})

$$\text{Impedance of the coil } Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 235.62^2} = 239\ \Omega$$

$$V_{coil} = I\,Z_{coil} = 3.92 \times 239 = 936.88\ V$$

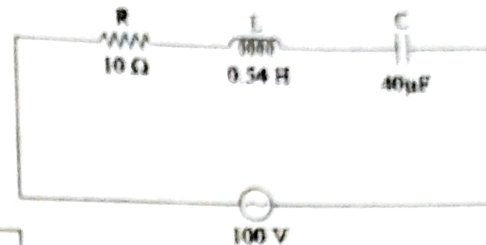
$$\boxed{V_{coil} = 936.88\ V}$$

EXAMPLE 26: *A series RLC circuit has $R = 10\ \Omega$, $L = 0.54\ H$ and $C = 40\ \mu F$. The applied voltage is 100 V. Find (i) resonant frequency (ii) quality factor (iii) upper (iv) lower half power frequencies (v) bandwidth (vi) current at resonance (vii) current at half-power points (viii) voltage across inductance at resonance.*

HOTS

(AU, Chennai/EEE - May 2009)

Solution :



(i) **Resonant frequency (f_r)**

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.54 \times 40 \times 10^{-6}}}$$

$$\boxed{f_r = 34.24\ \text{Hz}}$$

(ii) **Quality factor (Q)**

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.54}{40 \times 10^{-6}}}$$

$$\boxed{Q = 11.62}$$

(iii) **Upper half power frequency (f_2)**

$$f_2 = f_r + \frac{R}{4\pi L} = 34.24 + \frac{10}{4\pi \times 0.54}$$

$$\boxed{f_2 = 35.71\ \text{Hz}}$$

(iv) **Lower half power frequency (f_1)**

$$f_1 = f_r - \frac{R}{4\pi L} = 34.24 - \frac{10}{4\pi \times 0.54}$$

$$\boxed{f_1 = 32.76\ \text{Hz}}$$

(v) **Bandwidth (BW)**

$$BW = f_2 - f_1 = 35.71 - 32.76$$

$$\boxed{BW = 2.95\ \text{Hz}}$$

(vi) **Current at resonance (I)**

$$I = \frac{V}{R} = \frac{100}{10} = 10\ \text{A}$$

$$\boxed{I = 10\ \text{A}}$$

(vii) Current at half-power points

$$I = 0.707\, I_m = 0.707 \times 10 = 7.07 \text{ A}$$

(viii) Voltage across inductance at resonance (V_L)

Inductive reactance $X_L = 2\pi f_r L = 2\pi \times 34.24 \times 0.54 = 116.17\,\Omega$

$V_L = IX_L = 10 \times 116.17 = 1161.7$ V

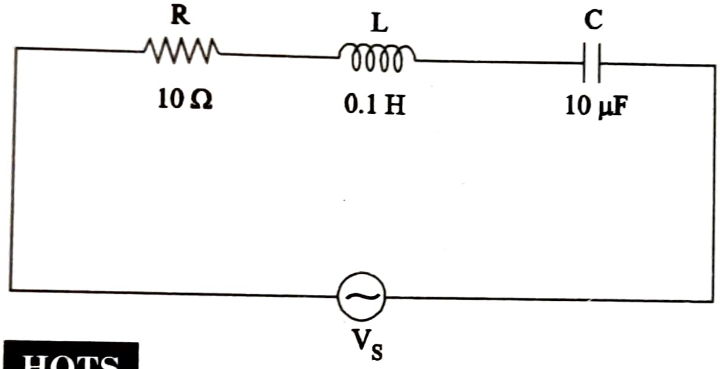
$$\boxed{V_L = 1161.7 \text{ V}}$$

EXAMPLE 27: *For the given circuit of figure, find the impedance*

(i) At resonant frequency

(ii) 20 Hz above resonant frequency and 20 Hz below resonant frequency

(iii) Quality factor of the coil



(AU, Tirunelveli/EEE - May 2009)

HOTS

Solution :

$$\text{Resonant frequency } f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 10 \times 10^{-6}}} = 159.15 \text{ Hz}$$

Impedance at resonance $= R = 10\,\Omega$

At 20 Hz below $f_r = 159.15 - 20 = 139.15$ Hz

At 20 Hz above $f_r = 159.15 + 20 = 179.15$ Hz

Capacitive reactance at 139.15 Hz is

$$X_{C_1} = \frac{1}{2\pi f_1 C} = \frac{1}{2\pi \times 139.15 \times 10 \times 10^{-6}} = 114.37\,\Omega$$

Inductive reactance at 139.15 Hz, $X_{L_1} = 2\pi f_1 L = 2\pi \times 139.15 \times 0.1 = 87.43\,\Omega$

Impedance at 139.15 Hz, $Z_1 = \sqrt{R^2 + (X_{L_1} - X_{C_1})^2} = \sqrt{10^2 + (87.43 - 114.37)^2}$

$$\boxed{Z_1 = 28.736\,\Omega}$$

Capacitive reactance at 179.15 Hz, X_{C_2}

$$X_{C_2} = \frac{1}{2\pi f_2 C} = \frac{1}{2\pi \times 179.15 \times 10 \times 10^{-6}} = 88.83\ \Omega$$

Inductive reactance at 179.15 Hz, X_{L_2}

$$X_{L_2} = 2\pi f_2 L = 2\pi \times 179.15 \times 0.1 = 112.56\ \Omega$$

Impedance at 179.15 Hz, $Z_2 = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2} = \sqrt{10^2 + (112.56 - 88.83)^2}$

$$\boxed{Z_2 = 25.75\ \Omega}$$

EXAMPLE 28: *A series RLC circuit with $R = 10\ \Omega$, $L = 0.2$ mH and a variable capacitor has to resonate at 200 kHz. Find the value of C at resonance.*

(AU, Trichy/ECE - Dec 2008)

LOTS

Solution :

Inductive reactance $X_L = 2\pi f_r L$

$$= 2\pi \times 200 \times 10^3 \times 0.2 \times 10^{-3} = 251.32\ \Omega$$

At resonance condition, capacitance reactance is equal to inductive reactance i.e.

$$X_C = X_L = 251.32\ \Omega$$

$$X_C = \frac{1}{2\pi f_r C}$$

$$251.32 = \frac{1}{2\pi \times 200 \times 10^3 \times C}$$

$$\boxed{C = 0.0031\ \mu F}$$

EXAMPLE 29: *A voltage $v(t) = 50 \sin \omega t$ is applied to a series RLC circuit. At the resonant frequency, the maximum voltage across the capacitor is found to be 400 V. If the bandwidth offered by the circuit is 500 rad/s and the impedance at resonance is 100 Ω, find (i) the resonant frequency (ii) half power frequencies (iii) quality factor at resonance (iv) the component values.*

(AU, Chennai/EEE - May 2011), (AU, Trichy/ECE - Dec 2008)

HOTS

Solution :

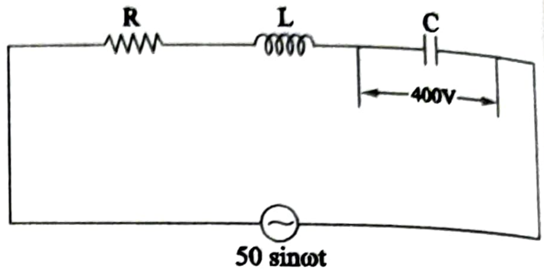
Bandwidth = 500 rad/second

Impedance at resonance, $R = 100\ \Omega$

Supply voltage $V = \frac{V_m}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35$ V

Maximum current corresponds to resonant condition

$$I_{max} = \frac{V}{R} = \frac{35.35}{100} = 0.3535 \text{ A}$$



$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

$$500 = \frac{100}{2\pi L}$$

$$L = 31.83 \text{ mH}$$

At resonance condition, voltage across the capacitor $V_C = 400$ V

$$V_C = IX_L$$

$$X_L = \frac{V_L}{I} = \frac{400}{0.3535} = 1131.54\ \Omega$$

$$X_L = 2\pi f_r L$$

$$f_r = \frac{X_L}{2\pi L} = \frac{1131.54}{2\pi \times 31.83 \times 10^{-3}}$$

$$\boxed{f_r = 5657.87 \text{ Hz}}$$

Half power frequencies

$$f_1 = f_r - \frac{R}{4\pi L} = 5657.87 - \frac{100}{4\pi \times 31.83 \times 10^{-3}}$$

$$\boxed{f_1 = 5407.86 \text{ Hz}}$$

$$f_2 = f_r + \frac{R}{4\pi L} = 5657.87 + \frac{100}{4\pi \times 31.83 \times 10^{-3}}$$

$$\boxed{f_2 = 5907.87 \text{ Hz}}$$

$$Q \text{ - factor } = \frac{f_r}{BW} = \frac{5657.87}{500} = 11.31$$

$$\boxed{Q = 11.31}$$

Components values

$$R = 100\ \Omega$$

$$L = 31.8 \text{ mH}$$

Capacitive reactance $X_C = X_L = 1131.54\ \Omega$

$$X_C = \frac{1}{2\pi f_r C}$$

$$1131.54 = \frac{1}{2\pi \times 5657.87 \times C}$$

$$\boxed{C = 0.0248\ \mu F}$$

EXAMPLE 30: *A series circuit with R = 10 Ω, L = 0.1 H and C = 50 μ F has an applied voltage V = 50 ∠0° with a variable frequency. Find the resonant frequency, the value of frequency at which maximum voltage occurs across the inductor and the value of frequency at which maximum voltage occurs across the capacitor. Explain what do you infer from the results.*

(AU, Trichy/EEE - June 2009)

HOTS

Solution :

The frequency at which maximum voltage occurs across the inductor is

$$f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{\left(1 - \frac{R^2 C}{2L}\right)}}$$

$$= \frac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}}\sqrt{\frac{1}{1 - \left(\frac{10^2 \times 50 \times 10^{-6}}{2 \times 0.1}\right)}}$$

$$\boxed{f_L = 72.08\ \text{Hz}}$$

The frequency at which maximum voltage occurs across the capacitor is

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 50 \times 10^{-6}} - \frac{10^2}{2 \times 0.1}}$$

$$\boxed{f_C = 71.08\ \text{Hz}}$$

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}} = 71.18 \text{ Hz}$$

$$\boxed{f_r = 71.18 \text{ Hz}}$$

From the above results, it is clear that the maximum voltage across the capacitor occurs below the resonant frequency and the maximum inductor voltage occurs above the resonant frequency.

4.2 RESONANCE IN PARALLEL A.C. CIRCUITS

Learning Objective (LO 2)

- ***Students will be able to analyze the resonance in parallel AC circuits with resonant frequency, bandwidth, Q factor and half power frequencies invovled in it.***

Let us consider a parallel circuit consisting of two branches as shown in the figure 4.8.

The impedance of branch 1 $= R_L + jX_L$

The impedance of branch 2 $= R_C - jX_C$

The admittance of the circuit is

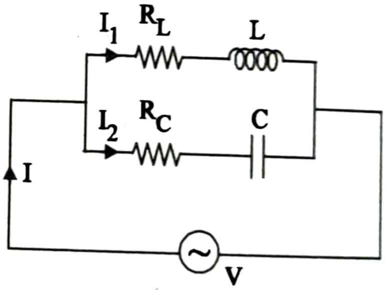


Fig. 4.8

$$Y = \frac{1}{Z} \text{ gives}$$

$$Y = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$$Y = \frac{R_L - jX_L}{(R_L + jX_L)(R_L - jX_L)} + \frac{R_C + jX_C}{(R_C + jX_C)(R_C - jX_C)}$$

$$Y = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$Y = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} - j\left(\frac{X_L}{R_L^2 + X_L^2} - \frac{X_C}{R_C^2 + X_C^2}\right)$$