4.1.6 Quality factor (Q) and its effect on bandwidth

The Quality factor Q is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

$$\text{The quality factor } Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

In an inductor, the maximum energy stored is given by

$$= \frac{1}{2} LI^2$$

$$\text{Energy dissipated per cycle} = \left(\frac{I}{\sqrt{2}}\right)^2 R \times T$$

$$= \frac{I^2RT}{2}$$

$$\text{Quality factor of the coil } Q = 2\pi \times \frac{\frac{1}{2}LI^2}{\frac{I^2R}{2} \times \frac{1}{f}}$$

$$\boxed{Q = \frac{2\pi fL}{R} = \frac{\omega L}{R}}$$

In a capacitor, the maximum energy stored is given by $= \frac{1}{2} CV^2$

$$\text{The energy dissipated per cycle} = \left(\frac{I}{\sqrt{2}}\right)^2 RT$$

Quality factor of the capacitor is

$$\frac{2\pi \times \frac{1}{2} C \left(\frac{I}{\omega C}\right)^2}{\frac{I^2}{2} R \times \frac{1}{f}} = \frac{1}{\omega CR}$$

$$\therefore \text{ In a series circuit Quality factor } \boxed{Q = \frac{\omega L}{R} = \frac{1}{\omega CR}}$$

4.1.7 Magnification in resonance

If in a series RLC circuit, the voltage applied is V, and the current at resonance is I, then the voltage across inductor L is $V_L = IX_L = \left(\frac{V}{R}\right)\omega_r L$

Similarly, the voltage across capacitor C

$$V_c = IX_c = \frac{V}{R\omega_r C}$$

$$Q = \frac{1}{\omega_r CR} = \frac{\omega_r L}{R}$$

where ω_r is the frequency at resonance.

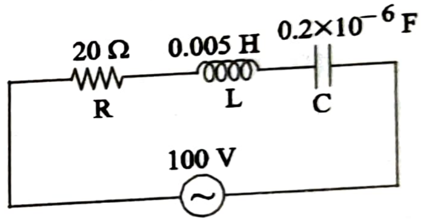
Therefore $V_L = VQ$

$V_C = VQ$

The ratio of voltage across either L or C to the voltage applied at resonance is defined as magnification.

Magnification $Q = \frac{V_L}{V}$ (or) $\frac{V_c}{V}$

EXAMPLE 12: *A series RLC circuit has $R = 20\ \Omega$, $L = 0.005\ H$ and $C = 0.2 \times 10^{-6}\ F$. It is fed from a 100 V variable frequency source. Find (i) frequency at which current is maximum (ii) impedance at this frequency and (iii) voltage across inductor at this frequency.*



(AU/EEE - Dec 2005)

HOTS

Solution :

(i) Maximum current

$$I_{max} = \frac{V}{R} = \frac{100}{20} = 5\text{ A}$$

Frequency at which current is maximum

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.005 \times 0.2 \times 10^{-6}}} = 5033\text{ Hz}$$

$$\boxed{f_r = 5033\text{ Hz}}$$

(ii) Impedance at this frequency

$$\boxed{Z = R = 20\ \Omega}$$

(iii) Voltage across inductor at this frequency $V_L = IX_L$

$$X_L = 2\pi f L = 2 \times \pi \times 5033 \times 0.005 = 158.11\ \Omega$$

$$V_L = 5 \times 158.11 = 790.55\ \text{V}$$

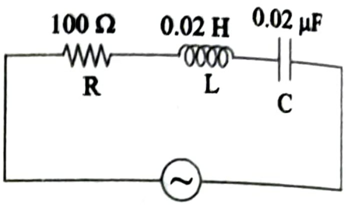
$$\boxed{V_L = 790.55\ \text{V}}$$

EXAMPLE 13: *A series RLC circuit consists of $R = 100\ \Omega$, $L = 0.02\,H$ and $C = 0.02\ \mu F$. Calculate the frequency of resonance.*

LOTS **(AU/EEE - Dec 2005)**

Solution :

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.02 \times 0.02 \times 10^{-6}}}$



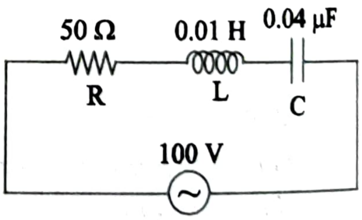
$$\boxed{f_r = 7957.74\ \text{Hz}}$$

EXAMPLE 14: *A series RLC circuit has $R = 50\ \Omega$, $L = 0.01\ H$ and $C = 0.04\ \mu F$. Find resonant frequency, circuit impedance and current under resonance condition, voltage across inductor under resonance when system voltage is 100 V.*

LOTS **(AU/ECE - Dec 2005)**

Solution :

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.01 \times 0.04 \times 10^{-6}}}$



$$\boxed{f_r = 7957.74\ \text{Hz}}$$

Current under resonance condition

$$I_{max} = \frac{V}{R} = \frac{100}{50} = 2\ \text{A}$$

Voltage across inductor $V_L = IX_L$

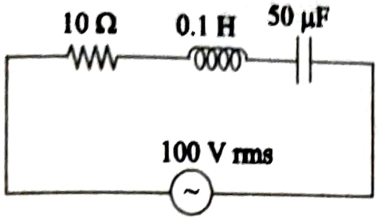
$$X_L = 2\pi f_r L = 2\pi \times 7957.74 \times 0.01$$

$$= 500\ \Omega$$

$$V_L = 2 \times 500 = 1000\ \text{V}$$

$$\boxed{V_L = 1000\ \text{V}}$$

EXAMPLE 15: *For the circuit shown in figure, determine the frequency at which the circuit resonates? Also, find the voltage across the inductor at resonance and the Q-factor of the circuit.*



(AU/EEE - Dec 2007)

HOTS

Solution :

Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}}$

$$\boxed{f_r = 71.18 \text{ Hz}}$$

At resonance, the current $I = \frac{V}{R} = \frac{100}{10} = 10 \text{ A}$

Inductive reactance $X_L = 2\pi f L = 2\pi \times 71.18 \times 0.1$

$= 44.72\ \Omega$

Voltage across inductor $V_L = I X_L = 10 \times 44.72$

$$\boxed{V_L = 447.2 \text{ V}}$$

$$Q \text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.1}{50 \times 10^{-6}}}$$

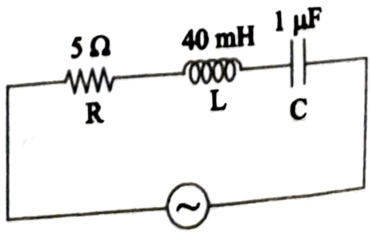
$$\boxed{Q = 4.472}$$

EXAMPLE 16: *A series RLC circuit with $R = 5\ \Omega$, $L = 40$ mH and $C = 1\ \mu F$. Calculate the Q of the circuit, the separation between half power frequencies, the resonant frequency and the half power frequencies.*

(AU/EEE - Dec 2007)

HOTS

Solution :



$$Q - \text{factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{5}\sqrt{\frac{40 \times 10^{-3}}{1 \times 10^{-6}}}$$

$$\boxed{Q = 40}$$

The separation between half power frequencies

$$f_2 - f_1 = \frac{R}{2\pi L} = \frac{5}{2\pi \times 40 \times 10^{-3}} = 20 \text{ Hz}$$

$$\boxed{f_2 - f_1 = 20 \text{ Hz}}$$

Resonant frequency

$$f_r = Q\,(f_2 - f_1) = 40 \times 20 = 800 \text{ Hz}$$

$$\boxed{f_r = 800 \text{ Hz}}$$

Half power frequencies

$$f_1 = f_r - \frac{R}{4\pi L} = 800 - \frac{5}{4\pi \times 40 \times 10^{-3}}$$

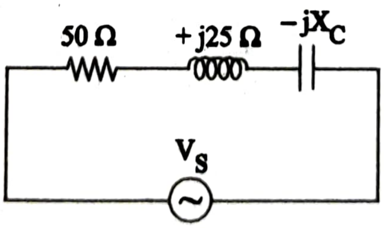
$$= 800 - 9.95 = 790.05 \text{ Hz}$$

$$\boxed{f_1 = 790.05 \text{ Hz}}$$

$$f_2 = f_r + \frac{R}{4\pi L} = 800 + \frac{5}{4\,\pi \times 40 \times 10^{-3}} = 809.95$$

$$\boxed{f_2 = 809.95 \text{ Hz}}$$

EXAMPLE 17: *For the circuit shown in figure, determine the value of capacitive reactance and impedance at resonance.*



(AU/ECE - Dec 2006)

LOTS

Solution :

At resonance condition, inductive reactance is equal to capacitive reactance i.e.,

$$X_L = X_C = 25\ \Omega$$

$$\boxed{X_C = 25\ \Omega}$$

Impedance at resonance

$$\boxed{Z = R = 50\ \Omega}$$

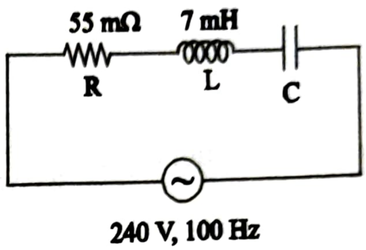
EXAMPLE 18: ***A 240 V, 100 Hz ac source supplies a series RLC circuit consisting of a capacitor and a coil. If the coil has 55 m Ω resistance and 7 mH inductance, calculate the value of the capacitor at 100 Hz resonance frequency, the Q – factor and the half power frequencies of the circuit.***

HOTS

(AU/EEE - Dec 2006)

Solution :

$$f_r = 100 \text{ Hz}$$



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$C = \frac{1}{f_r^2 L (2\pi)^2}$$

$$= \frac{1}{100^2 \times 7 \times 10^{-3} \times (2\pi)^2} = 3.6186 \times 10^{-4} \text{ F}$$

$$\boxed{C = 361.86 \ \mu\text{F}}$$

$$Q - \text{factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{55 \times 10^{-3}}\sqrt{\frac{7 \times 10^{-3}}{361.86 \times 10^{-6}}}$$

$$\boxed{Q = 80}$$

Half power frequencies

$$f_1 = f_r - \frac{R}{4\pi L} = 100 - \frac{55 \times 10^{-3}}{4\pi \times 7 \times 10^{-3}}$$

$$\boxed{f_1 = 99.374 \text{ Hz}}$$

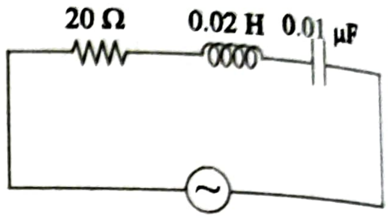
$$f_2 = f_r + \frac{R}{4\pi L} = 100 + \frac{55 \times 10^{-3}}{4\pi \times 7 \times 10^{-3}}$$

$$\boxed{f_2 = 100.625 \text{ Hz}}$$

EXAMPLE 19: *An inductive coil having a resistance of 20 Ω and an inductance of 0.02 H is connected in series with 0.01 μF capacitor. Calculate: (a) Q of the coil (b) Resonant frequency of the circuit.*

LOTS **(AU/ECE - May 2006)**

Solution :



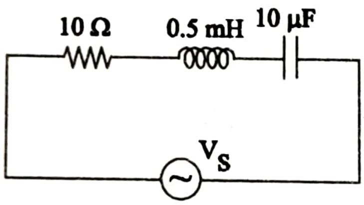
(a) $Q - \text{factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{20}\sqrt{\frac{0.02}{0.01 \times 10^{-6}}}$

$$\boxed{Q = 70.71}$$

(b) Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02 \times 0.01 \times 10^{-6}}}$

$$\boxed{f_r = 11254 \text{ Hz}}$$

EXAMPLE 20: *Determine the resonant frequency for the circuit.*



LOTS **(AU/EEE - May 2004)**

Solution :

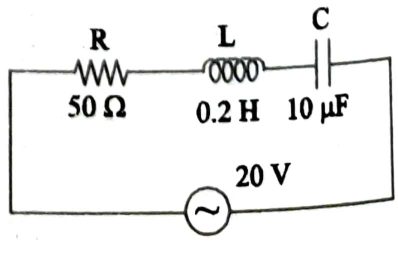
Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5 \times 10^{-3} \times 10 \times 10^{-6}}}$

$$\boxed{f_r = 2250 \text{ Hz}}$$

EXAMPLE 21: *A series RLC circuit consists of 50 Ω resistance, 0.2 H inductance and 10 μF capacitance with the applied voltage of 20 V. Determine the resonant frequency. Find the Q factor of the circuit. Compute the lower and upper frequency limits and also find the bandwidth of the circuit.*

HOTS **(AU, Coimbatore/EEE - Dec. 2010) (AU/EEE - May 2004)**

Solution :



$Q - \text{factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{50}\sqrt{\frac{0.2}{10 \times 10^{-6}}} = 2.82$

$$\boxed{Q = 2.82}$$

Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 10 \times 10^{-6}}}$

$$\boxed{f_r = 112.54 \text{ Hz}}$$

Lower frequency limit $f_1 = f_r - \frac{R}{4\pi L} = 112.54 - \frac{50}{4\pi \times 0.2}$

$$\boxed{f_1 = 92.64 \text{ Hz}}$$

Upper frequency limit $f_2 = f_r + \frac{R}{4\pi L} = 112.54 + \frac{50}{4\pi \times 0.2}$

$$\boxed{f_2 = 132.43 \text{ Hz}}$$

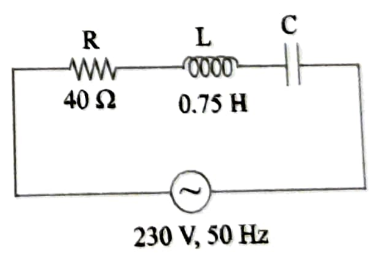
Bandwidth $BW = f_2 - f_1 = 132.43 - 92.64$

$$\boxed{BW = 39.79 \text{ Hz}}$$

EXAMPLE 22: *A coil of inductance 0.75 H and a resistance 40 Ω is a part of a series resonant circuit having a resonant frequency of 160 Hz. If the supply voltage is 230 V, 50 Hz, find (i) current (ii) power factor (iii) voltage across the coil*

HOTS **(AU/ECE - Dec 2004)**

Solution :



First find the capacitor value.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f_r)^2 L} = \frac{1}{(2\pi \times 160)^2 \times 0.75}$$

$$\boxed{C = 1.32\ \mu\text{F}}$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.75 = 235.62\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 1.32 \times 10^{-6}} = 2411.43\Omega$$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (235.62 - 2411.43)^2}$

$= 2176.17\ \Omega$ (capacitive)

(i) Current

$$I = \frac{V}{Z} = \frac{230}{2176.17}$$

$$\boxed{I = 0.1056 \text{ A}}$$

(ii) Power factor

$$\cos\phi = \frac{R}{Z} = \frac{40}{2176.17}$$

$$\boxed{\cos\phi = 0.0183 \text{ (leading)}}$$

(iii) Impedance of the coil

$$Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 235.62^2} = 239\ \Omega$$

Voltage across the coil

$$V_{coil} = I\,Z_{coil} = 0.1056 \times 239$$

$$\boxed{V_{coil} = 25.23 \text{ V}}$$

EXAMPLE 23: *A series RLC circuit has Q = 75 and a pass band (between half power frequencies)of 160 Hz. Calculate the resonant frequency and the upper and lower frequencies of the pass band.*

(AU/EEE - May 2007)

HOTS

Solution :

$$Q = 75$$

$$BW = f_2 - f_1 = 160 \text{ Hz} \quad \text{... (i)}$$

$$f_r = Q\,(f_2 - f_1) = 75 \times 160 = 12000 \text{ Hz}$$

$$\boxed{f_r = 12000 \text{ Hz}}$$

We know that $f_1 f_2 = f_r^2 = (12000)^2$

$$f_1 + f_2 = \sqrt{(f_2 - f_1)^2 + 4 f_1 f_2} = \sqrt{(160)^2 + 4 \times (12000)^2} = 24000 \quad \text{... (ii)}$$

From equations (i) and (ii)

$$f_2 - f_1 = 160$$

$$f_2 + f_1 = 24000$$

$$2 f_2 = 24160$$

$$f_2 = \frac{24160}{2} = 12080 \text{ Hz}$$

$$\boxed{f_2 = 12080 \text{ Hz}}$$

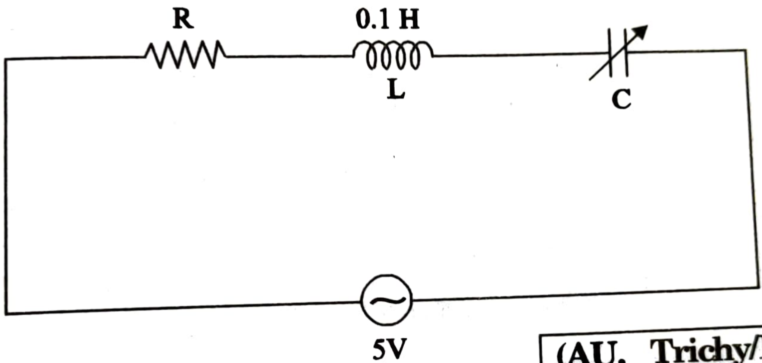
$$f_2 - f_1 = 160$$

$$\therefore f_1 = 12080 - 160 = 11920 \text{ Hz}$$

$$\boxed{f_1 = 11920 \text{ Hz}}$$

EXAMPLE 24: *In the circuit shown in figure, a maximum current of 0.1 A flows through the circuit when the capacitor is at 5 μF with a fixed frequency and a voltage of 5V. Determine the frequency at which the circuit resonates, the bandwidth, the quality factor Q and the value of resistance of resonance frequency.*

HOTS



(AU, Trichy/EEE - June 2009)

Solution :

At resonance condition, $Z = R$

$$R = \frac{V}{I_{max}} = \frac{5}{0.1} = 50\ \Omega$$

$$\text{Resonant frequency } f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 5 \times 10^{-6}}} = 225.08\text{Hz}$$

$$\boxed{f_r = 225.08 \text{ Hz}}$$

$$\text{Quality factor } Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{2\pi \times 225.08 \times 0.1}{50} = 2.828$$

$$\boxed{Q = 2.828}$$

$$\text{Bandwidth } BW = \frac{f_r}{Q} = \frac{225.08}{2.828} = 79.62 \text{ Hz}$$

$$\boxed{BW = 79.59 \text{ Hz}}$$