WHY LINEARIZATION

Errors in the sensor transfer curve

offset: when a zero (or minimum) physical input signal is applied, the measured output signal is not zero (or minimum of output range) but shows an 'offset' value.

gain, range, or full-scale error: the sensitivity of the sensor is not as intended, the maximum physical input signal does not match with the maximum electrical output signal.

nonlinearity: the sensor output does not change linearly with the physical input signal.

.

cross-sensitivity: the transfer curve changes when measured at different ambient conditions (temperature, for example), thus the sensor is not only sensitive to the input signal but also to other parameters.

hysteresis: the sensor transfer curve is different for decreasing physical signals than for increasing signals, once the signal passes a certain level.

drift: the sensor transfer curve changes (slowly) in time

Linearization

we will explain different **Linearization techniques** which can **be used to calibrate the offset, gain, and Linearity errors in the sensor transfer**. In the last part we will propose and explain a polynomial calibration method which can be used to calibrate and Iinearize the sensor transfer in a step-by-step approach.

Linearization

- Some types of sensors show a characteristic nonlinear transfer, which can be explained from a physical model, or which can be reproduced consistently. In that case, the required calibration can be simplified by first applying a systematic linearization.
- The remaining random variations of the nonlinearity can then be calibrated by using a general linearization method.
- Such methods are applied directly for sensors with a more or less linear transfer, showing only linearity errors that vary randomly from device-to-device.

➢The following sections will explain several such linearization methods. **All those methods are based on the use of calibration measurements of the sensor output signal.**

➢Besides for linearizing the signal transfer, **the calibration measurements are also used for correcting the offset and gain error**.

1. Sensor characteristic linearization

➢Certain kinds of sensors always show a typical non linear transfer curve. The shape of the nonlinearity is reproducible and can often be traced in the model of the sensor.

➢Such a **systematic error should preferably be corrected by a systematic linearization**, rather than by means of interpolation of multiple calibration measurements. A sensor may, for example, typically show a logarithmic transfer which could be written in the form of equation with a, b, c, and d as sensor-dependent constants:

$$e = f(\phi) = a + b \cdot \ln(c + d \cdot \Phi) \quad \text{----------} \; 1$$

➢Such a logarithmic transfer curve can easily be **linearized by using an electronic circuit with an exponential transfer function.**

e_s

- We could use the exponential relation between base-emitter voltage and collector current of a bipolar transistor.
- The idea is illustrated in below Fig. The sensor output signal e_s is applied at the base-emitter voltage.
- The collector current Ic is converted into a output voltage Vout by an IV -converter. For the overall transfer of sensor and circuit, we achieve:

$$V_{out} = h(\varphi) = R \cdot I_C = R \cdot e^{K_T V_{be}}$$
$$= R \cdot e^{K_T \{a + b \ \ln(c + d \cdot \varphi)\}}$$
$$= R \cdot e^{aK_T} \cdot e^{bK_T} \cdot (c + d \cdot \varphi)$$

which can be written in a simplified form as:

$$h(x) = A + B \cdot x$$

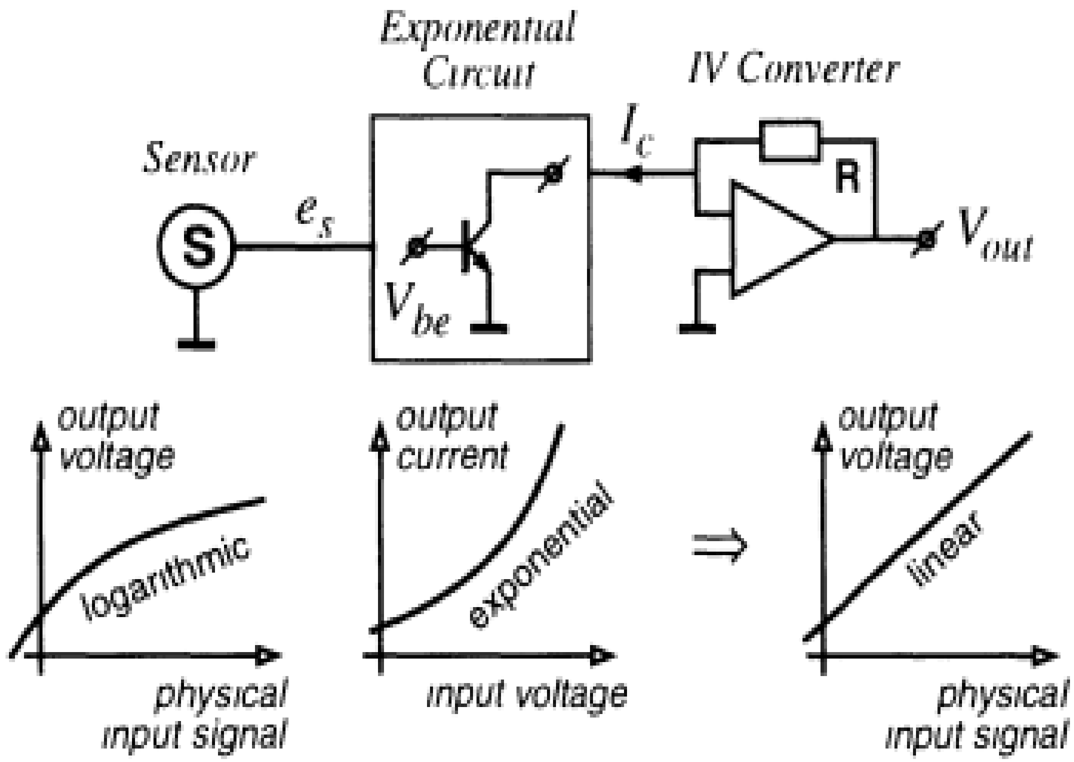


Fig : Linearization of a typical logarithmic transfer function.

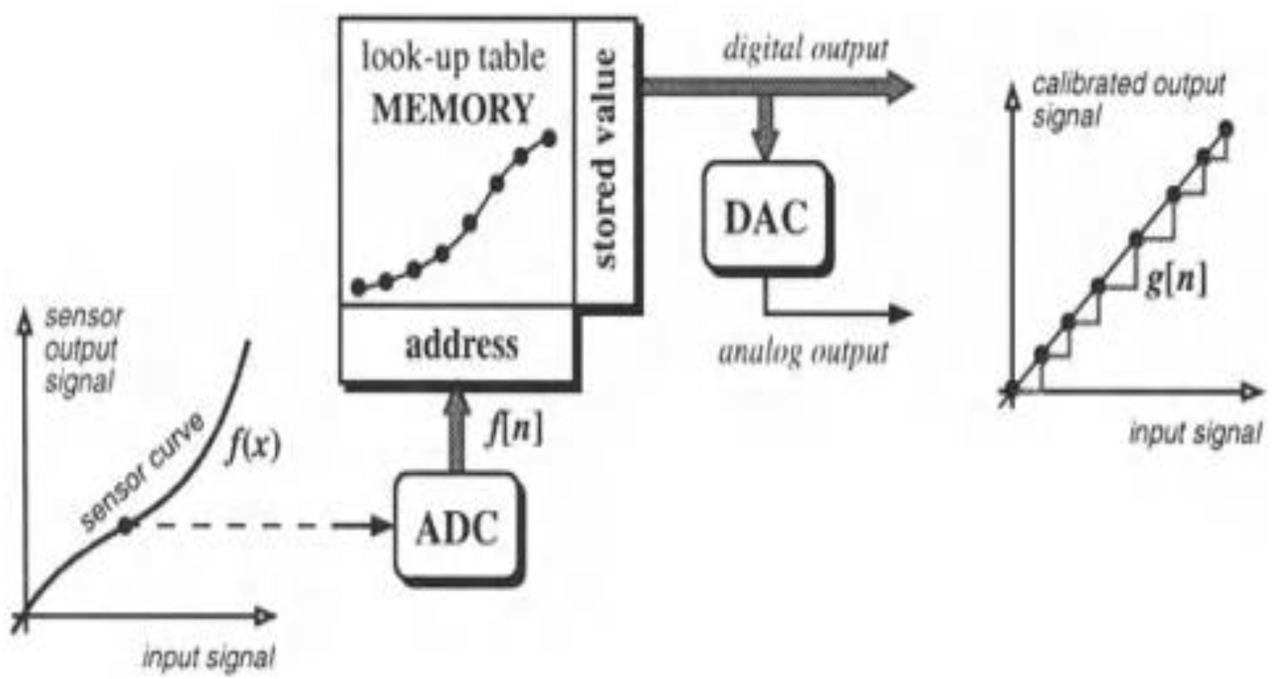
- Due to **the inverse transfer curve of the transistor** with respect to the transfer of the sensor, **the output is now linearly proportional to the physical input signal Φ**, and only the gain and offset errors remain to be calibrated.
- **Attention** should be paid **to the temperature sensitivity introduced by KŢ = q/kT**. This temperature sensitivity could be cancelled easily by multiplying the sensor signal e_s with a PTAT (proportional to 1) signal, before applying it to the transistor
- PTAT-PROPORTIONAL TO ABSOLUTE TEMPERATURE

Demerits:

- It may not always find a way to integrate the inverse function of sensor curve in an electronic circuits.
- The compensation applied may not sufficiently reduce the nonlinearity. In those cases, additional linearization can be improved,

2. Linearization based on a look-up table

➢A simple approach to correct any type of transfer is to store the complete inverse function of the sensor transfer in a table and to look up the corrected output value corresponding to the sensor output signal



- An **Analog-to-Digital converter (ADC) is used to quantize the sensor output signal into a digital number,** which can be **used as an address of the memory** containing the look-up table.

- At the memory address, **the corrected digital value for the desired transfer curve can be read out**. The digital output can be converted into an analog signal by means of a Digital-to-Analog converter (DAC)

- During the calibration phase, the **correct output signals must be stored in the memory at the correct addresses.** This can be done by **(slowly) scanning through the range of the physical input signal of the sensor,** and storing the desired digital output values in the memory, at each single address change.

- The correct digital output values could be obtained from a reference sensor with an ADC
- Filling the complete look-up table **requires a large series or 'scan' of calibration measurements**
  - a nonlinear transfer may locally show a low sensitivity, which means less ADC quantization levels will be available for a certain range of input signals.

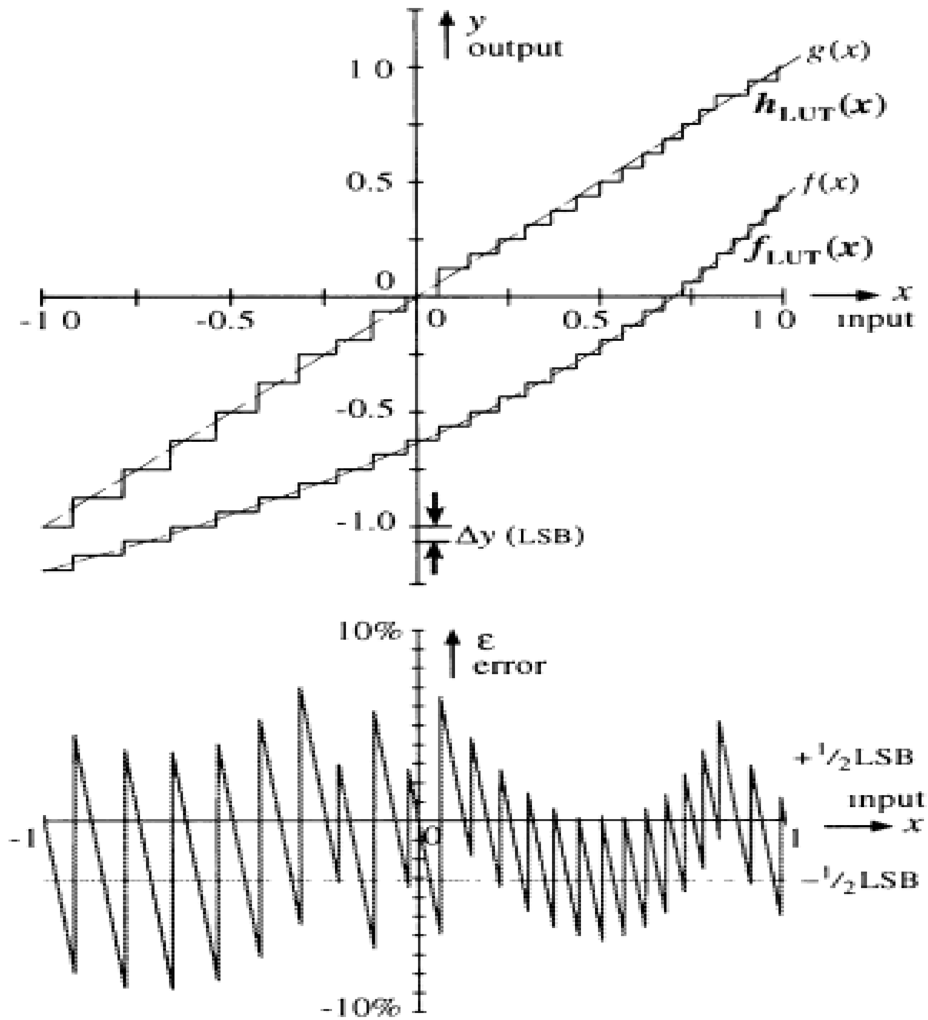


Fig : Look-up table linearization of the transfer curve, and corresponding error curve.

The disadvantages of using a look-up table :

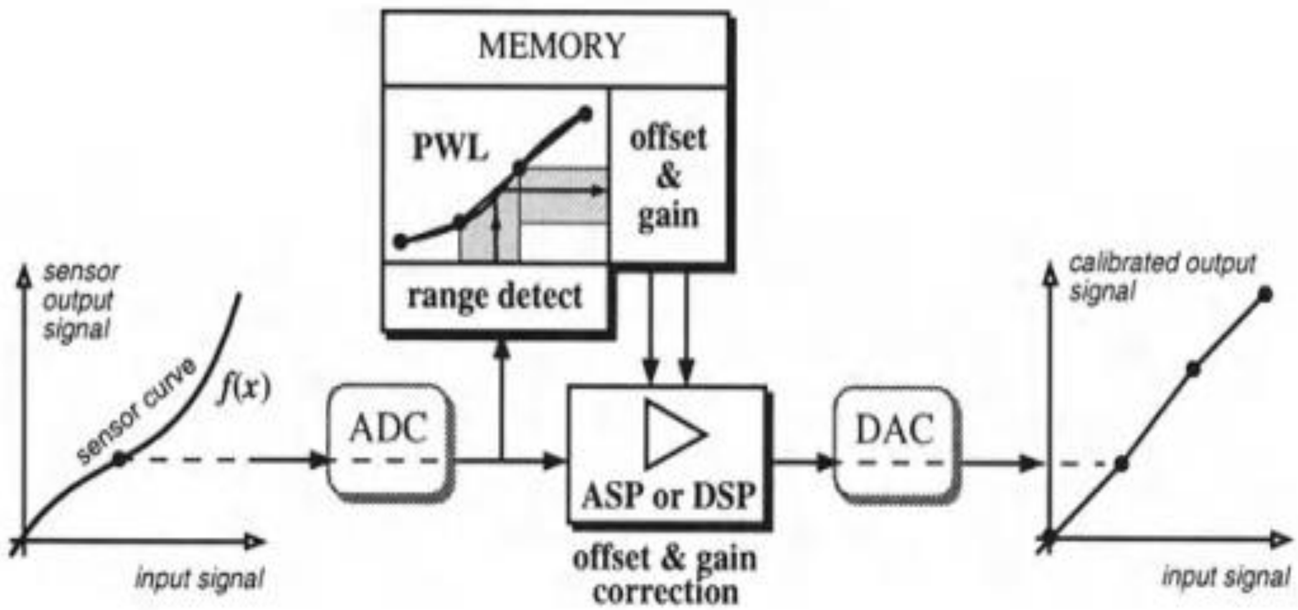
- requirement of a large memory: for an 8-bit quantization 256 bytes are needed, for J 6-bit J 35 kilobytes are needed
- many calibration measurements needed or a good interpolation to obtain all values for the look-up table
- quantization of the (nonlinear) sensor output signal is needed

Advantages of the method can be summarized as:

- once the look-up table is filled, a very fast correction of the sensor signal is possible, as no signal processing is needed
- the look-up table calibration can be realized in a simple straight-forward hardware or software implementation
- the calibration includes the errors of the sensor interface circuit and the AD-converter

3. Linearization based on piecewise-linear interpolation

➢Instead of storing the complete sensor transfer curve in a large memory, we could use a small memory and store only the coefficients of a model which describes the sensor transfer.



➢These calibration measurements represent 'knots' on the sensor transfer curve. Between all adjacent knots we can draw straight lines described by

$$y = an + b\,nX\,.$$

➢These lines compose the piecewise-linear (PWL) interpolation o f_{pwl} (x) of the sensor transfer curve.

➢Based on these **linear 'portions'**, the sensor output signal can be processed in a simple way to **obtain a linearized transfer curve.**

- In each sub-range an 'offset' and a 'gain' correction will be applied.
- Only the values **indicating the sub-ranges and the required offset and gain corrections for each sub-range need to be stored in a memory**.
- **a range detection is sufficient** to determine what offset and gain correction must be applied to the sensor signal.
- Although a digital implementation is the most obvious, the signal processing can also be done in the analog signal domain.
- Hence the **dotted print used for the ADC and the DAC**
- **The effect of quantization is not con**sidered because it is not essential for the PWL to quantize the sensor output signal

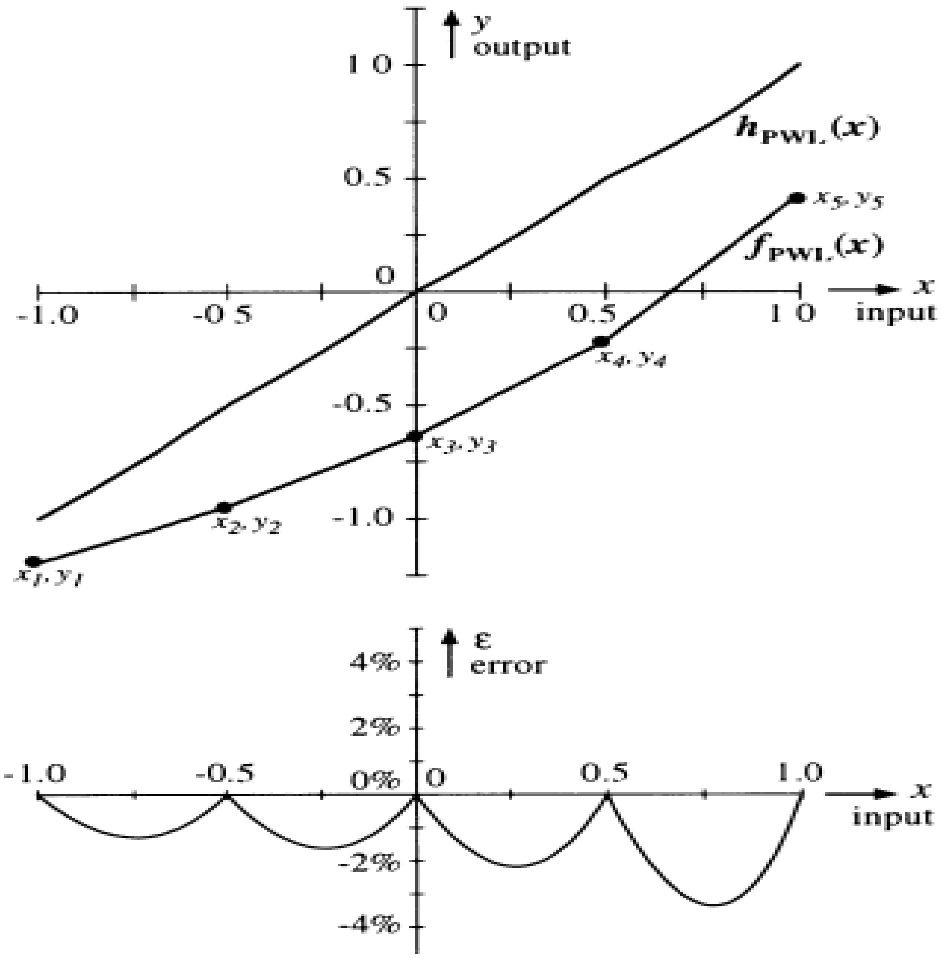


Fig: Piecewise-linear interpolation, and corresponding linearization and error curve,

- It can be proved simply by verifications with x=x/ and x=x2.
- With N knots the complete PWL curve can be described mathematically by the sum of the N-1 linear components fn(x):

$$f_{pwl}(x) = \sum_{n=1}^{N-1} f_n(x)$$

- we can describe the components hn(x) which transform the output signal f(x) as follows:
- The linearization function hpwl(x) is then defined as the sum of the sub-range components:

$$h_{pwl}(x) = \sum_{n=1}^{N-1} h_n(x),$$

The advantages and the disadvantages of the PWL calibration and linearization method can be summarized as:

- low memory requirements
- a reasonably small number of calibration measurements is required
- fast and simple signal correction; no advanced signal processing is required, only gain and offset correction
- reduction of the linearity error is limited; very nonlinear transfer curves still require a large number of calibration steps
- separate calculation of offset and gain correction is required for each subsection of the signal range
- large discontinuities in the derivative of the linearized signal

4. *Linearization based on curve fitting*

- it is possible to counteract the nonlinear transfer of the sensor with an inversely nonlinear transfer when the expression for the sensor transfer curve is known
- It is also possible when the sensor transfer function is not precisely known but only characterized by some calibration measurements [1,2,4,5,15,16,17].
- Curve-fitting techniques make it possible to compute a mathematical function which intersects the actual sensor function in the measurement points.
- This is usually done by defining a weighted sum of expressions based on the sensor output y=f(x).
- The weight factors are calculated on the basis of the measurements f(Xn) of the sensor transfer function.

➢The curve fitting then refers to matching the resulting transfer function h(x)=H {f(x)} to the desired linear transfer function g(x).

The linearization function can usually be expressed in the following form

$$h(x) = \sum_{n=0}^{N} c_n \cdot E_n \{f(x)\}$$

➢En depends on the specific curve-fit technique use like an exponential, sinusoidal, or polynomial function of increasing order (n)

➢The weight factors Cn' also referred to as calibration coefficients

Linearization based on curve fitting

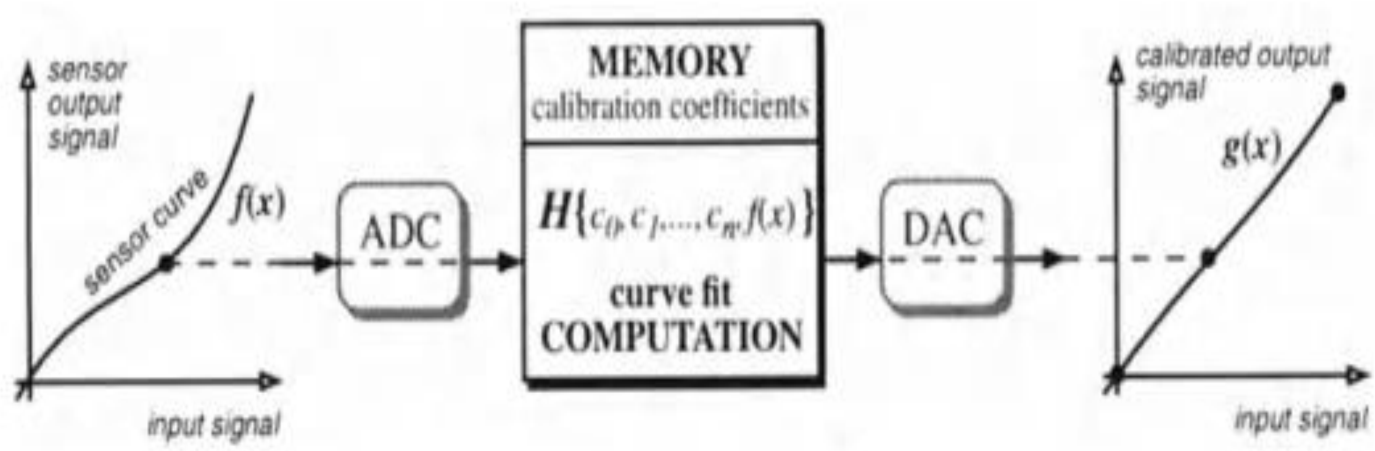


Fig: Inverse transfer function method for linearity calibration.

- The computations required can be **split in two groups**.
- First, the calculation of the **correct values for calibration coefficients during or after the calibration phase.**
- This may be done **externally of the sensor (on a computer).**
- Second, calculation of the **Linearized output signal based on the sensor output and the calibration coefficients**.
- This correction must be **integrated with the smart sensor**, either in analog or digital hardware, or in software in a microcontroller interfacing the sensor.
- **Memory** must be present to **store the calibration coefficient**

Advantages of linearization using curve fitting

- low number of calibration measurements results in good linearization
- low memory requirements because of a low number of coefficients
- one single correction formula for the complete signal range

Disadvantages of linearization using curve fitting

- higher order polynomials might require high-accuracy computations (floating-point operations)
- rather complex calculation of calibration coefficients may be needed

5. Progressive polynomial calibration method

- The polynomial calibration and linearization method we propose operates on the principle that **each calibration measurement can be used directly to calculate one programmable coefficient in the correction functio**n.
- The correction is then immediately applied to modify the sensor output.
- The next calibration step makes use of this corrected sensor signal.
- Each succeeding correction step is applied in such a way that the previous calibrations remain undisturbed.

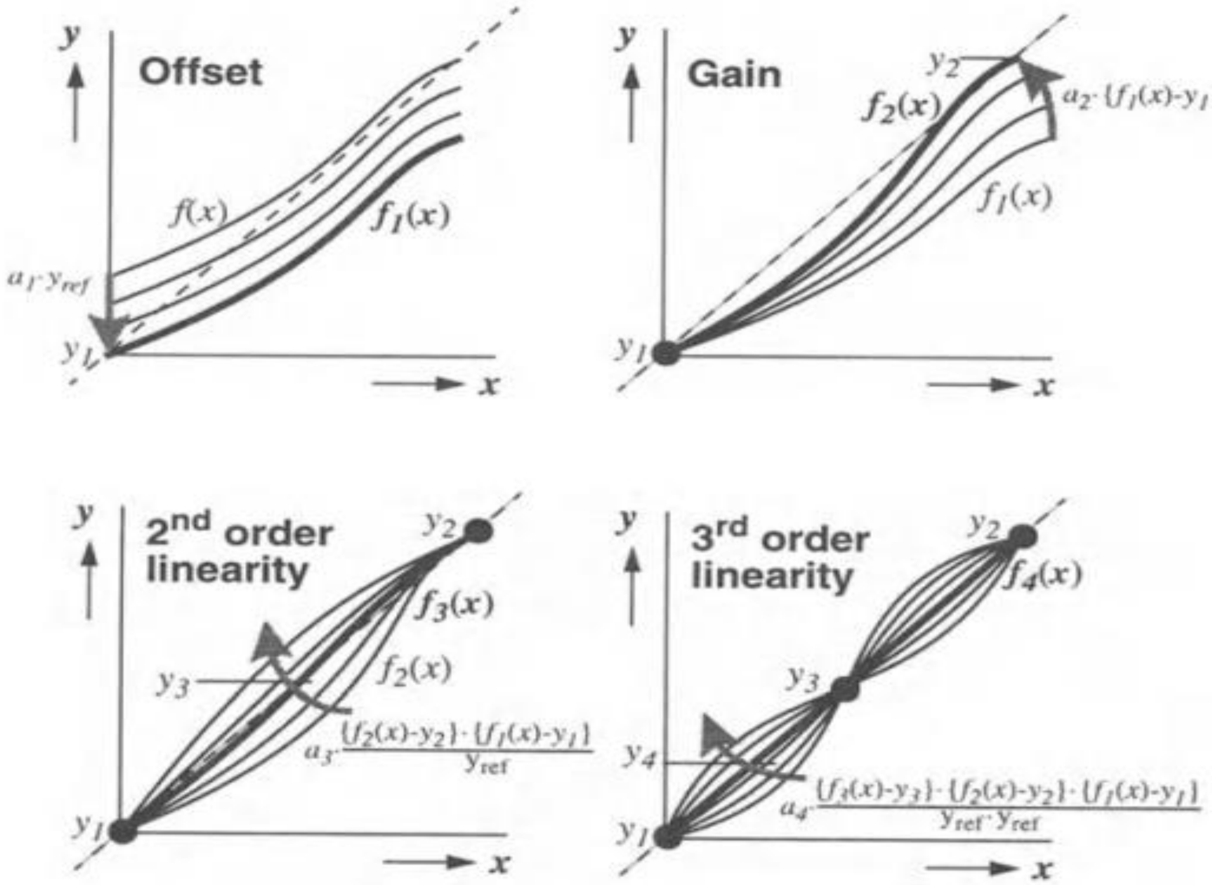


Fig. 3-13 *Four steps of the progressive polynomial calibration/ linearization method.*

Four steps to linearize the transfer function using progressive polynomial function

1. The first measurement is done to calibrate the offset which means the **transfer function is shifted or translated**.

2. A following calibration measurement is used to **correct the g**ain error, without affecting the offset calibration. This is **achieved by rotating the function around the previous calibration point.**

3. The next calibration measurements are used to correct linearity, which is achieved by **'bending' the functio**n in such a way that the previous calibration points stay fixed. The third picture in Fig.3-13 shows the second order linearity correction

4. the last picture of Fig. 3-13 shows the third order linearization. More calibrations can be done in the same manner to further linearize the sensor transfer function

- In fact, at each calibration step, ***the calibrated transfer function progresses towards the desired transfer, hence the label progressive polynomial calibration***

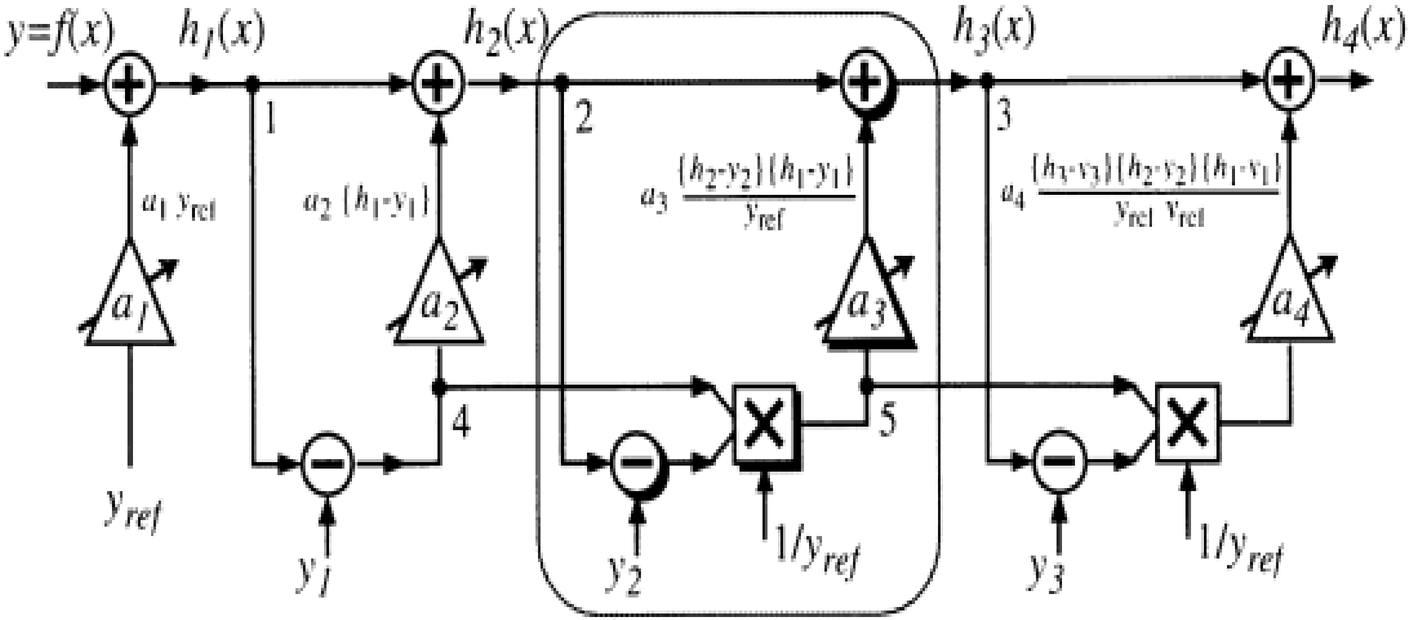


Fig. 3-16 *Graphical diagram of the polynomial calibration method.*

- When taking only one calibration step, the calibration measurement is best taken with a reference input signal xl at the middle of the input range.
- When taking two calibration steps, the calibration measurements should be done for input signals at about 25% and 75% of the input range.
- For three or more calibration steps, generally a good linearization is obtained when the calibration points are selected in the following sequence: the first point at one end of the sensor range of operation, the second point at the other end of the range, and further calibration points halfway between two previously selected points.

The proposed polynomial calibration method is valuable for sensors which may show an arbitrary linearity error.

Advantages of the proposed method are:

- a good linearization is obtained through a minimum number of calibration measurements
- a low number of calibration coefficients, thus only a small memory is required
- the required calibration coefficients can be obtained directly, without mathematical iterations
- a systematic step-by-step calibration, using a repetitive algorithm


Thank you