Topic: 1.7 – DIAGONALIZATION OF MATRICES

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Working Rule For Diagonalisation:

* To find the characteristic equation.
* To solve the characteristic equation.
* To find the Eigen values
* To find the Eigen vectors
* If the Eigen vectors are orthogonal, then form a normalized modal N.
* Normalised form is $N = \begin{bmatrix} x_1/l \\ x_2/l \\ x_3/l \end{bmatrix}$ where $l = \sqrt{x_1^2 + x_2^2 + x_3^2}$
* Find N^T
* Calculate AN
* Calculate $D = N^T A N$

Problems:

1. Diagonalise the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ +2 & -4 & 3 \end{bmatrix}$

$$\text{Let } A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Characteristic equation $\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$

Eigen values are $0, 3, 15$

Eigen vectors i) If $\lambda = 0$, then Eigen vector is $[1 \quad 2 \quad 2]^T$

ii) If $\lambda = 3$, then Eigen vector is $[2 \quad 1 \quad -2]^T$

iii) If $\lambda = 15$, then Eigen vector is $[2 \quad -2 \quad 1]^T$

To find the Eigen vectors are orthogonal to each other.

$$X_1^T X_2 \Rightarrow 2+2-4 = 0$$

$$X_2^T X_3 \rightarrow 4-2-2 = 0$$

$$X_1^T X_3 \Rightarrow 2-4+2 = 0$$

They are orthogonal to each other.

To form the Normalised Matrix:

$$N = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

To find the transpose of normalised matrix

$$N^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\frac{1}{3}$$

Calculate AN

$$AN = \frac{1}{3}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 10 \\ 0 & 1 & -10 \\ 0 & -2 & 5 \end{bmatrix}$$

Calculate Diagonised Matrix D

$$D = NAN^T$$

$$= \frac{1}{3}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\cdot\frac{1}{3}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 10 \\ 0 & 1 & -10 \\ 0 & -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$