Problem - 3; $R = 15\,\Omega$, $L = 0.2H$, $C = 3\mu F$, $V = 400.\cos[5\omega t + \frac{\pi}{4}]$

UNIT-V

Analysing Three - Phase Circuits

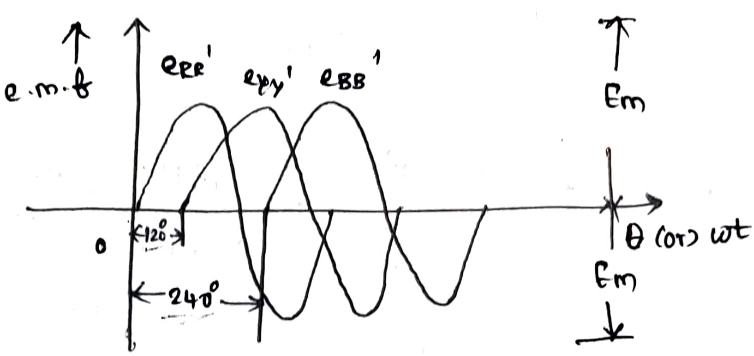
* Generation of 3-phase voltage :-

* The equations for the instantaneous voltages of 3-phases are given below,

$$e_{RR'} = E_m . \sin\omega t \quad \text{—} \ (1)$$

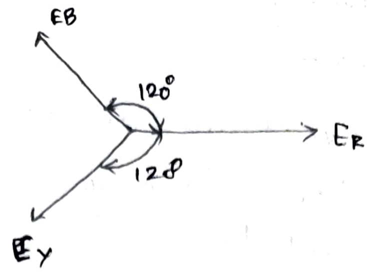
$$e_{YY'} = E_m . \sin(\omega t - 120^\circ) \quad \text{—} \ (2)$$

$$e_{BB'} = E_m . \sin(\omega t - 240^\circ) \quad \text{—} \ (3)$$



$$\therefore E_{r.m.s} = \frac{E_m}{\sqrt{2}}$$

Voltage	polar form	Rectangular form
E_R	$E\angle 0^\circ$	$E(1+j0)$
E_Y	$E\angle -120^\circ$	$E(-0.5 - j0.8660)$
E_B	$E\angle 120^\circ$	$E(-0.5 + j0.8660)$



$\therefore E_R + E_Y + E_B = 0$

* phase Sequence :-

* The order in which the Alternating quantities attain their maximum values is called the "phase sequence"

→ RYB - Positive Sequence
→ RBY - Negative Sequence

* Interconnection of the phases:-

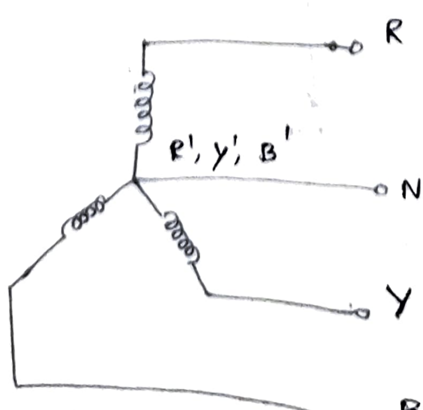
* The three phases are interconnected, to

(i) Achieve the economy
(ii) Reduce the number of conductors and space
(iii) reduce the complexity of the circuit

* There are two types of interconnection. They are,

(i) star (or) Wye (Y) connection
(ii) Delta (or) (Δ) mesh connection

(i) star (or) Wye (Y) connection :-



(a) 3 - phase star connection

(a) Line voltage and phase voltage in star-system:-

* The voltage across each coil is called "phase voltage" and it is denoted by E_p (or) $E\phi$. It is also the voltage between each line and neutral

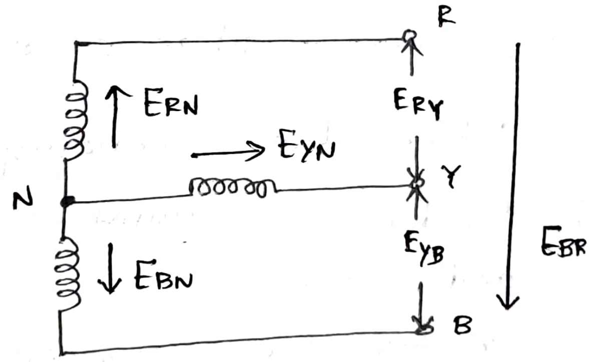
$$|E_{RN}| = |E_{YN}| = |E_{BN}| = |E\phi|$$

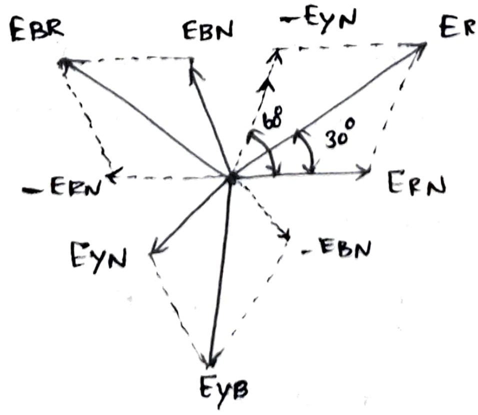
$$E_{RN} = E\phi \angle 0^\circ$$

$$E_{YN} = E\phi \angle -120^\circ$$

$$E_{BN} = E\phi \angle 120^\circ$$

* The phase and line voltage in star-connection is given by,





(b) Relation between E_L and E_ϕ in balanced 3-phase system:-

1. $E_{RY} = E_{RN} - E_{YN}$

$= E_\phi \angle 0^\circ - E_\phi \angle -120^\circ$

$= \sqrt{3} \cdot E_\phi$

$E_{RY} = 440 \angle 30^\circ$ (V)

$E_{YB} = 440 \angle -90^\circ$ (V)

$E_{BR} = 440 \angle 150^\circ$ (V)

2. $E_{YB} = E_{YN} - E_{BN} = E_\phi \angle -120^\circ - E_\phi \angle 120^\circ$

$= \sqrt{3}\, E_\phi \angle -90^\circ$

3. $E_{BR} = E_{BN} - E_{RN}$

$= E_\phi \angle 120^\circ - E_\phi \angle 0^\circ$

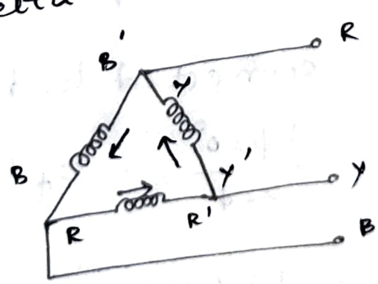
$= \sqrt{3}\, E_\phi \angle 150^\circ$

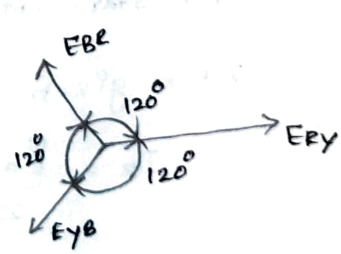
$\boxed{\therefore \; E_L = \sqrt{3}\, E_\phi}$ $\boxed{I_L = I_{ph}}$

* Three-phase power, $P = \sqrt{3}\, V_L . I_L . \cos\phi$

$$\boxed{\begin{aligned} E_{RY} &= E_R - E_Y \\ E_{YB} &= E_Y - E_B \\ E_{BR} &= E_B - E_R \end{aligned}}$$

(ii) Delta connection:- (or) mesh-connection:-





$|E_{RY}| = |E_{YB}| = |E_{BR}| = E_L = E_\phi$ — (1)

$E_{RY} = E_\phi \angle 0^\circ = E_\phi [1 + j0]$

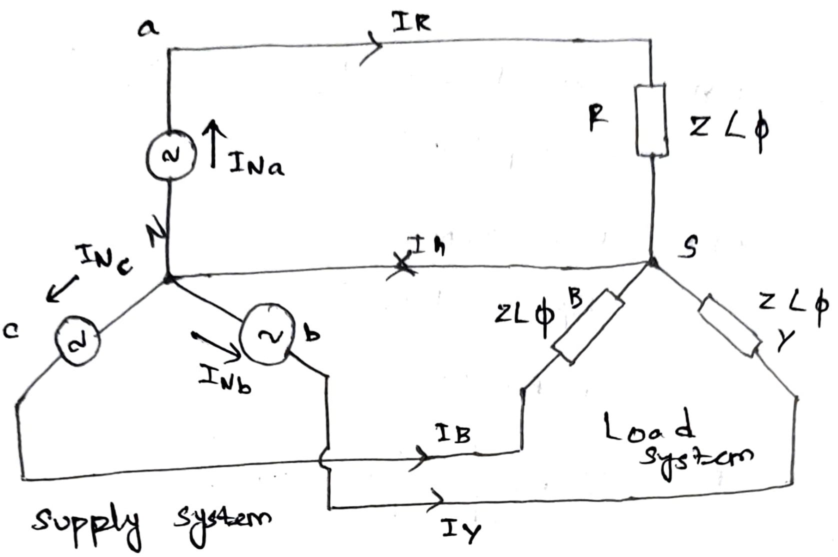
$E_{YB} = E_\phi \angle -120^\circ = E_\phi [-0.5 - j \frac{\sqrt{3}}{2}]$

$E_{BR} = E_\phi \angle 120^\circ = E_\phi [-0.5 + j \frac{\sqrt{3}}{2}]$

* The phasor sum of line voltages,

$$= E_{RY} + E_{YB} + E_{BR} = 0$$

* Balanced star - connected load:-



* In 3-phase, 4-wire system, the neutral point-N of the supply is connected.

$$I_{Na} = I_R = I_{RS}$$

$$I_{Nb} = I_y = I_{yS}$$

$$I_{Nc} = I_B = I_{BS}$$

For star-connection, $I_L = I_\phi$

* For given star-connected load,

$$I_R = \frac{E_{aN}}{Z} = \frac{E_\phi \angle 0^\circ}{Z\angle\phi} = I_\phi \angle \phi$$

$$I_y = \frac{E_{bN}}{Z} = \frac{E_\phi \angle -120^\circ}{Z\angle\phi} = I_\phi \angle -\phi-120^\circ$$

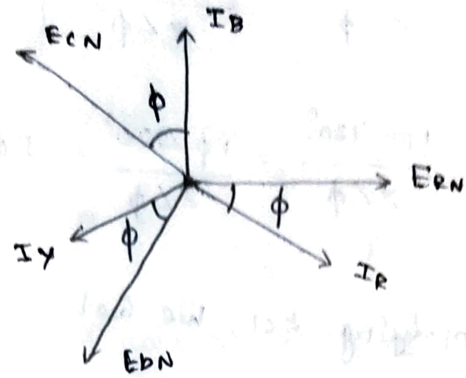
$$I_z = \frac{E_{cN}}{Z} = \frac{E_\phi \angle 120^\circ}{Z\angle\phi} = I_\phi \angle -\phi+120^\circ$$

* By Applying, - KCL at star point - S,

$$I_n = I_R + I_y + I_B$$

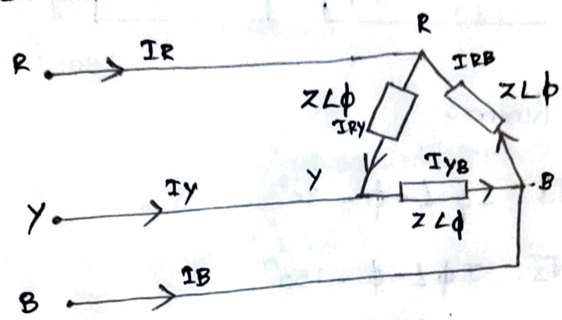
system, $I_n = 0$

$\boxed{\therefore I_R + I_Y + I_B = 0}$

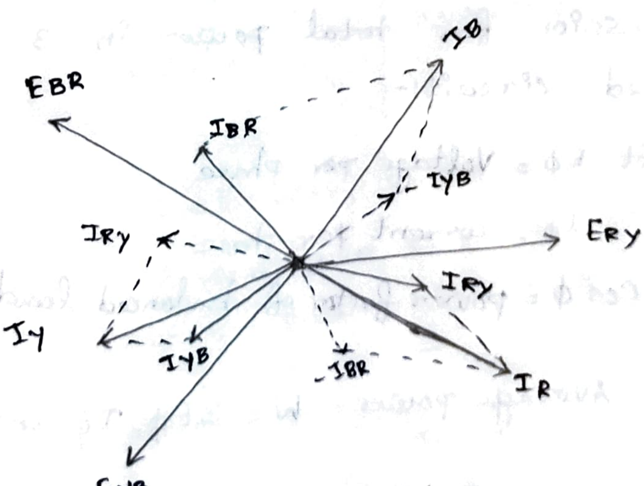


phasor diagram

* Balanced Delta connected load:-



$\therefore$ Load , $z = Z\angle\phi$



phasor diagram

* Assume the phase sequence to be - RYB.

Taking - ERY as reference

$$I_{RY} = \frac{E_{RY}\angle 0^\circ}{Z\angle\phi} = \frac{E_\phi}{Z}\angle -\phi = I_\phi \angle -\phi$$

$\therefore I_{YB} = \frac{E_{YB} \angle -120^\circ}{Z \angle \phi} = \frac{E_\phi \angle -120^\circ}{Z \angle \phi} = I_\phi \angle -\phi - 120^\circ$

$I_{BR} = \frac{E_{BR} \angle 120^\circ}{Z \angle \phi} = \frac{E_\phi \angle 120^\circ}{Z \angle \phi} = I_\phi \angle -\phi + 120^\circ$

$\therefore$ By Applying - KCL, We Get,

$I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$

$\boxed{\therefore I_L = \sqrt{3}\, I_\phi}$ $\boxed{V_L = V_{ph}}$

$\therefore$ In polar form,

$I_R = \sqrt{3} \cdot I_\phi \angle -\phi - 30^\circ$

$I_Y = \sqrt{3} \cdot I_\phi \angle -\phi - 150^\circ$

$I_B = \sqrt{3} \cdot I_\phi \angle -\phi + 90^\circ$

* Expression for total power in 3-phase Balanced circuit:-

Let E_ϕ = Voltage per phase

I_ϕ = current per phase

$\cos \phi$ = power factor of balanced load

$\therefore$ Total Average power = $W = 3 . E_\phi . I_\phi . \cos\phi$

case-1:- Star-connected load:-

Let, $\boxed{E_L = \sqrt{3} . E_\phi}$

$\rightarrow E_\phi = \frac{E_L}{\sqrt{3}}$

$\boxed{I_L = I_\phi}$

$\therefore W = 3 . \frac{E_L}{\sqrt{3}} . I_L . \cos\phi = \sqrt{3} . V_L . I_L . \cos\phi$

Case-2 :- Delta-connected load :-

Let, $\boxed{E_L = E_\phi}$ $\boxed{I_L = \sqrt{3} \cdot I_\phi}$

$$\therefore W = 3 \cdot E_\phi \cdot I_\phi \cdot \cos\phi$$

$$W = 3 \cdot E_L \cdot \frac{I_L}{\sqrt{3}} \cdot \cos\phi$$

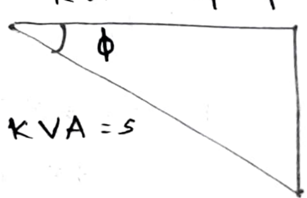
$$\therefore W = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos\phi$$

$\div 1000$; $\dfrac{W}{1000} = \dfrac{\sqrt{3} \cdot V_L \cdot I_L \cdot \cos\phi}{1000}$

$\dfrac{W}{1000}$ - denoted in ("kW")

$\dfrac{\sqrt{3} \cdot V_L \cdot I_L}{1000}$ - Apparent power and measured in -
-KV amperes ("KVA")

KVA $\cos\phi$ = P



KVA $\sin\phi$ = Q

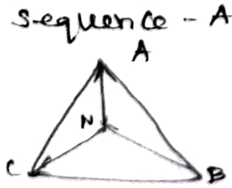
KVA = S

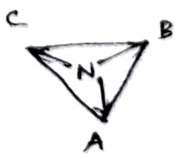
$\therefore$ Real power, $P = V \cdot I \cdot \cos\phi$ (kW)

Apparent power, $S = V \cdot I$ (KVAR)

Reactive power = $V \cdot I \cdot \sin\phi$

Sequence - ABC :-





$-30° + 120°$

$V_{AB} = V_L \angle 120°$

$V_{BC} = V_L \angle 0°$

$V_{CA} = V_L \angle 240°$

$V_{AN} = \frac{V_L}{\sqrt{3}} \angle 90°$

$V_{BN} = \frac{V_L}{\sqrt{3}} \angle -30°$

$V_{CN} = \frac{V_L}{\sqrt{3}} \angle -150°$

$V_{AB} = V_L \angle 240°$

$V_{BC} = V_L \angle 0°$

$V_{CA} = V_L \angle 120°$

$V_{AN} = \frac{V_L}{\sqrt{3}} \angle -90°$

$V_{BN} = \frac{V_L}{\sqrt{3}} \angle 30°$

$V_{CN} = \frac{V_L}{\sqrt{3}} \angle 150°$

Problem -1 :- A balanced Star - connected load of $(8+j6)\ \Omega$ / phase is connected to a 3 - Phase, 50 Hz supply. Find the line current, power factor, power, reactive volt Amperes and total volt Amperes

$$V_L = 230\,V$$

$$\therefore V_\phi = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 133\,V$$

$$\text{Impedance}, Z = 8+j6$$
$$= 10\angle 36.9^\circ$$

$$|Z_\phi| = 10, \quad \phi = 36.9^\circ$$

(i) for star, $I_L = I_\phi = \dfrac{|V_\phi|}{|Z_\phi|} = \dfrac{133}{10} = 13.3\,A$

(ii) $\text{Power factor} = \cos\phi = \cos 36.9^\circ = 0.8 \text{ lag}$

(or) $\cos\phi = \dfrac{|R_\phi|}{|Z_\phi|} = \dfrac{8}{10} = 0.8$

(iii) Apparent power $= \sqrt{3}.V_L.I_L. = \sqrt{3}\times\sqrt{3}.V_\phi \times I_\phi$

$$= 3\times V_\phi \times I_\phi$$
$$= \sqrt{3}\times 230\times 13.3 = 5298\ VA$$

(iv) Real power $= \sqrt{3}.V_L.I_L.\cos\phi = 5298\times 0.8 = 4239\ W$

(v) Reactive power $= \sqrt{3}.\times V_L \times I_L \times \sin\phi$

$$= 5298\times 0.6 = 3179\ VAR$$

Problem - 2 :- A three - phase, four - wire, ~~ABC~~ CBA - system, with effective line voltage of 120V, has three impedance of $20\angle -30^\circ\ \Omega$ in Y - connection. Determine the line currents

$V_L = 120V$, $\quad Z\phi = 20\angle -30^\circ$

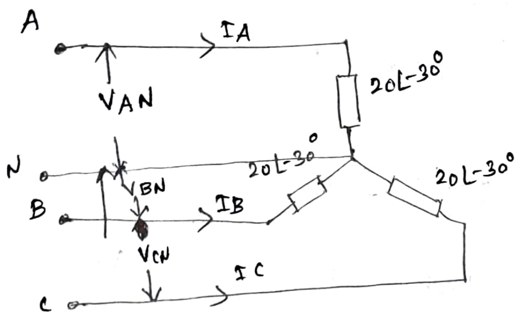
$V\phi = \frac{V_L}{\sqrt{3}} = \frac{120}{\sqrt{3}} = 69.28V$

$\therefore I_L = I\phi = \frac{V\phi}{Z\phi} =$

$\therefore$ Effective line voltage, $= 120V$

$\therefore V_L$ = Maximum line voltage $= 120 \times \sqrt{2} = 169.70V$

$\therefore V\phi = \frac{V_L}{\sqrt{3}} = \frac{169.70}{\sqrt{3}} = 97.97V \simeq 98.0V$



$\therefore V_{AN} = 98.0\angle -90^\circ V$; $\quad V_{BN} = 98.0\angle 30^\circ V$; $\quad V_{CN} = 98.0\angle 150^\circ V$

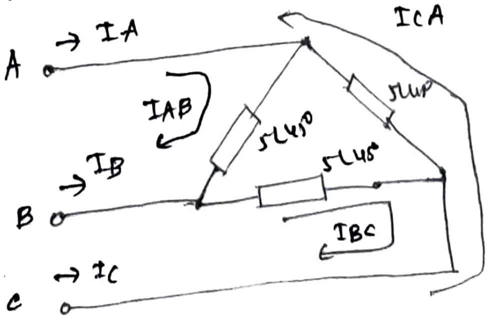
$\therefore I_A = \frac{V_{AN}}{Z} = \frac{98.0\angle -90^\circ}{20\angle -30^\circ} = 4.90\angle -60^\circ A$

$I_B = \frac{V_{BN}}{Z} = \frac{98.0\angle 30^\circ}{20\angle -30^\circ} = 4.90\angle 60^\circ A$

$I_C = \frac{V_{CN}}{Z} = \frac{98.0\angle 150^\circ}{20\angle -30^\circ} = 4.90\angle 180^\circ A$

Problem-2:- A three-phase, three-wire ABC system, with an effective line voltage of 120V, has three impedance $5.0\angle 45^\circ$. Determine line currents

$V_L = V_{eff} \times \sqrt{2} = 169.70V$; $\quad V_L = V\phi = 169.70V$



$I_A = I_{AB} + \cancel{I_{BC}}$

$= \frac{169.70\angle \cancel{120}}{5\angle 45^\circ} + \frac{169.70\angle \cancel{120}\, 0^\circ}{5\angle 45^\circ}$

$= 33.94 \angle \cancel{45}^\circ + 33.94\angle -165^\circ$

$= 58.7\angle 45^\circ$ (A)

$V_{BC} = 169.70\angle 0^\circ$

$V_{AB} = 169.70\angle 120^\circ$

$V_{BA} = 169.70\angle 240^\circ$

$I_B = I_{BC} + I_{CA}$

$= 58.7\angle -75^\circ$ (A)

$I_C = I_{AB} + I_{BC}$

$= 58.7\angle 165^\circ$ (A)

$I_A = I_{AB} - I_{CA}$

$I_B = \cancel{I_{AB} - I_{BC}}\ \ I_{BC} - I_{AB}$

$I_C = I_{CA} - I_{BC}$

$I_{AB} = \frac{169.70\angle 0^\circ}{5\angle 45^\circ} = 33.94\angle -45^\circ$

$I_{BC} = \frac{169.70\angle -120^\circ}{5\angle 45^\circ} = 33.94\angle -165^\circ$

$I_{CA} = \frac{169.70\angle -240^\circ}{5\angle 45^\circ} = 33.94\angle -285^\circ$

$I_A = 58.785\angle -75^\circ$

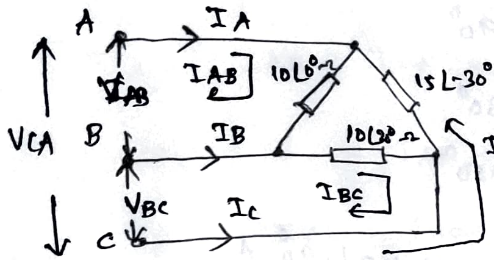
$I_B = 58.785\angle 165^\circ$.

$I_C = 58.785\angle 45^\circ$

Problem-3:- A three-phase, 4-wire, ~~CBA~~ ABC system, with ~~effective~~ has voltage of ~~120V~~ 339.4V, has ~~three~~

$Z_{AB} = 10\angle 0^\circ \,\Omega$, $Z_{BC} = 10\angle 30^\circ \,\Omega$, $Z_{CA} = 15\angle -30^\circ \,\Omega$.

obtain phase and line currents



$V_{AB} = 339.4\angle 120^\circ$

$V_{BC} = 339.4\angle 0^\circ$

$V_{CA} = 339.4\angle 240^\circ$

$\therefore V_L = V_\phi = 339.4 \text{ V}$

$$I_{AB} = \frac{339.4\angle 120^\circ}{10\angle 0^\circ} = 33.94\angle 120^\circ \text{ A}$$

$$I_{BC} = \frac{339.4\angle 0^\circ}{10\angle 30^\circ} = 33.94\angle -30^\circ \text{ A}$$

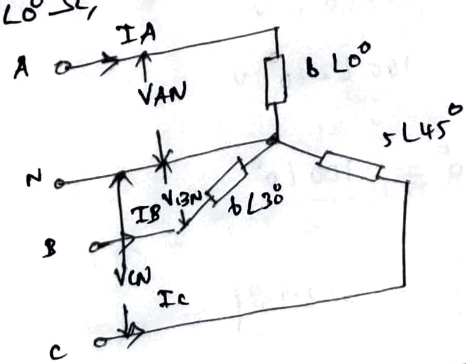
$$I_{CA} = \frac{339.4\angle 240^\circ}{15\angle -30^\circ} = 22.63\angle 270^\circ \text{ A}$$

$$\therefore I_A = I_{AB} - I_{CA} = 54.72\angle 108.1^\circ \text{ A}$$

$$I_B = I_{BC} - I_{AB} = 65.56\angle -45^\circ \text{ A}$$

$$I_C = I_{CA} - I_{BC} = 29.93\angle -169.1^\circ \text{ A}$$

Problem-4:- A three-phase, four-wire, 150V, CBA-system has Y-connected, with $Z_A = 6\angle 0^\circ \,\Omega$, $Z_B = 6\angle 30^\circ$, $Z_C = 5\angle 45^\circ \,\Omega$, obtain all line currents



$V_{AB} = 120 \angle 240^\circ$ V

$V_{BC} = 120 \angle 0^\circ$ V

$V_{CA} = 120 \angle 120^\circ$ V

$V_{AN} = 86.6 \angle -90^\circ$

$V_{BN} = 86.6 \angle 30^\circ$

$V_{CN} = 86.6 \angle 150^\circ$

$\therefore I_A = \dfrac{V_{AN}}{Z_A} = 14.43 \angle -90^\circ$ A

$I_B = \dfrac{V_{BN}}{Z_B} = 14.43 \angle 0^\circ$ A

$I_C = \dfrac{V_{CN}}{Z_C} = 17.32 \angle 105^\circ$ A

problem-4:- A balanced Δ- connected load has one phase current $I_{BC} = 2 \angle -90^\circ$A. Find the other phase currents and the three line currents if the system is an-ABC system. If the line voltage is 100 V, what is the load impedance

$I_{BC} = 2 \angle -90^\circ$

$\therefore I_{BC} = \dfrac{V_{BC}}{Z}$

$V_{AB} = 100 \angle 120^\circ$ V

$V_{BC} = 100 \angle 0^\circ$ V

$V_{CA} = 100 \angle 240^\circ$ V

$2 \angle -90^\circ = \dfrac{100 \angle 0^\circ}{Z}$

$\boxed{Z = 50 \angle 90^\circ}$

$\therefore\ I_{AB} = \frac{V_{AB}}{Z} = \frac{100 \angle 120^\circ}{50 \angle 90^\circ} = 2 \angle 30^\circ$

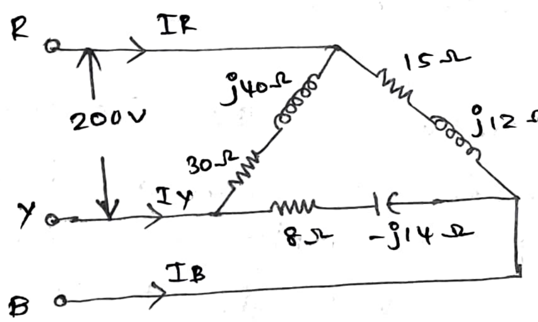
$\therefore\ I_{CA} = \frac{V_{CA}}{Z} = \frac{100 \angle 240^\circ}{50 \angle 90^\circ} = 2 \angle 150^\circ$

$\therefore\ I_A = I_{AB} - I_{CA} = 2\angle 30^\circ - 2\angle 150^\circ = 2\sqrt{3}$ A

$I_B = I_{BC} - I_{AB} = 2\angle -90^\circ - 2\angle 30^\circ = 2\sqrt{3} \angle -120^\circ$ A

$I_C = I_{CA} - I_{BC} = 2\angle 150^\circ - 2\angle -90^\circ = 2\sqrt{3} \angle +120^\circ$ A

Problem-5 :- Determine the line currents for the unbalanced delta connected load shown below. Phase sequence - RYB



$V_{RY} = 200 \angle 120^\circ$ V

$V_{YB} = 200 \angle 0^\circ$ V

$V_{RB} = 200 \angle 240^\circ$ V

$I_{RY} = \frac{V_{RY}}{Z} = \frac{200 \angle 120^\circ}{30 + j40} = 4 \angle 66.866^\circ$ A

$I_{YB} = \frac{V_{YB}}{Z} = \frac{200 \angle 0^\circ}{\cancel{30+j40}\ 8 - j14} = 12.40 \angle 60.25^\circ$ A $\cancel{4\angle -53.13^\circ A}$

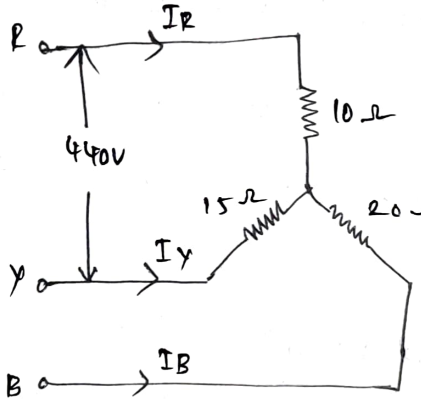
$I_{RB} = \frac{V_{RB}}{Z} = \frac{200 \angle 240^\circ}{15 + j12} = 10.41 \angle -158.65^\circ$ A

$\therefore\ I_R = I_{RY} - I_{RB} = 14.07 \angle 35.01^\circ$ A

$I_Y = I_{YB} - \cancel{I_{RY}} = 8.43 \angle 57.12^\circ$ A

$I_B = I_{BA} - I_{YB} = 21.51 \angle -137.43^\circ$ A

Problem-6:- A unbalanced, Star-connected load is supplied from a 3ϕ, 440V, symmetrical system. Determine the line currents and power input to the circuit. Assume-RYB sequence. Take V_{RN} as reference



$P = I^2 R = 13418$ W

Problem-7:- Determine the line currents and the total power for the unbalanced-Δ connected load. A 3-phase supply, with effective line voltage of 240V is given to the circuit