$$= \sqrt{3}|\bar{V}_L||\bar{I}_L| \text{ (VA)}$$

Therefore, power factor $= \dfrac{P}{S} = \cos\phi$

Phasor Diagram

The phasor diagram for a balanced, delta-connected source is shown in Fig. 5.11.

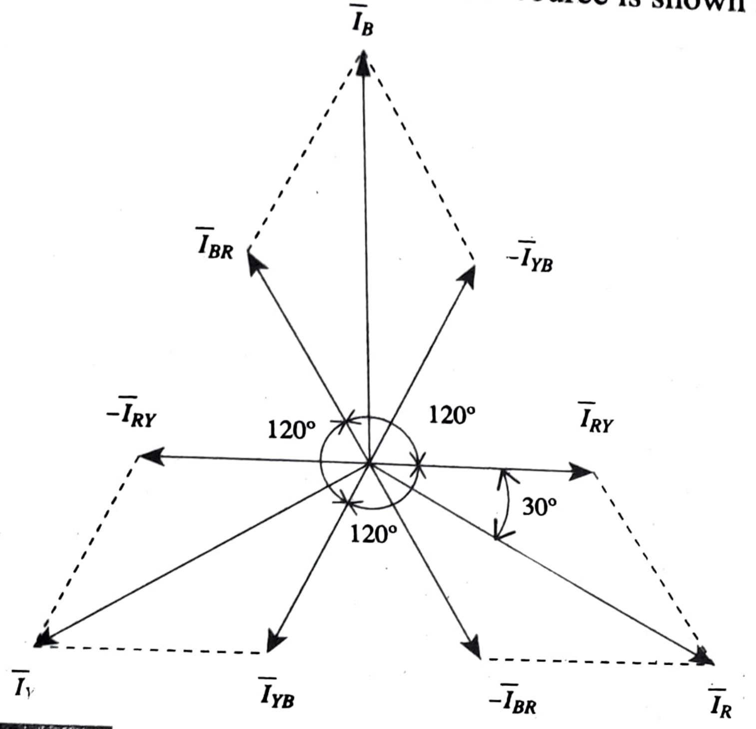

FIG. 5.11 Phasor diagram for a balanced, delta-connected source

5.5 THREE-PHASE, BALANCED, STAR-CONNECTED LOAD

The circuit diagram for a three-phase, balanced, star-connected load is shown in Fig. 5.12.

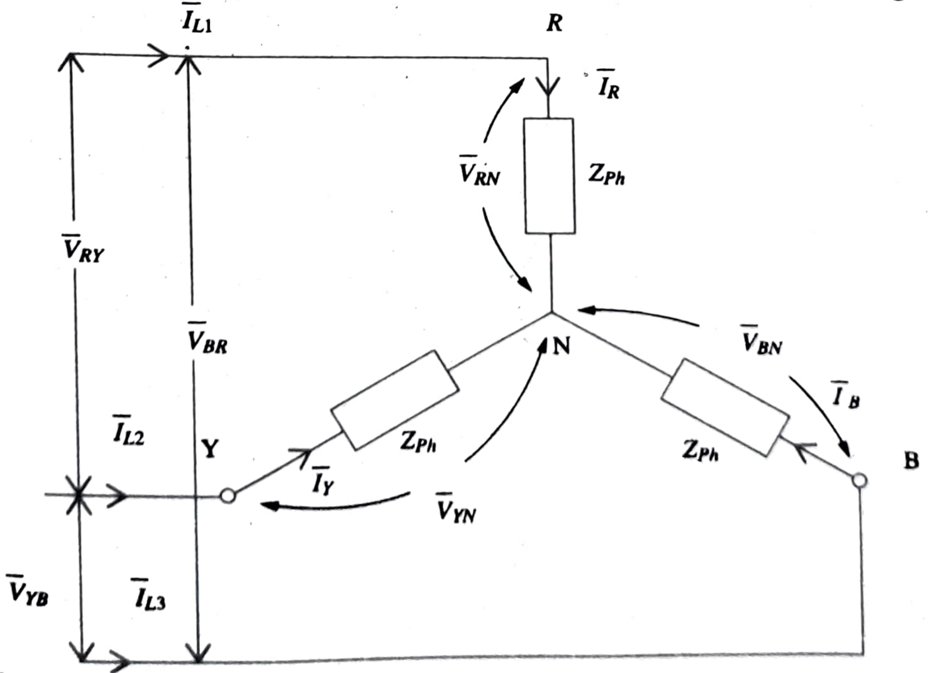

FIG. 5.12 Circuit diagram for a three-phase, balanced, star-connected load

Analysis

The notations used for the analysis of three-phase, balanced, star-connected load are:

$\bar{V}_{RN}, \bar{V}_{YN}, \bar{V}_{BN}$: Phase voltages of R, Y and B phases

$\bar{I}_R, \bar{I}_Y, \bar{I}_B$: Phase currents of R, Y and B phases

$\bar{V}_{RY}, \bar{V}_{YB}, \bar{V}_{BR}$: Line voltages across R, Y and B lines

$\bar{I}_{L1}, \bar{I}_{L2}, \bar{I}_{L3}$: Line currents of R, Y and B lines

Z_{ph} : Load impedance per phase

In a balanced system,

$$|\bar{V}_{RN}| = |\bar{V}_{YN}| = |\bar{V}_{BN}| = |\bar{V}_{ph}|; \quad |\bar{V}_{RY}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = |\bar{V}_L|$$

$$|\bar{I}_R| = |\bar{I}_Y| = |\bar{I}_B| = |\bar{I}_{ph}| \quad ; \quad |\bar{I}_{L1}| = |\bar{I}_{L2}| = |\bar{I}_{L3}| = |\bar{I}_L|$$

Current Relationship

Applying Kirchhoff's current law at nodes R, Y and B in Fig. 5.12, we get

$$\bar{I}_R = \bar{I}_{L1}; \quad \bar{I}_Y = \bar{I}_{L2}; \quad \bar{I}_B = \bar{I}_{L3}$$

This means that in a balanced, star-connected system, phase current equals the line current,

$$\bar{I}_{ph} = \bar{I}_L$$

where $\quad \bar{I}_R = \dfrac{\bar{V}_{RN}}{Z_{ph}}; \quad \bar{I}_Y = \dfrac{\bar{V}_{YN}}{Z_{ph}}; \quad \bar{I}_B = \dfrac{\bar{V}_{BN}}{Z_{ph}}$

Voltage Relationship

Applying Kirchhoff's voltage law to the loop consisting of voltages, $\bar{V}_{RN}, \bar{V}_{RY}$ and $\bar{V}_{YN}$, in Fig. 5.12, we have $\bar{V}_{RN} - \bar{V}_{YN} = \bar{V}_{RY}$

Using parallelogram law of addition,

$$|\bar{V}_{RY}| = \sqrt{|\bar{V}_{RN}|^2 + |\bar{V}_{YN}|^2 + 2|\bar{V}_{RN}||\bar{V}_{YN}|\cos 60^\circ}$$

$$= \sqrt{|\bar{V}_{ph}|^2 + |\bar{V}_{ph}|^2 + 2|\bar{V}_{ph}||\bar{V}_{ph}|(0.5)}$$

$$|\bar{V}_{RY}| = \sqrt{3}|\bar{V}_{ph}|$$

Therefore, $\quad |\bar{V}_{ph}| = \dfrac{|\bar{V}_{RY}|}{\sqrt{3}}$

Similarly, $\quad \bar{V}_{YN} - \bar{V}_{BN} = \bar{V}_{YB} \quad$ and $\quad \bar{V}_{BN} - \bar{V}_{RN} = \bar{V}_{BR}$

$$|\bar{V}_{YB}| = \sqrt{3}|\bar{V}_{ph}| \quad \text{and} \quad |\bar{V}_{BR}| = \sqrt{3}|\bar{V}_{ph}|$$

Therefore, $\quad |\bar{V}_{ph}| = \dfrac{|\bar{V}_{YB}|}{\sqrt{3}}$

and $\quad |\bar{V}_{ph}| = \dfrac{|\bar{V}_{BR}|}{\sqrt{3}}$

Thus, $\quad |\bar{V}_L| = \sqrt{3}|\bar{V}_{ph}|$

i.e., Line voltage = $\sqrt{3}$ Phase voltage

Therefore, $|\bar{V}_{ph}| = \dfrac{|\bar{V}_L|}{\sqrt{3}}$

i.e., Phase voltage = $\dfrac{\text{Line voltage}}{\sqrt{3}}$

Load Impedance

If the load has lagging power factor (inductive) in nature, then the load impedance is given by

$$Z_{ph} = R_{ph} + jX_{Lph}$$

If the load has leading power factor (capacitive) in nature, then the load impedance is given by

$$Z_{ph} = R_{ph} - jX_{Cph}$$

Power Relationship

The power factor of the system is $\cos\phi$.

Real power per phase, $P = |\bar{V}_{ph}||\bar{I}_{ph}|\cos\phi$

Total real power, $P = 3|\bar{V}_{ph}||\bar{I}_{ph}|\cos\phi$

$$= 3\frac{|\bar{V}_L|}{\sqrt{3}}|\bar{I}_L|\cos\phi$$

$$= \sqrt{3}|\bar{V}_L||\bar{I}_L|\cos\phi\,(\text{W})$$

Reactive power per phase, $Q = |\bar{V}_{ph}||\bar{I}_{ph}|\sin\phi$

Total reactive power, $Q = 3|\bar{V}_{ph}||\bar{I}_{ph}|\sin\phi$

$$= 3\frac{|\bar{V}_L|}{\sqrt{3}}|\bar{I}_L|\sin\phi$$

$$= \sqrt{3}|\bar{V}_L||\bar{I}_L|\sin\phi\,(\text{VAR})$$

Apparent power per phase, $S = |\bar{V}_{ph}||\bar{I}_{ph}|$

Total apparent power, $S = 3|\bar{V}_{ph}||\bar{I}_{ph}|$

$$= 3\frac{|\bar{V}_L|}{\sqrt{3}}|\bar{I}_L|$$

$$= \sqrt{3}|\bar{V}_L||\bar{I}_L|(\text{VA})$$

Therefore, power factor $= \dfrac{P}{S} = \cos\phi$

Phasor Diagram

The phasor diagram for a three-phase, balanced, star-connected load with lagging power factor load (inductive load) is shown in Fig. 5.13.

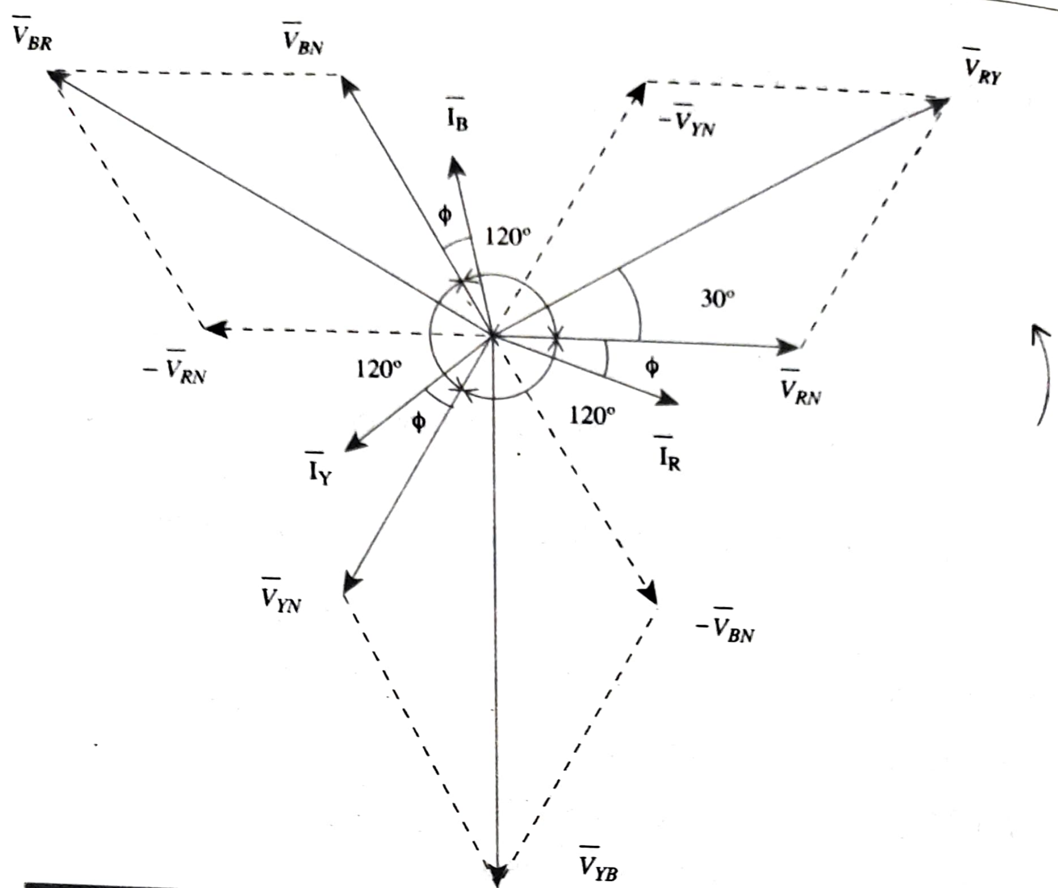

FIG. 5.13 Phasor diagram for a three-phase, balanced, star-connected load with lagging power factor (inductive load)

The phasor diagram for a three-phase, balanced, star-connected load with leadi power factor load (capacitive load) is shown in Fig. 5.14.

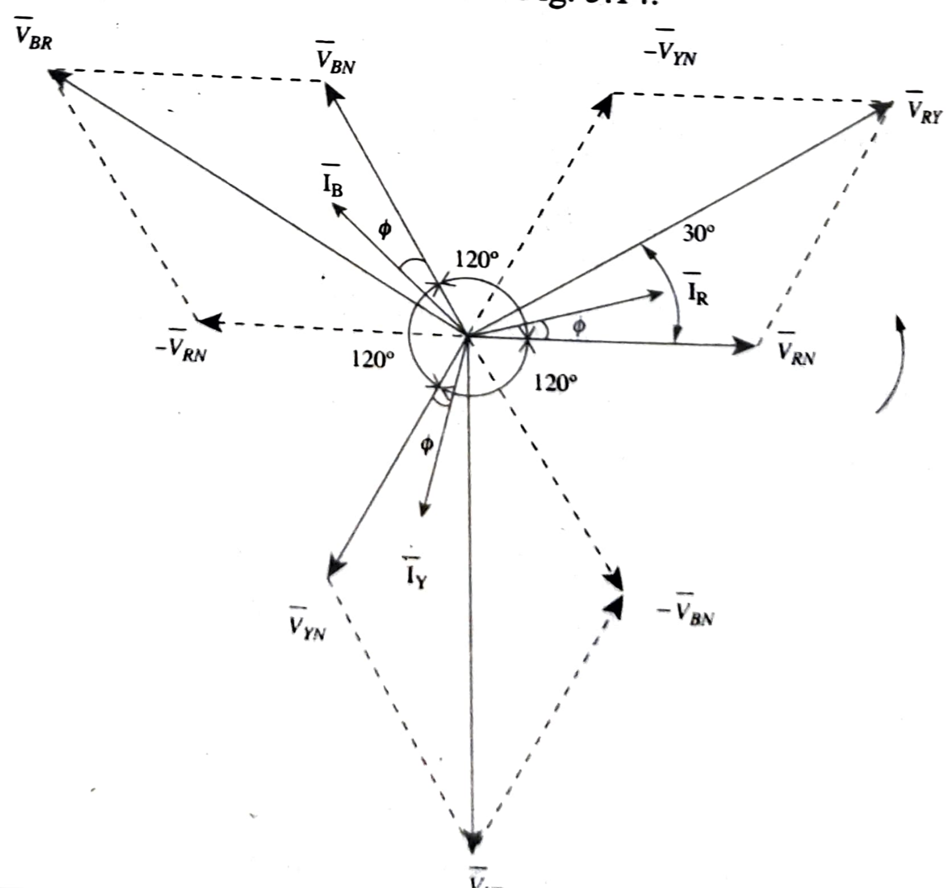

FIG. 5.14 Phasor diagram for a three-phase, balanced, star-connected load

5.6 THREE-PHASE, BALANCED, DELTA-CONNECTED LOAD

The circuit diagram for a three-phase, balanced, delta-connected load is shown in Fig. 5.15.

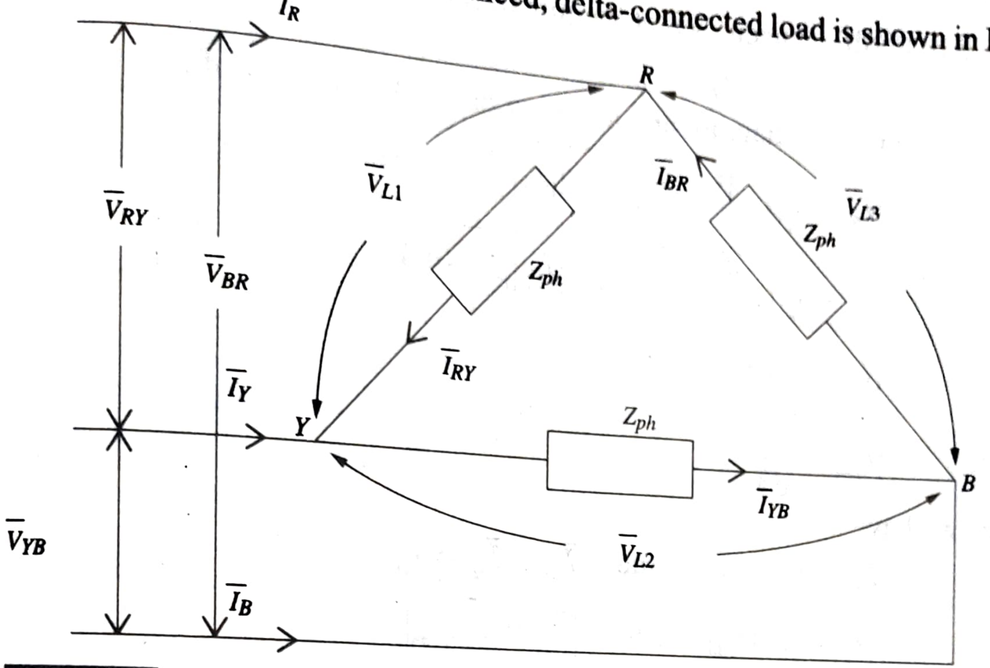

FIG. 5.15 Circuit diagram for a three-phase, balanced, delta-connected load

Analysis

The notations used for the analysis of three-phase, balanced, delta-connected load are:

$\bar{V}_{RY}, \bar{V}_{YB}, \bar{V}_{BR}$	: Phase voltages of R, Y and B phases
$\bar{I}_{RY}, \bar{I}_{YB}, \bar{I}_{BR}$	: Phase currents of R, Y and B phases
$\bar{V}_{L1}, \bar{V}_{L2}, \bar{V}_{L3}$	: Line voltages across R, Y and B lines
$\bar{I}_R, \bar{I}_Y, \bar{I}_B$	: Line currents of R, Y and B lines
Z_{ph}	: Load impedance per phase

In a balanced system,

$$|\bar{V}_{RN}| = |\bar{V}_{YN}| = |\bar{V}_{BN}| = |\bar{V}_{ph}| \quad ; \quad |\bar{V}_{RY}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = |\bar{V}_L|$$

$$|\bar{I}_R| = |\bar{I}_Y| = |\bar{I}_B| = |\bar{I}_{ph}| \quad ; \quad |\bar{I}_{L1}| = |\bar{I}_{L2}| = |\bar{I}_{L3}| = |\bar{I}_L|$$

Voltage Relationship

Applying Kirchhoff's voltage law to the loop consisting of $\bar{V}_{L1}$ and $\bar{V}_{RY}$ in Fig. 5.15, we have

$$\bar{V}_{L1} = \bar{V}_{RY}$$

Similarly, $\bar{V}_{L2} = \bar{V}_{YB}$ and $\bar{V}_{L3} = \bar{V}_{BR}$

Thus, $\bar{V}_{ph} = \bar{V}_L$

i.e., Phase voltage = Line voltage

Current Relationship

Applying Kirchhoff's current law at the junction R in Fig. 5.15, we have

$$\bar{I}_{RY} - \bar{I}_{BR} = \bar{I}_R$$

Referring to the phasor diagram and applying parallelogram law of addition, we have

$$|\bar{I}_R| = \sqrt{|\bar{I}_{RY}|^2 + |\bar{I}_{BR}|^2 + 2|\bar{I}_{RY}||\bar{I}_{BR}|\cos 60^\circ}$$

$$= \sqrt{|\bar{I}_{ph}|^2 + |\bar{I}_{ph}|^2 + 2|\bar{I}_{ph}||\bar{I}_{ph}|(0.5)}$$

$$|\bar{I}_R| = \sqrt{3}|\bar{I}_{ph}|$$

Therefore, $$|\bar{I}_{ph}| = \frac{|\bar{I}_R|}{\sqrt{3}}$$

Similarly, we have

$$\bar{I}_{YB} - \bar{I}_{RY} = \bar{I}_Y \quad \text{and} \quad \bar{I}_{BR} - \bar{I}_{YB} = \bar{I}_B$$

$$|\bar{I}_Y| = \sqrt{3}|\bar{I}_{ph}| \quad \text{and} \quad |\bar{I}_B| = \sqrt{3}|\bar{I}_{ph}|$$

Therefore, $$|\bar{I}_{ph}| = \frac{|\bar{I}_Y|}{\sqrt{3}} \quad \text{and} \quad |\bar{I}_{ph}| = \frac{|\bar{I}_B|}{\sqrt{3}}$$

Thus, $$|\bar{I}_L| = \sqrt{3}|\bar{I}_{ph}|$$

i.e., Line current $= \sqrt{3}$ Phase current

Therefore $$|\bar{I}_{ph}| = \frac{|\bar{I}_L|}{\sqrt{3}}$$

i.e., $$\text{Phase current} = \frac{\text{Line current}}{\sqrt{3}}$$

$$\bar{I}_{Ry} = \frac{\bar{V}_{RY}}{Z_{ph}}; \quad \bar{I}_{YB} = \frac{\bar{V}_{YB}}{Z_{ph}}; \quad \bar{I}_{BR} = \frac{\bar{V}_{BR}}{Z_{ph}}$$

Load Impedance

If the load has lagging power factor (inductive) in nature, then the load impedance is given by

$$Z_{ph} = R_{ph} + jX_{Lph}$$

If the load has leading power factor (capacitive) in nature, then the load impedance is given by

$$Z_{ph} = R_{ph} - jX_{Cph}$$

Power Relationship

The power factor of the system is $\cos\phi$.

Real power per phase, $$P = |\bar{V}_{ph}||\bar{I}_{ph}|\cos\phi$$

Total real power, $$P = 3|\bar{V}_{ph}||\bar{I}_{ph}|\cos\phi$$

$$= 3|\bar{V}_L|\frac{|\bar{I}_L|}{\sqrt{3}}\cos\phi$$

$$= \sqrt{3}|\bar{V}_L||\bar{I}_L|\cos\phi\,(\text{W})$$

Reactive power per phase, $$Q = |\bar{V}_{ph}||\bar{I}_{ph}|\sin\phi$$

Total reactive power, $$Q = 3|\bar{V}_{ph}||\bar{I}_{ph}|\sin\phi$$

$$= 3|\bar{V}_L|\frac{|\bar{I}_L|}{\sqrt{3}}\sin\phi$$

$$= \sqrt{3}|\bar{V}_L||\bar{I}_L|\sin\phi\ (\text{VAR})$$

Apparent power per phase, $$S = |\bar{V}_{ph}||\bar{I}_{ph}|$$

Total apparent power, $$S = 3|\bar{V}_{ph}||\bar{I}_{ph}|$$

$$= 3|V_L|\frac{|\bar{I}_L|}{\sqrt{3}}$$

$$= \sqrt{3}|\bar{V}_L||\bar{I}_L|(\text{VA})$$

Therefore, power factor $= \dfrac{P}{S} = \cos\phi$

Phasor Diagram

The phasor diagram for a three-phase, balanced, delta-connected load with lagging power factor load (inductive load) is shown in Fig. 5.16.

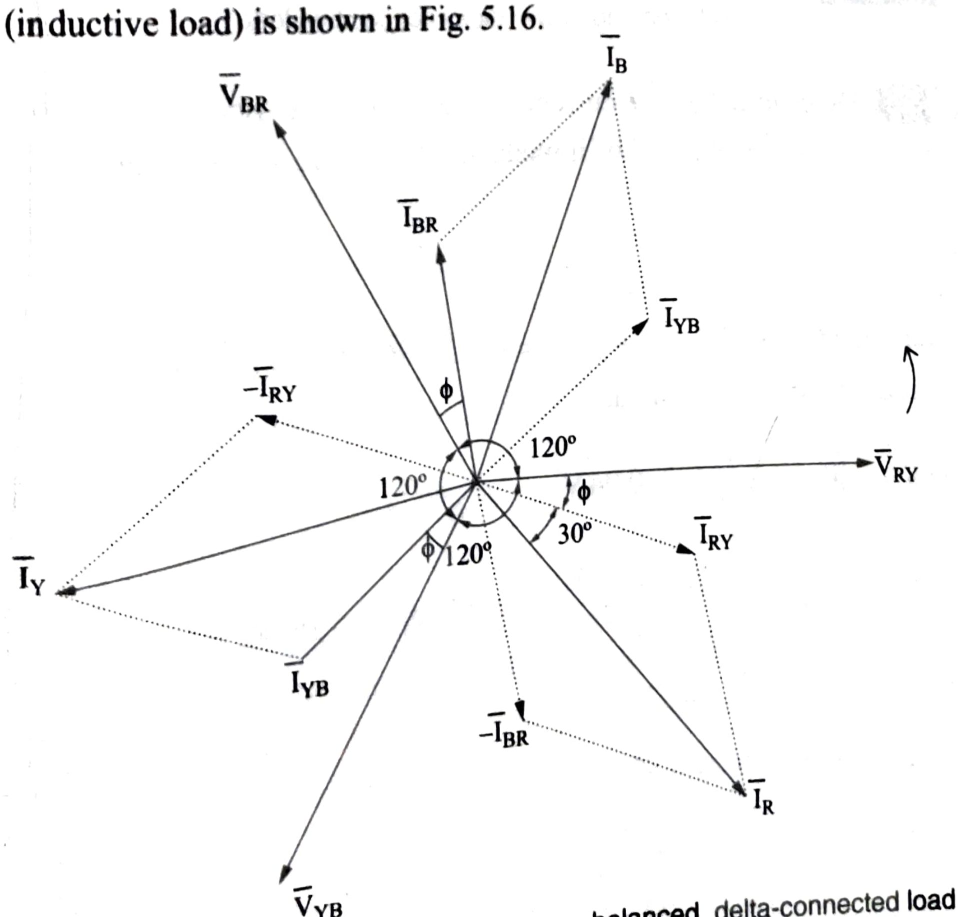

FIG. 5.16 Phasor diagram for a three-phase, balanced, delta-connected load with lagging power factor (inductive load)

The phasor diagram for a three-phase, balanced, delta-connected load with leading power factor load (capacitive load) is shown in Fig. 5.17.

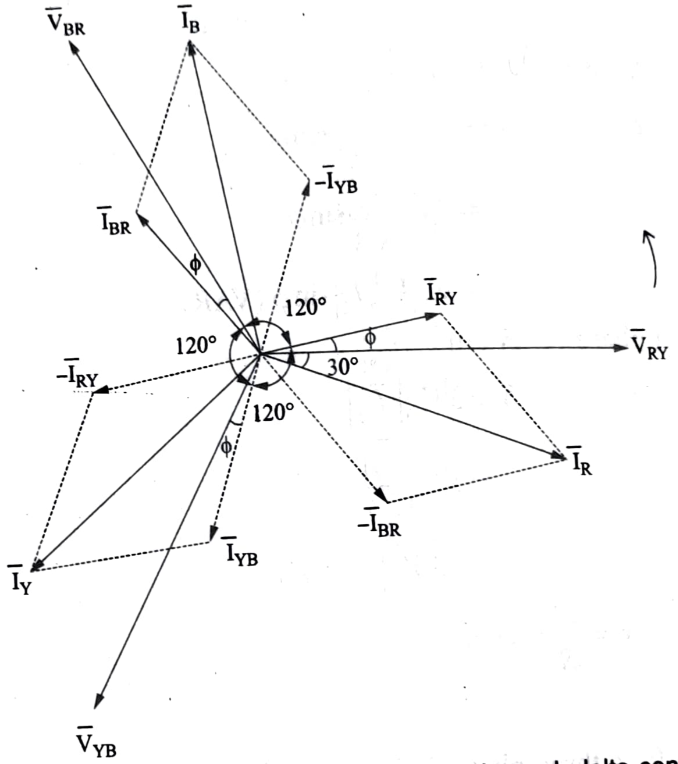

FIG. 5.17 Phasor diagram for a three-phase, balanced, delta-connected load with leading power factor (capacitive load)

Example 5.1 Determine the line current, power factor and total power when a 3ϕ, 400 V supply is given to a balanced load of impedance (8 + j6) Ω, if each branch is connected in star form.

Solution

Given: Load impedance, $Z_{ph} = 8 + j6\Omega = 10\angle 36.86°\Omega$ and line voltage, $|\bar{V}_L| = 400\text{V}$

For a balanced, star-connected load,

Phase voltage, $$|\bar{V}_{ph}| = \frac{|\bar{V}_L|}{\sqrt{3}} = 230.94\,\text{V}$$

Phase current, $$\bar{I}_{ph} = \frac{\bar{V}_{ph}}{Z_{ph}} = \frac{230.94\angle 0°}{10\angle 36.860°} = 23.094\angle -36.86°\text{A}$$

Line current, $$|\bar{I}_L| = |\bar{I}_{ph}| = 23.094\text{A}$$

Power factor, $$\cos\phi = \cos(36.86°) = 0.8\,\text{lagging}$$

Total power, $$P = \sqrt{3}|\bar{V}_L||\bar{I}_L|\cos\phi$$

$$= \sqrt{3} \times 400 \times 23.094 \times 0.8 = 12799.994\text{W} = 12.8\text{kW}$$

Example 5.2 The power consumed in a 3ϕ, balanced, star-connected load is 2 kW at a power factor of 0.81 lagging. The supply voltage is 400 V, 50 Hz. Calculate the resistance and reactance of each phase.

Solution

Given: Power consumed, P = 2 kW, power factor, $\cos\phi = 0.8$ lagging and line voltage, $|\bar{V}_L| = 400\,\text{V}$

For a balanced, star-connected load,

$$|\bar{V}_{ph}| = \frac{|\bar{V}_L|}{\sqrt{3}} = 230.94\,\text{V}$$

Power consumed,

$$P = \sqrt{3}|\bar{V}_L||\bar{I}_L|\cos\phi$$

Substituting the values of p, $|\bar{V}_L|$ and ϕ,

$$2\times10^3 = \sqrt{3}\times400\times|\bar{I}_L|\times0.8$$

Therefore, $|\bar{I}_L| = 3.6084\,\text{A}$

For a balanced, star-connected load

$$|\bar{I}_{ph}| = |\bar{I}_L|$$

Therefore, $|\bar{Z}_{ph}| = \dfrac{|\bar{V}_{ph}|}{|\bar{I}_{ph}|} = \dfrac{230.94}{3.6084} = 64\,\Omega$

and phase angle, $\phi = \cos^{-1}0.8 = 36.869°$

Therefore, $Z_{ph} = |Z_{ph}|\angle\phi = 64\angle36.869°$

$$= 51.2 + j38.4\,\Omega = R_{ph} + jX_{Lph}$$

Therefore, $R_{ph} = 51.2\,\Omega$, $X_{Lph} = 38.4\,\Omega = 2\pi\times f\times L_{ph}$

and $L_{ph} = \dfrac{38.4}{2\pi\times50} = 0.1222\,\text{H}$

Example 5.3 A 3ϕ balanced, delta-connected load of $(4+j8)\,\Omega$ is connected across a 400 V, 3ϕ balanced supply. Determine the phase currents and line currents. Assume RYB phase sequence. Also, calculate the power drawn by the load.

Solution

Given: Load impedance, $Z_{ph} = 4 + j8\,\Omega = 8.944\angle63.43°\,\Omega$ and line voltage, $|\bar{V}_L| = 400\,\text{V}$

For delta connected, $|\bar{V}_L| = |\bar{V}_{ph}| = 400\,\text{V}$

The phase current, $\left|\bar{I}_{ph}\right| = \dfrac{\left|\bar{V}_{ph}\right|}{\left|Z_{ph}\right|} = \dfrac{400}{8.944} = 44.7227\text{ A}$

The line current, $\left|\bar{I}_L\right| = \sqrt{3}\left|\bar{I}_{ph}\right| = \sqrt{3} \times 44.7227 = 77.462\text{ A}$

Taking $\bar{V}_{ph} = \bar{V}_{RN} = \bar{V}_{RY}$ as the reference phasor, we have

$\bar{I}_{RY} = \bar{I}_{ph}$ which lags $\bar{V}_{RY}$ by phase angle $63.43°$

Therefore, $\bar{I}_{RY} = 44.7227\angle -63.43°\text{A}$

Similarly, $\bar{I}_{YB} = 44.7227\angle -63.43° - 120° = 44.7227\angle -183.43°\text{A}$

and $\bar{I}_{BR} = 44.7227\angle -183.43° - 120° = 44.7227\angle -303.43°\text{A}$

For a delta-connected load, the line current lags the respective phase current by 30°.

Therefore, $\bar{I}_R = 77.462\angle -63.43° - 30° = 77.462\angle -93.43°\text{A}$

Similarly, $\bar{I}_Y = 77.462\angle -93.43° - 120° = 77.462\angle -213.43°\text{A}$

and $\bar{I}_B = 77.462\angle -213.43° - 120° = 77.462\angle -333.43°\text{A}$

The power drawn by the load, $P = \sqrt{3}\left|\bar{V}_L\right|\left|\bar{I}_L\right|\cos\phi = \sqrt{3} \times 400 \times 77.462 \times \cos(63.43°)$

$= 24004.868\text{ W} = 24.004\text{ kW}$

Example 5.4 A symmetric 3ϕ, 400 V system supplies a balanced, delta-connected load as shown in Fig. 5.18. The current in each branch of the circuit is 20 A and phase angle 40° lagging. Calculate the line current and the total power.

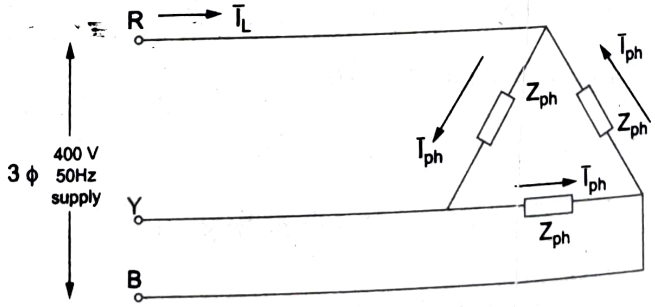

FIG. E5.4

Solution

Given: Line voltage, $\left|\bar{V}_L\right| = 400\text{V}$, phase current, $\bar{I}_{ph} = 20\angle -40°\text{ A}$ and phase angle, $\phi = 40°$

For a balanced-delta connected load, $\left|\bar{V}_{ph}\right| = \left|\bar{V}_L\right| = 400\text{ V}$

The line current is given by $\left|\bar{I}_L\right| = \sqrt{3}\left|\bar{I}_{ph}\right| = 34.641\text{ A}$

The total power is given by $P = \sqrt{3}\left|\bar{V}_L\right|\left|\bar{I}_L\right|\cos\phi$

Substituting $|\overline{V}_L|, |\overline{I}_L|$ and ϕ, we get

$$= \sqrt{3} \times 400 \times 34.641 \times \cos(40°) = 18385.058 \text{ W} = 18.385 \text{kW}$$

Example 5.5 A 3ϕ balanced, star-connected load of $2 + j4\,\Omega$ is connected to a 3ϕ balanced, star-connected source with a phase to neutral voltage of 110 V. Determine the line voltage, phase voltage across the load, line currents and phase currents in the load. Assume RYB phase sequence. Also calculate the power drawn by the load.

Solution

Given: Load impedance, $Z_{ph} = 4 + j8\Omega = 8.944\angle 63.43°\Omega$

Phase voltage, $|\overline{V}_{ph}|_{(load)} = 110\text{V} = |\overline{V}_{ph}|_{(source)}$

For a balanced, star-connected load,

Line voltage $|\overline{V}_L| = \sqrt{3}|\overline{V}_{ph}|_{(load)} = \sqrt{3} \times 110 = 190.525\text{V}$

The phase current, $|\overline{I}_{ph}|_{(load)} = \dfrac{|\overline{V}_{ph}|_{(load)}}{|Z_{ph}|} = \dfrac{110}{4.472} = 24.597\text{A}$

The line current, $|\overline{I}_L| = |\overline{I}_{ph}| = 24.597\text{A}$

Taking $\overline{V}_{ph} = \overline{V}_{RN}$ as the reference phasor, we have

$\overline{I}_R = \overline{I}_{ph}$ which lags $\overline{V}_{RN}$ by phase angle 63.435°

Therefore, $\overline{I}_{R(load)} = 24.597\angle -63.435°\text{A}$

Similarly, $\overline{I}_{Y(load)} = 24.597\angle -63.435° - 120° = 24.597\angle -183.435°\text{A}$

and $\overline{I}_{B(load)} = 24.597\angle -183.435° - 120° = 24.597\angle -303.435°\text{A}$

For a balanced, star-connected load, the line voltage leads the respective phase voltage by 30°.

Therefore, $\overline{V}_{RY} = 190.525\angle 0° + 30° = 190.525\angle 30°\text{V}$

Similarly, $\overline{V}_{YB} = 190.525\angle 30° - 120° = 190.525\angle -90°\text{V}$

and $\overline{V}_{BR} = 190.525\angle -90° - 120° = 190.525\angle -210°\text{V}$

The power drawn by the load,

$$P = \sqrt{3}|\overline{V}_L||\overline{I}_L|\cos\phi$$

$$= \sqrt{3} \times 190.525 \times 24.597 \times \cos(63.435°) = 3630.019\text{ W} = 3.63\text{kW}$$