$$1.5 = \frac{V_x}{150} + 0.5 + I$$

Substituting $V_x = 1.333\,\text{V}$, we get

$$1.5 = \frac{1.333}{150} + 0.5 + I$$

Therefore, $I = 0.991\,\text{A}$

The voltage across the unknown circuit element $V_x = 1.333\,\text{V}$ and the current

$$I = 0.991\,\text{A}$$

Therefore, power $P = 1.333 \times 0.991 = 1.321\,\text{W}$

Power in AC Circuits

An AC circuit consists of AC sources and components. Table 1.1 shows different components used in AC circuits and their relationships among voltage, current and power.

TABLE 1.1

Circuit element	Impedance	Voltage (V)	Current (A)	Instantaneous power (W)
Resistor R (Ω)	R	$v = Ri$	$i = \frac{v}{R}$	$p = i^2 R$
Inductor L (H)	$j\omega L$	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + i(0^+)$ where $i(0+)$ is the initial current	$p = Li\frac{di}{dt}$
Capacitor C (F)	$\frac{-j}{\omega C}$	$v = \frac{1}{C}\int i\,dt + v(0^+)$ where $v(0^+)$ is the initial voltage	$i = C\frac{dv}{dt}$	$p = Cv\frac{dv}{dt}$

A series RLC circuit consisting of all the circuit elements R, L and C connected in series is shown in Fig. 1.32(a). As illustrated in the circuit, the resistor dissipates the power in the form of heat, and pure inductors or capacitors store the supplied energy in the form of magnetic or electric field.

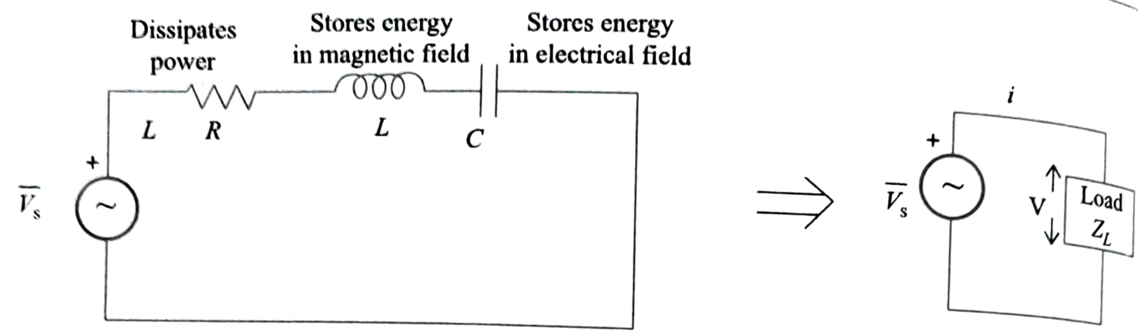


FIG. 1.32 (a) A series RLC circuit with AC excitation, and (b) its equivalent circuit

The circuit can be simplified with a single load impedance Z_L as shown in Fig. 1.32(b)

where, $Z_L = R + j\omega L - \dfrac{j}{\omega c}$.

In the representation of voltage and current in AC circuits, v and i are no longer considered as DC constants, and are represented as complex sinusoidal quantities consisting of both magnitudes and phase angles.

An alternating current is represented as

$$i = I_{max} \sin \omega t$$

Similarly, in representing an AC voltage, if ϕ is the phase angle between the current and voltage, then $v = V_{max} \sin(\omega t \pm \phi)$. Here, ϕ is assigned with a + or – sign depending on whether the voltage v is leading or lagging the current i.

If the load is a pure resistor, then $\phi = 0°$, i.e., v is in phase with i.

If the load is a pure inductive, then $\phi = 90°$, i.e., v leads i.

If the load is a pure capacitive, then $\phi = -90°$, i.e., v lags i.

Substituting the equations for v and i in the equation for power,

$$p = vi, \text{ we get}$$

$$p = V_{max} \sin(\omega t + \phi) I_{max} \sin \omega t$$

Power in an AC circuit is often expressed in three forms: (*i*) apparent power, (*ii*) average or active power, and (*iii*) reactive power.

(i) Apparent Power

Apparent power in the AC circuit is defined as the product of applied voltage v and current i. It is called apparent power because it is simply calculated from the multiplication of known voltage and current values indicated by the voltmeter and ammeter readings. The type of load connected to the circuit is usually not taken into consideration in the calculation of apparent power. Apparent power is symbolically represented by S, and its unit of measurement is volt-ampere (VA).

Apparent power is also called 'complex power' and is expressed as

$$S = VI$$

$$= \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}}$$

$$= \frac{V_{max} I_{max}}{2}$$

(ii) Average or Active power

The active or average power is obtained by the product of apparent power (i.e., V_{rms} and I_{rms}) and the cosine of the angle between the voltage and current. Active power is actually the 'real' or 'true' power of the circuit. It is represented by the symbol P and its unit is watt (W)

$$P = V_{max} I_{max} \sin \omega t \sin(\omega t + \phi)$$

Using trigonometric identities and converting V_{max} and I_{max} values to the corresponding V_{rms} and I_{rms} values, the above equation can be rewritten as

$$P = V_{rms} I_{rms} \cos\phi(1 - \cos 2\omega t) + V_{rms} \sin\theta(\sin 2)\omega t$$

If $V = V_{rms}$ and $I = I_{rms}$), then

$$P = \underbrace{VI \cos\phi}_{1} - \underbrace{VI \cos\phi}_{2} \cos 2\omega t + \underbrace{VI \sin\phi}_{3} \sin 2\omega t$$

where (1) indicates the average power and (2) and (3) indicate the peak power, as

Average power, $P_{av} = VI \cos\phi$, and

Peak power, $P_{peak} = VI \cos\phi$ or $VI \sin\phi$

Through careful inspection of the equation for these terms, it can be inferred that the average power is time-independent. Both forms of peak power indicated by (2) have similar formats, and vary with a frequency twice that of applied voltage or current.

Average power can be written with respect to apparent power as

$$P = S \cos\phi$$

$$= V_{rms} . I_{rms} . \cos\phi$$

$$= \frac{V_{max} I_{max}}{2} \cos\phi$$

(iii) Reactive power

Reactive power is obtained by the product of apparent power (i.e., V_{rms} and I_{rms}) and the sine of the angle between voltage and current. It is represented by the symbol Q and its unit is *volt-ampere reactive* (VAR).

$$Q = S \sin\phi$$

$$V_{rms} I_{rms} \sin\phi = \frac{V_{max} I_{max}}{2} \sin\phi$$

Power Triangle

In a vector domain, the equations associated with three types of power, namely apparent (S), average (P) and reactive (Q) can be related to each other by

$$S = P + jQ$$

Case (i) Resistive load on active power

If the load is pure resistive, then $\phi = 0°$, and $P = |P|\angle\phi° = |P|\angle 0°$

Case (ii) Reactive load on reactive power:

If the load is pure inductive, then $\phi = 90°$ and $Q_L = |Q_L|\angle\phi° = |Q_L|\angle 90°$

If the load is pure capacitive, then $\phi = -90°$ and $Q_C = |Q_C|\angle\phi° = |Q_C|90°$

The phasor power S for inductive load is given by

$$S = P + jQ_L$$

This relationship can be graphically represented in vector domain as in Fig. 1.33(a).

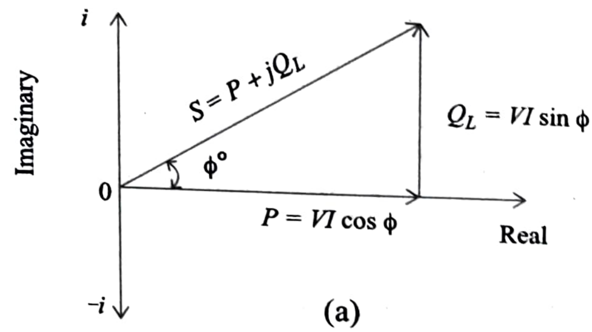


FIG. 1.33(a)

Similarly, for the capacitive load X_C, the phase power S can be written as

$$S = P - jQ_C, \text{ (since } Q_C\angle -90° = -jQ_C)$$

This relationship can be graphically represented in vector domain as in Fig. 1.33(b).

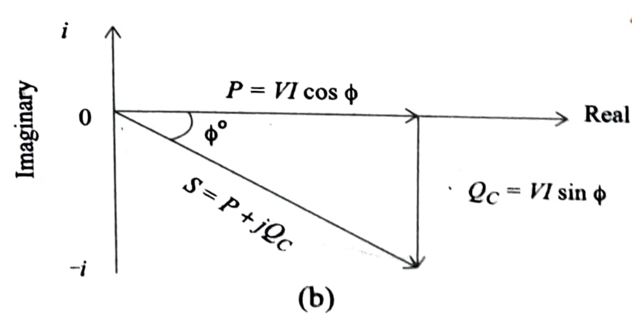


FIG. 1.33(b)

If the circuit has both inductive-load reactive power Q_L and capacitive-load reactive power Q_C then the difference between the reactive powers $Q_L - Q_C$ will be used in obtaining the reactive component Q of the power triangle.

For example, in a series RLC circuit, consisting of both reactive terms X_L and X_C and corresponding reactive components Q_L and Q_C if $Q_L > Q_C$ then the resultant Q will be in the direction of Q_L as shown in Fig. 1.33(c).

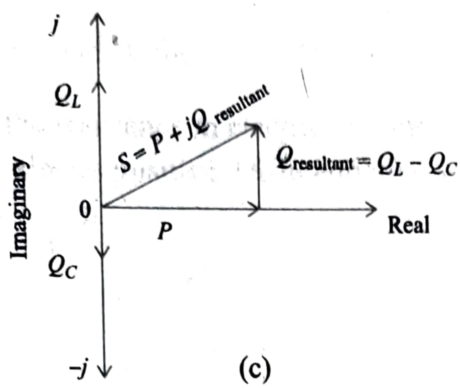


FIG. 1.33(c)

In a more generalized form, the power triangle can be represented as in Fig. 1.33(d)

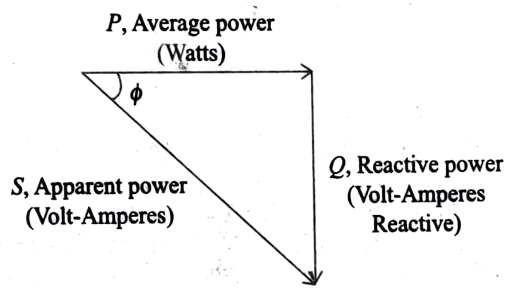


FIG. 1.33(d)

where Average power $P = S\cos\phi$

Reactive power $Q = S\sin\phi$

Note: The direction of Q is subject to change depending on the type of load used and the domination of either inductance or capacitance in the load.

Power Factor

Power factor is defined as the cosine of the phase angle difference between the voltage and current. It is denoted as PF. If ϕ is the phase angle between the voltage and current, then

$$\text{PF} = \cos\phi$$

where $\phi = |\phi_v - \phi_i|$

$\phi_v =$ phase angle of voltage, V

$\phi_i =$ phase angle of current, I

While calculating the phase difference between V and I, the symbol | | indicates that only the difference in magnitude is taken into consideration by neglecting its polarity. This implies that the calculation of power factor does not depend on whether V leads I or I leads V.

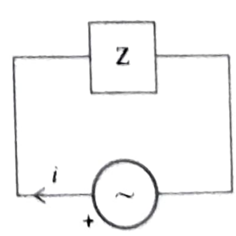


FIG. 1.34 An example AC circuit

The power factor is practically used to obtain the average power or true power P.

$$P = V_{rms} \times I_{rms} \times \cos\phi$$

Derivation of this relationship can be explained with the following steps. Consider the circuit shown in Fig. 1.34. It consists of an AC load impedance and a voltage v with initial phase angle ϕ_v.

$$\text{The applied voltage } v = V_{max} \sin(\omega t + \phi_v)$$

Due to this applied voltage, a current i flows through the load impedance with the phase angle ϕ_i.

Current $\quad i = I_{max} \sin(\omega t + \phi_i)$

Hence, the instantaneous power in the circuit is given by

$$P = vi$$

$$= V_{max} \sin(\omega t + \phi_v) I_{max} \sin(\omega t + \phi_i)$$

$$= V_{max} I_{max} \left[\sin\left(\underbrace{\omega t + \phi_v}_{A} \right) \times \sin\left(\underbrace{\omega t + \phi_i}_{B} \right) \right]$$

Applying the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

$$P = V_{max} I_{max} \left[\frac{\cos(\omega t + \phi_v - \omega t - \phi_i) - \cos(\omega t + \phi_v + \omega t + \phi_i)}{2} \right]$$

$$= \frac{V_{max} I_{max}}{2} [\cos(\phi_v - \phi_i) - \cos(2\omega t + \phi_v + \phi_i)]$$

$$= \left[\underbrace{\frac{V_{max} I_{max}}{2} \cos(\phi_v - \phi_i)}_{\text{time independent term}} \right] - \left[\underbrace{\frac{V_{max} I_{max}}{2} \cos(2\omega t + \phi_v + \phi_i)}_{\text{time dependent term}} \right]$$

As the first term of the above equation is independent of time, it represents the net transfer of energy, or average power P,

$$P = \frac{V_{max} I_{max}}{2} \cos(\phi_v - \phi_i)$$

$$= \frac{V_{max} I_{max}}{2} \cos\phi, \text{ where } \phi = \phi_v - \phi_i$$

Since, $\quad V_{rms} = \frac{V_{max}}{\sqrt{2}}, \; V_{max} = \sqrt{2} V_{rms}$

Similarly, $\quad I_{rms} = \frac{I_{rms}}{\sqrt{2}}$

and $$I_{max} = \sqrt{2} I_{rms}$$

Hence, $$P = \frac{\sqrt{2} I_{rms} \sqrt{2} V_{rms}}{2} \cos\phi$$

$$= \frac{2 V_{rms} I_{rms}}{2} \cos\phi$$

$$= V_{rms} I_{rms} \cos\phi$$

In AC circuits, the average power is dissipated only by ohmic resistances; the elements such as inductors and capacitors do not result in power dissipation.

To illustrate this, let us consider three cases:

Case (i) In case the load impedance consists of resistor alone, then as v and i are in phase,

$$|\phi_v - \phi_i| = 0°$$

Therefore, $$\phi = 0°$$

and $$\cos\phi = \cos 0° = 1$$

Therefore, $$P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms}$$

In terms of resistor R, the V_{rms} can be written as

$$V_{rms} = I_{rms} R$$

Therefore, $$P = I_{rms}^2 R$$

Case (ii) If the load impedance consists of a pure inductive load, then v leads i by $90°$.

Therefore, $$|\phi_v - \phi_i| = 90°$$

or $$\phi = 90°$$

$$\cos\phi = \cos 90° = 0$$

Therefore, $$P = V_{rms} I_{rms} \cos\phi = 0$$

Case (iii) If the load impedance consists of a pure capacitor load, then v lags i by $90°$.

Therefore, $$|\phi_v - \phi_i| = |-90°| = 90°$$

or $$\phi = 90°$$

$$\cos\phi = 0$$

$$P = V_{rms} I_{rms} \cos\phi = 0$$

Example 1.85 A current of repetitive function $i = 10^5 t$ A is applied through a resistor of 10 Ω. Determine the value of power between 0 and 4 ms.

Solution

According to Ohm's law,

$$v = Ri = 10 \times 10^5 t \text{ V}$$

Therefore, the instantaneous power, $p = vi = 10 \times 10^5 \times 10^5 t = 10^{11} t^2$ W.

The average power between a time period is

$$P_{av} = \frac{1}{T}\int_0^T p\,dt$$

$$= \frac{1}{4\times10^{-3}}\int_0^{4\times10^{-3}} 10^{11} t^2 dt$$

$$= \frac{10^{11}}{4\times10^{-3}}\int_0^{4\times10^{-3}} t^2 dt$$

$$= \frac{10^{11}}{4\times10^{-3}}\left(\frac{t^3}{3}\right)_0^{4\times10^{-3}}$$

$$= \frac{10^{11}}{4\times10^{-3}\times3}\left(4\times10^{-3}\right)^3$$

$$= 533.33 \text{ kW}.$$

Example 1.86 The voltage across a 2 Ω resistor is given by

$$v = 10\left[1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right] \text{V}$$

Find the power dissipation, and RMS power.

Solution

Given, $v = 10\left[1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right]$ V and $R = 2\ \Omega$.

Since, $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$ is a cosine series, $v = 10\cos\theta$ V.

According to Ohm's law,

$$i = \frac{v}{R} = \frac{10\cos\theta}{2} = 5\cos\theta \text{ A}$$

Hence, the power dissipation, $p = vi = 10\cos\theta \times 5\cos\theta = \left(50\cos^2\theta\right)$ W

The RMS power is $P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = \frac{V_m I_m}{2} = \frac{50}{2} = 25\text{W}$

Example 1.87 A voltage of $v = 5\sin 314t\,\text{V}$ is applied to an inductance of 4 mH. Find the value of current, instantaneous power, and average power.

Solution

The current flow through the inductance is

$$i = \frac{1}{L}\int v\,dt = \frac{1}{4\times 10^{-3}}\int 5\sin 314t\,dt$$

$$= \frac{5}{4\times 10^{-3}}\left[\frac{-\cos 314t}{314}\right]$$

$$= (-3.981\cos 314t)\,\text{A}$$

The instantaneous power is

$$P = vi = (5\sin 314t)(-3.981\cos 314t)$$

$$= -19.9.4\sin 314t\cos 314t\ \text{W}$$

Apparently, the average power P in an inductance is zero.

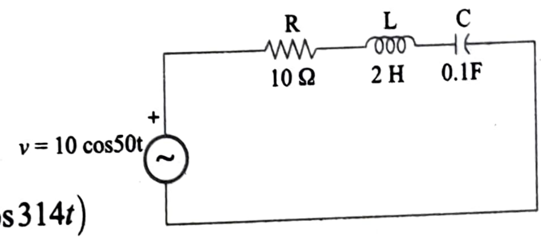


FIG. E1.88

Example 1.88 For the RLC circuit shown in Fig. E1.88, determine the circuit impedance and current i. Also, find the voltage v_L across the inductor, voltage v_C across the capacitor, and the apparent power of the circuit.

Solution

The input voltage in phasor form is

$$\bar{V} = 10\angle 0^\circ\,\text{V}$$

The circuit impedance is

$$\bar{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= 10 + j\left(50\times 2 - \frac{1}{50\times 0.1}\right) = 10 + j99.8\ \Omega$$

$$= \sqrt{10^2 + 99.8^2}\ \tan^{-1}\left(\frac{99.8}{10}\right)$$

$$= 100.29\angle 84.27^\circ\ \Omega$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{10\angle 0^\circ}{100.24\angle 84.27^\circ}$$

$$= 0.1\angle -84.27^\circ\,\text{A}$$

Therefore, the voltage across capacitor, $v_C = 0.1\cos(50t - 84.27^\circ)\,\text{V}$

Hence, the current $i(t)$ leads the voltage $v_C(t)$ by 90°.

Apparent power of the circuit $S = |\bar{V}||\bar{I}| = 10\times 0.1 = 1\,\text{VA}$

Example 1.89 The voltage of a circuit is $v = 200\sin(\omega t + 30°)\text{V}$ and the current $i = 50\sin(\omega t + 60°)\text{A}$. Calculate: (a) the average power, (b) reactive power, (c) apparent power, (d) phasor diagram and power triangle, and (e) the circuit elements if $\omega = 100$ πrad/s.

Solution

Given, $v = 200\sin(\omega t + 30°)\text{V}$, $i = 50\sin(\omega t + 60°)\text{A}$

Therefore, $$V_{max} = \frac{200}{\sqrt{2}} = 141.42\,\text{V}$$

$$I_{max} = \frac{50}{\sqrt{2}} = 35.35\,\text{A},\ \theta = 60° - 30° = 30°$$

(a) The average power, $P = V_{max} I_{max} \cos\theta$

$$= 141.42 \times 35.35 \times \cos(60° - 30°) = 4330\,\text{W}$$

(b) The reactive power, $Q = V_{max} I_{max} \sin\theta$

$$= 141.42 \times 35.35 \times \sin(60° - 30°) = 2500\,\text{VAR}$$

(c) The apparent power, $S = \dfrac{P}{\cos\theta} = \dfrac{4330}{\cos 30°} = 5000\,\text{VA}$

(d) Phasor diagram and power triangle for this circuit can be drawn as shown in Fig. E1.89(a) and Fig. E1.89(b), respectively.

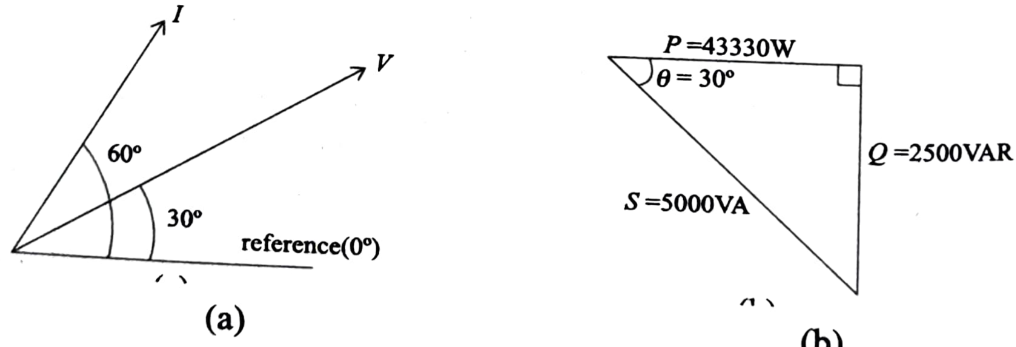


FIG. E1.89

Since the current leads the voltage, the circuit must consist of resistor and capacitor

$$\tan\theta = \frac{1}{\omega RC}$$

Therefore, $$\tan 30° = \frac{1}{100\pi \times RC}$$

$$RC = \frac{1}{100\pi \times \tan 30°} = 0.0055$$

According to Ohm's law,

$$|Z| = \frac{V_{\max}}{I_{\max}} = \sqrt{R^2\left(\frac{1}{\omega C}\right)^2}$$

Since $RC = 0.0055$, $C = \dfrac{0.0055}{R}$

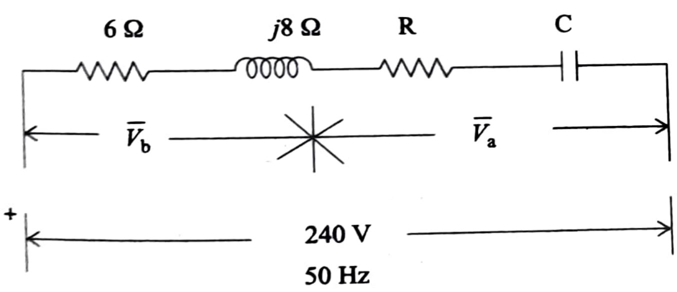
$$|Z| = \frac{200}{50}\sqrt{R^2\left(\frac{1}{\dfrac{\omega \times 0.0055}{R}}\right)^2}$$

$$\frac{200}{50} = \sqrt{R^2\left(\frac{R}{100\pi \times 0.0055}\right)^2}$$

Solving this, we get $R = 2.62\ \Omega$

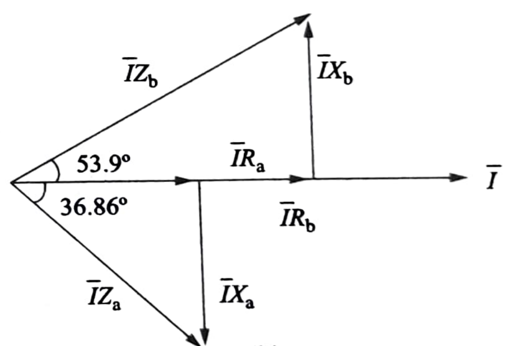
Therefore, $\quad C = \dfrac{0.0055}{2.62} = 2.0992\,\text{mF}$

Example 1.90 Find the values of R and C in the circuit shown in Fig. E1.90(a) so that $|\overline{V}|_b = 4|\overline{V}_a|$ and $\overline{V}_a$ and $\overline{V}_b$ are in phase quadrature.



(a)

FIG. E1.90(a)



(b)

FIG. E1.90(b)

Solution

From the given circuit, according to Ohm's law

$$|\overline{V}_a| = |\overline{I}|\sqrt{R^2 + X_C^2} \text{ and } |\overline{V}_b| = |\overline{I}|\sqrt{6^2 + 8^2} = 10|\overline{I}|$$

$$|Z_a| = \sqrt{R^2 + X_C^2} \text{ and } Z_b = 6 + j8\Omega = 10\angle 53.13°\Omega$$

Given, $\quad |\overline{V}|_b = 4|\overline{V}_a| \quad |\overline{V}_a| = |\overline{I}|\sqrt{R^2 + X_C^2}$

Therefore, $\quad 10|\overline{I}| = 4 \times |\overline{I}|\sqrt{R^2 + X_C^2}$,

$$\sqrt{R^2 + X_C^2} = 2.5 \text{ and } R^2 + X_C^2 = 6.25$$

Example 1.93 A current of 5 A flows through a non-inductive resistance in series w a chocking coil when supplied at 250 V, 50 Hz. If the voltage across the non-inducti resistance is 125 V and that across that coil 200 V, calculate the impedance, reactance a resistance of the coil and power absorbed by the coil.

Solution

The circuit with the given data can be drawn as shown in Fig. E1.93.

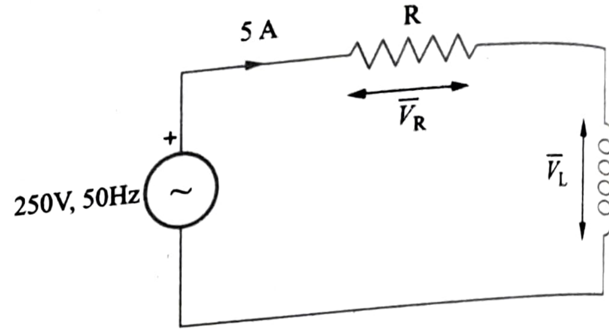


FIG E1.93

Given, the circuit current $\overline{I} = 5\,\text{A}$

Supply voltage $\overline{V}_s = 250\,\text{V}$

$f = 50\,\text{Hz}$

Voltage across resistor $\overline{V}_R = 125\,\text{V}$

Voltage across coil $\overline{V}_L = 200\,\text{V}$

According to Ohm's law,

$$\overline{V}_R = \overline{I}R = 125\,\text{V}$$

$$R = \frac{\overline{V}_R}{I} = \frac{125}{5} = 25\,\Omega$$

Similarly, $\overline{V}_L = \overline{I}X_L = \overline{I}(j\omega L) = 200\,\text{V}$

Since $\overline{I}X_L = 200\,\text{V}$, $X_L = \dfrac{200}{5} = 40\,\Omega$

$$\omega L = 40$$

$$(2\pi f)L = 40$$

$$(2\times\pi\times 50)L = 40$$

$$L = 0.1273\,\text{H}$$

$$Z = 25 + j40\,\Omega = 47.16\angle 57.99°\,\Omega$$

Power absorbed by the coil $= \dfrac{1}{2}LI^2$

$$= \frac{1}{2}\times 0.1273\times 25 = 1.59\,\text{W}$$

Example 1.94 In the circuit shown in Fig. E1.94(a), when 220 V_{AC} is applied across and B, the current drawn is 20A and the power input is 3000 W. Find the impedance Z o this circuit.

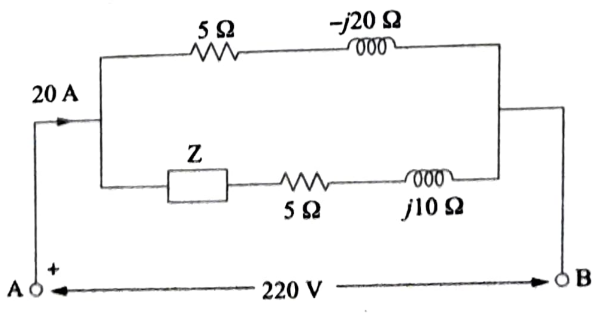
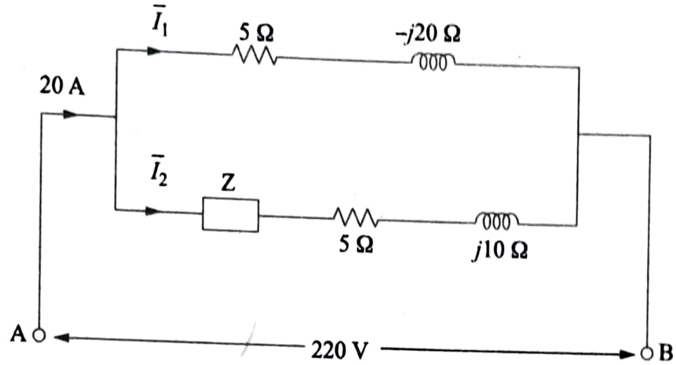


FIG. E1.94(a)

Solution

As shown in Fig. E1.94(b), let the branch currents be $\bar{I}_1$ and $\bar{I}_2$.



(b)

FIG. E1.94(b)

According to Ohm's law, the branch current is

$$\bar{I}_1 = \frac{220}{5+j20} = 2.588 - j10.352\,\text{A}$$

According to KCL, the total current $\bar{I}_T = \bar{I}_1 + \bar{I}_2$

$$\bar{I}_1 + \bar{I}_2 = 20\,\text{A}$$

$$\bar{I}_2 = 20 - \bar{I}_1$$

$$\bar{I}_2 = 20 - \frac{200}{5+j20}$$

And according to Ohm's law,

$$\bar{I}_2 = \frac{220}{Z+5+j10}$$

Solving the two equations for $\bar{I}_2$, we get

$$20 - \frac{220}{5+j20} = \frac{220}{Z+5+j10}$$

$$Z = 4.33 - j15.55\Omega = 16.14\angle -74.424°\Omega$$

Example 1.95 The efficiency of a 1/5 HP induction motor is 75%. If the operating power factor is 0.5 lagging, calculate the reactive power consumed.

Solution

From the given data, $\frac{1}{5}\text{HP} = \frac{746}{5} = 149.2\,\text{W}$

Efficiency, $\eta = \frac{P_{out}}{P_{in}} = 75\%$, where P_{out} and P_{in} are output and input powers, respectively.