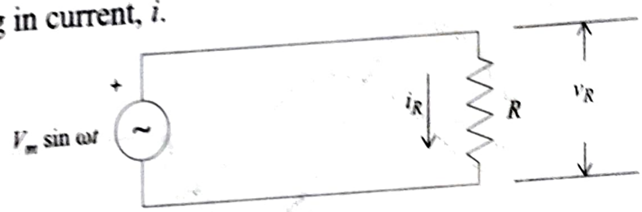
Impedance

In a network, impedance is the measure of opposition to the flow of current or applied voltage. It is the extension of the concept of resistance to AC circuits. However, unlike resistance, which has only magnitude, the impedance possesses both magnitude and phase. When a DC current is supplied the impedance cannot be distinguished from the resistance. Therefore, with a DC current, the resistance can be treated as impedance with zero phase angle.

Impedance of a pure resistive circuit

Consider the circuit shown in Fig. 1.23(a) consisting of a resistor, R, connected across an alternating voltage source. Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across the resistance, resulting in current, i.



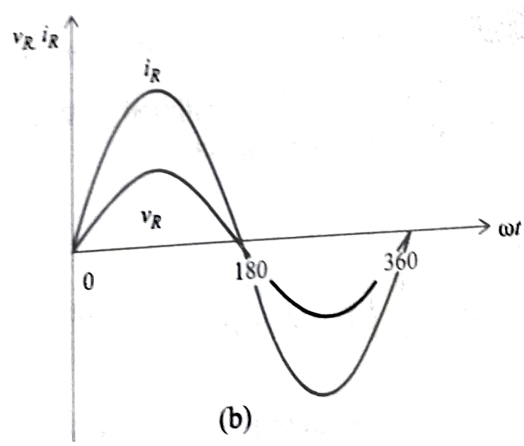


FIG. 1.23 (a) A pure resistor circuit, (b) v and i waveforms

According to Ohm's law, $v = iR$, and

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where $I_m = \dfrac{V_m}{R}$ represents the peak value of the circuit current.

From this relationship it can be inferred that the current in the resistor is in phase with the voltage as shown in Fig. 1.23(b).

As impedance in an AC circuit is the ratio of the voltage to current, we have

$$Z = \frac{v}{i} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m \sin \omega t}{(V_m / R) \sin \omega t} = R$$

Impedance of a pure inductive circuit

Consider the circuit shown in Fig. 1.24(a) which consists of a pure inductor of s inductance L henry (L H) and is connected across an alternating voltage source.

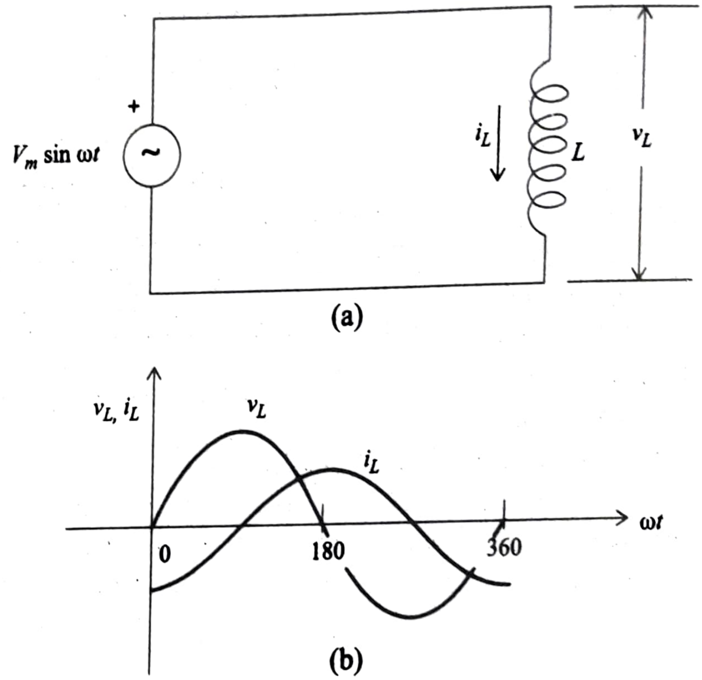


FIG. 1.24 (a) A pure inductor circuit, (b) v and i waveforms

Let $v = V_m \sin \omega t$ is the sinusoidal voltage applied across the inductor, resulting in a current i. The emf induced in the inductor is given by

$$v_L = L\frac{di}{dt}$$

Therefore, $$i_L = \frac{1}{L}\int v dt = \frac{1}{L}\int V_m \sin \omega t dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\cos \omega t = I_m \sin(\omega t - \pi/2)\text{, where } I_m = \frac{V_m}{\omega L}.$$

From this relationship, it can be inferred that the current in the inductor lags the voltage by an angle $\pi/2$ as shown in Fig. 1.24(b).

The impedance, $Z = \frac{V_m}{I_m} = \frac{V_m}{(V_m/\omega L)} = \omega L$

The term $Z = \omega L$ denotes the inductive reactance (X_L).

Impedance of a pure capacitive circuit

Consider the circuit shown in Fig. 1.25(a) as consisting of a pure capacitor of value C farad (C F) and connected across an alternating voltage source.

Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across the capacitor, resulting in a current i. The voltage across the capacitor is given by

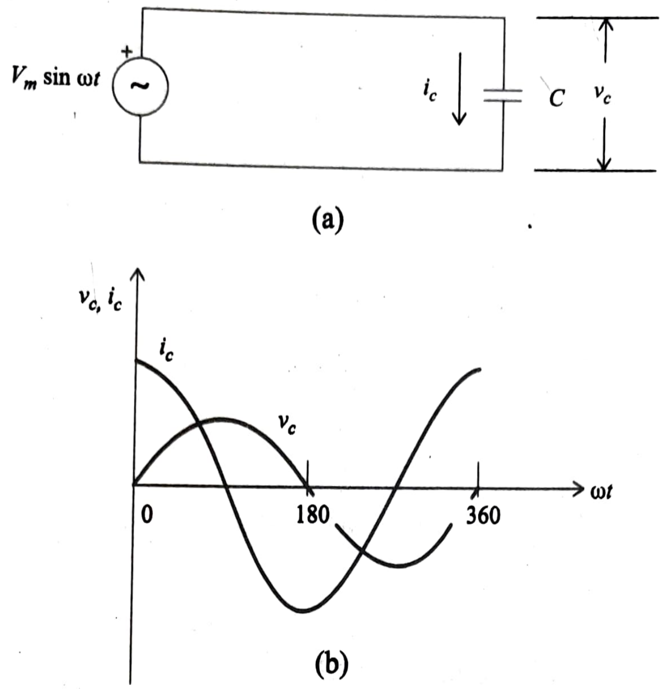


FIG. 1.25 (a) A pure capacitor circuit, (b) *v* and *i* waveforms

$$v_c = \frac{1}{C}\int idt$$

Therefore,
$$i_c = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t)$$
$$= \omega C V_m \cos \omega t = I_m \cos \omega t, \text{ where } I_m = \omega C V_m$$
$$i_c = \sin(\omega t + \pi/2)$$

From this relationship it can be inferred that the current in the capacitor leads the voltage by an angle $\pi/2$ as shown in Fig. 1.25(b).

The impedance $Z = \dfrac{V_m}{I_m} = \dfrac{V_m}{(\omega C V_m)} = \dfrac{1}{\omega C} = X_C$

The term $Z = \dfrac{1}{\omega C}$ denotes the capacitive reactance (X_C).

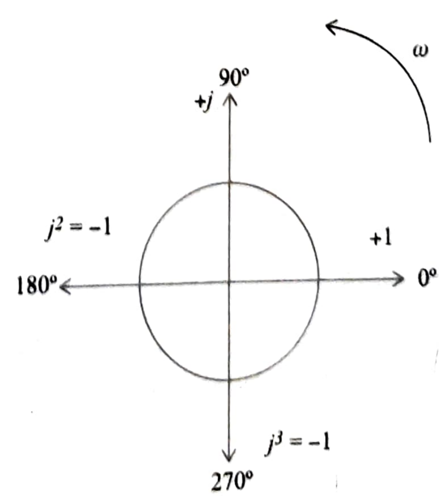


FIG. 1.26 Rotation of a quantity using the *j* operator

The *j* Operator

As the alternating current and voltage have both magnitude and phase, it is called a phasor or complex quantity. According to Ohm's law, the resultant impedance is also a complex quantity. Hence, the analysis of the AC circuit requires the knowledge of complex variables involving the *j* operator. The symbol *j* represents the imaginary part of a complex quantity; it is assigned with a value of $\sqrt{-1}$ or a phase angle

$3.935 + j0.715$ (Rectangular form)

To convert the rectangular form back to the polar form,

$$c = \sqrt{a^2 + b^2}$$

$$\sqrt{3.935^2 + 0.715^2} = 4 \quad \text{(Polar magnitude)}$$

$$\arctan \frac{0.715}{3.935} = 10.3^\circ \quad \text{(Polar angle)}$$

$$4\angle 10.3^\circ \quad \text{(Polar form)}$$

Complex Arithmetic

To add or subtract two phasors we must convert them into rectangular form. Similarly, to perform multiplication or division of two phasors we should first convert them to polar form to make things simpler.

Addition

$$C_1 = A_1 + jB_1$$
$$C_2 = A_2 + jB_2$$
$$C_1 + C_2 = (A_1 + A_2) + j(B_1 + B_2)$$

Subtraction

$$C_1 = A_1 + jB_1$$
$$C_2 = A_2 + jB_2$$
$$C_1 - C_2 = (A_1 - A_2) + j(B_1 - B_2)$$

Multiplication

The product in polar form is simply the product of their magnitudes, and the phase is the sum of their phases.

$$C_1 = |M_1|\angle\phi_1$$
$$C_2 = |M_2|\angle\phi_2$$
$$C_1 \times C_2 = |M_1 \times M_2|\angle(\phi_1 + \phi_2)$$

Division

The division in polar form is simply the division of the magnitudes, and the phase is the subtraction of the phases.

$$C_1 = |M_1|\angle\phi_1$$
$$C_2 = |M_2|\angle\phi_2$$
$$\frac{C_1}{C_2} = \left|\frac{M_1}{M_2}\right|\angle(\phi_1 - \phi_2)$$