$$20\times10^{-3}+5\times10^{-3}=0.3V_1+V_1\times0.5+V_1\times2$$

$$2.8V_1=25\times10^{-3}$$

Therefore, $V_1 = 8.929$ mV

1.9 FUNDAMENTALS OF AC CIRCUITS

AC Quantity

Unlike DC quantity which has a constant magnitude all the time, an AC quantity has a varying magnitude and angle with respect to time.

Waveform

Waveform is a graphical representation of instantaneous value of any quantity plotted against time as shown in Fig. 1.20.

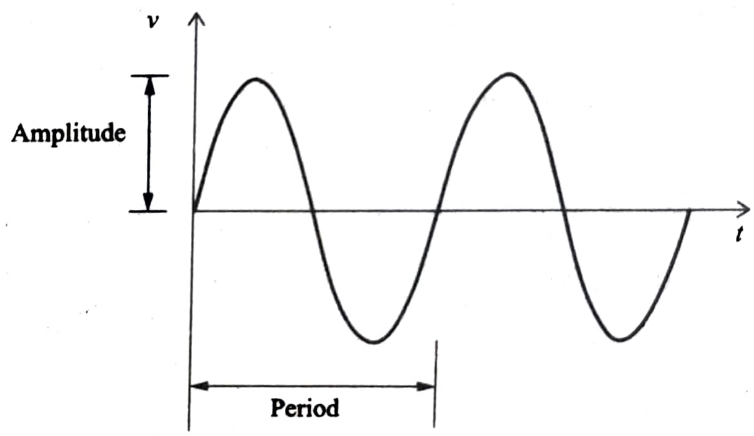

FIG. 1.20 A waveform

Alternating Current

The current wave reversing its direction at regularly repeating intervals is called alternating current.

Cycle

One complete cycle consists of a set of positive and negative halves.

Amplitude

The maximum positive or negative value of an alternating quantity is called amplitude or magnitude.

Frequency

The number of cycles per second of an alternating quantity is known as frequency (f) whose unit is *cycles/second*, or *hertz* (Hz).

Period (T)

The time period of an alternating quantity is the time taken to complete one cycle. Time period is equal to the reciprocal of frequency, whose unit is seconds (s).

Phase

The phase denotes a particular point in the cycle of a waveform, measured as an angle in *degrees*, as shown in Fig. 1.21.

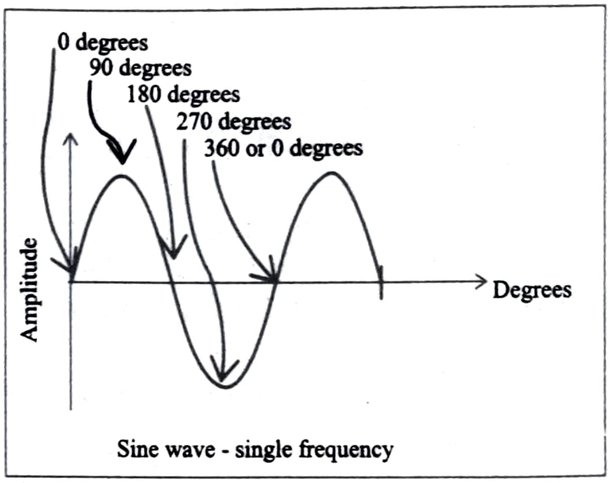

FIG. 1.21 Phase measurement in a single waveform

Phase Difference

The term phase difference is used to compare the phases of two waveforms or alternating quantities.

Phase difference between two sinusoidal waveforms that have the same frequency is illustrated in Fig. 1.22. The phase angle can be considered as a measure of the time delay between two periodic signals expressed as a fraction of the wave period. This fraction is normally expressed in units of angle, with a full cycle corresponding to 360°. For example, inspecting the waveforms shown in Fig. 1.22 reveals that since the voltage v_1 passes through zero cycle before v_2, the former leads the later by a phase difference of $\theta = \frac{\pi}{4} = \frac{360°}{8} = 45°$.

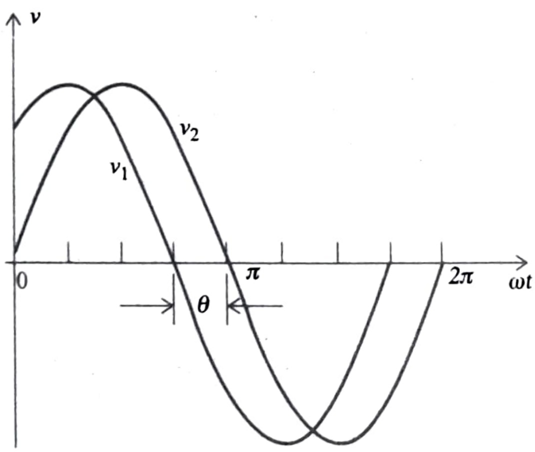

FIG. 1.22. Phase difference of two waveforms

If the alternating voltage, v_2 is taken as a reference waveform, it can be expressed mathematically as

$$v_2 = V_m \sin \omega t$$

where V_m is the magnitude of the waveform, and $\omega = (2\pi f)$ is the wavelength.

As v_1 leads v_2 by θ, it can be expressed as

$$v_1 = V_m \sin(\omega t + \theta)$$

Effective, or RMS, Value

When the RMS value of an AC and the steady value of the DC flow through a given circuit for equal time they produce the equal amount of heat.

RMS value of a wave can be obtained by the formula,

$$\text{RMS value} = \sqrt{\frac{\text{Area under the square curve for one cycle}}{\text{Time Period}}}$$

RMS value of the alternating sinusoidal current is

$$I_{\text{RMS}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, RMS value of an AC voltage is

$$V_{\text{RMS}} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

Average Value

The average value of an AC is given by that steady current which transfers across a circuit the same charge as would be transferred by the DC across the same circuit in the same time. This can be obtained simply by first obtaining the average value for a small interval of time and then integrating it over the curve.

$$I_{av} = \frac{1}{T}\int_0^T i\,dt$$

For an alternating sinusoidal current

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m,$$ where I_m is the maximum value of the current.

For an alternating sinusoidal voltage

$$V_{av} = \frac{2V_m}{\pi} = 0.637 V_m,$$ where V_m is the maximum value of the voltage.

Form Factor and Peak Factor

Form factor (K_f) is defined as the ratio of RMS value to the average value.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

Impedance

In a network, impedance is the measure of opposition to the flow of current or applied voltage. It is the extension of the concept of resistance to AC circuits. However, unlike resistance, which has only magnitude, the impedance possesses both magnitude and phase. When a DC current is supplied the impedance cannot be distinguished from the resistance. Therefore, with a DC current, the resistance can be treated as impedance with zero phase angle.

Impedance of a pure resistive circuit

Consider the circuit shown in Fig. 1.23(a) consisting of a resistor, R, connected across an alternating voltage source. Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across the resistance, resulting in current, i.

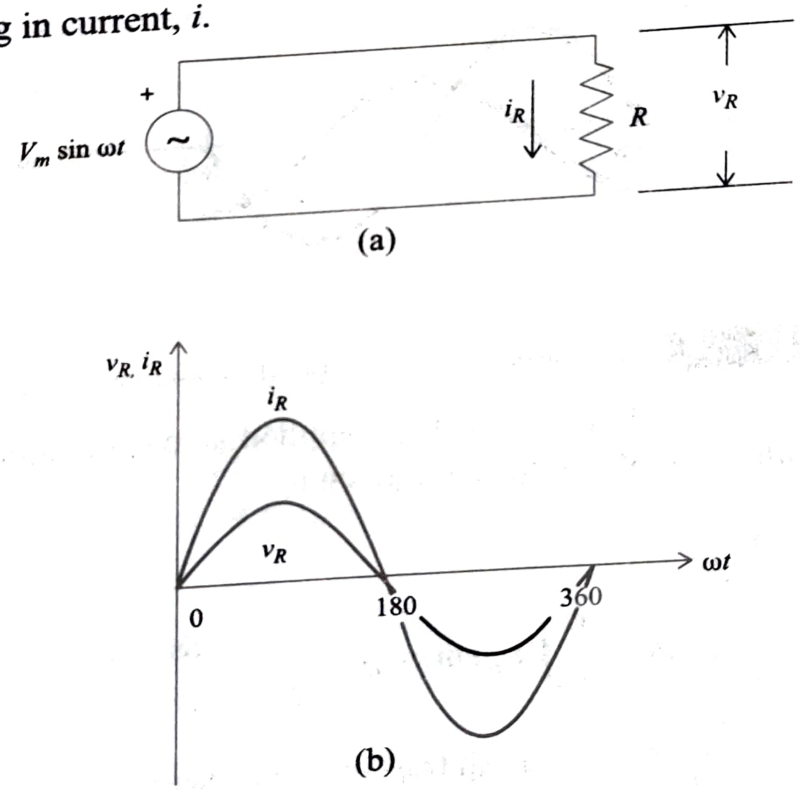

FIG. 1.23 (a) A pure resistor circuit, (b) *v* and *i* waveforms

According to Ohm's law, $v = iR$, and

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where $I_m = \dfrac{V_m}{R}$ represents the peak value of the circuit current.

From this relationship it can be inferred that the current in the resistor is in phase with the voltage as shown in Fig. 1.23(b).

As impedance in an AC circuit is the ratio of the voltage to current, we have

$$Z = \frac{v}{i} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m \sin \omega t}{(V_m / R) \sin \omega t} = R$$