Therefore, $Z_S = (5 + j5) + \dfrac{14.1421\angle 45^\circ \times 10.198\angle -11.3^\circ}{10 + j10 + 10 - j2}$

$$= (5 + j5) + \frac{144.2211\angle 33.7^\circ}{21.5406\angle 21.8^\circ}$$

$$= (5 + j5) + 6.6953\angle 11.9^\circ \Omega = (5 + j5) + 6.5514 + j\,1.3806$$

$$= 11.5514 + j\,6.3806\Omega$$

Hence, for maximum power transfer

$$Z_L = Z_S^* = 11.5514 - j\,6.3806\Omega.$$

2.10 RECIPROCITY THEOREM

The reciprocity theorem states that "*In any linear bilateral network, the ratio of a voltage V introduced in one mesh to the current I in any second mesh is the same as the ratio obtained if the positions of V and I are interchanged, other voltages are being removed*".

Reciprocity theorem is valid for a bilateral network, in which the analysis results obtained for transmission in one direction remain the same even if the transmission direction is reversed. This theorem is applicable to the circuit consisting of only bilateral elements such as resistors, inductors and capacitors, and is not applicable to those containing non-bilateral elements such as diodes, transistors, relays or other control devices.

In many electrical networks, it is found that if positions of the voltage source and response (ammeter) are interchanged, the reading of ammeter remains the same as shown in Fig. 2.13.

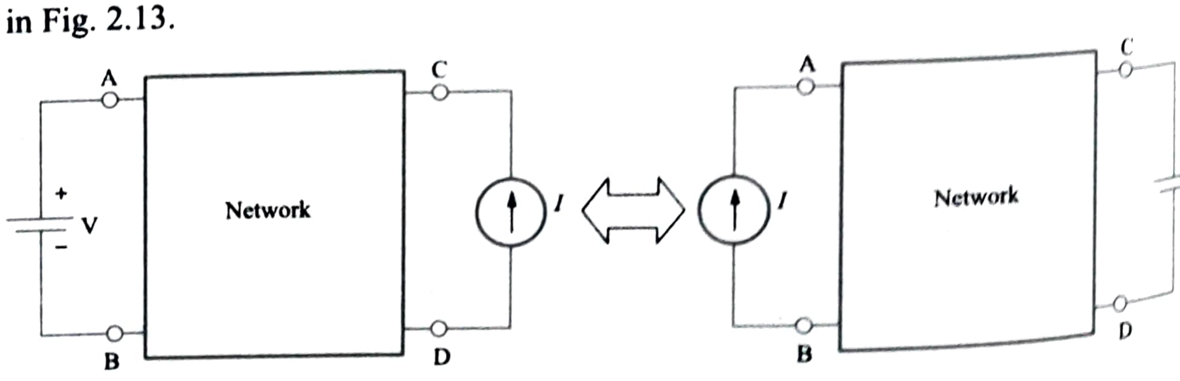

FIG. 2.13 Interchanging voltage source and response

Consider the example shown in Fig. 2.14(a) and (b). If a voltage source V_s in one branch produces a current I in another branch, then if that voltage source V_s is moved from the first branch to the second branch, it will cause the same current I in the first branch, where V_s has been replaced by a short circuit.

According to the reciprocity theorem $I' = I''$.

The circuit illustrated above is the example of reciprocity theorem. Solving these circuits in two cases will result in the current I' and I'' which will be equal in magnitude.

Similarly, in AC circuits with resistances replaced by impedances, and DC sources replaced by AC sources, the above stated relationship is valid.

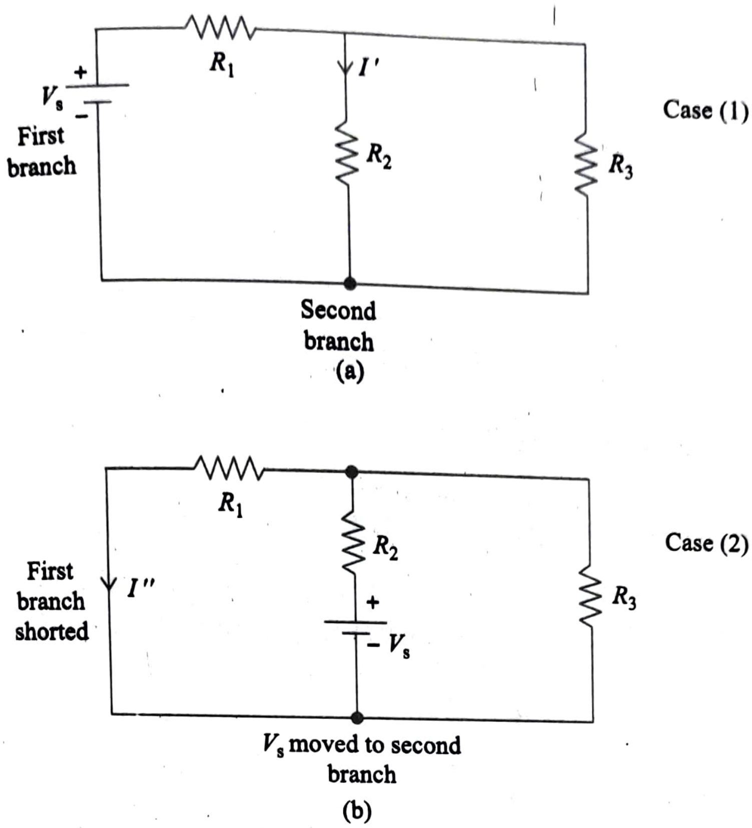

FIG. 2.14 (a) and (b)

Limitations of the Reciprocity Theorem

1. It is not applicable to circuits having more than one source.
2. It is not applicable to non-linear circuits.
3. It is not valid if the network consists of any time-varying elements.

Example 2.60 Verify reciprocity theorem for the circuit shown in Fig. E2.60(a).

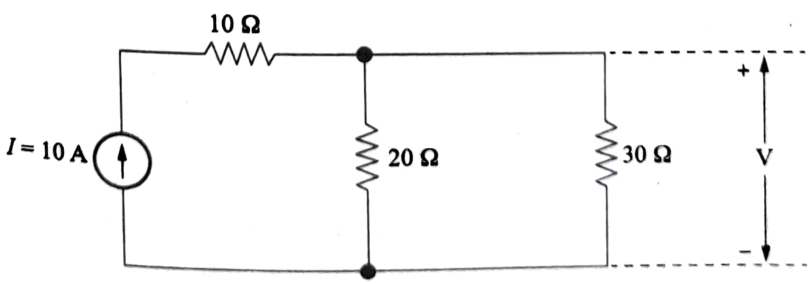

FIG. E2.60(a)

Solution

Assume that the current through 3Ω is I_1 as shown in Fig. E2.60(b).

Applying the current division rule, we get

$$I_1 = I \times \frac{20}{20+30} = \frac{10 \times 20}{50} = 4 \text{ A}$$

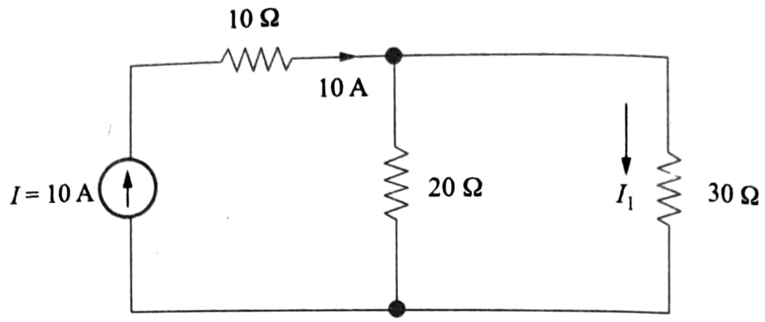

FIG. E2.60(b)

Therefore, $V = I_1 \times 30 = 4 \times 30 = 120\,\text{V}$

Hence, the voltage current ratio, $\dfrac{V}{I} = \dfrac{120}{10} = 12\Omega$

The circuit is redrawn after interchanging the positions of the voltage V and curre source I as shown in Fig. E2.60(c). Assume that the current through 20Ω is I_2.

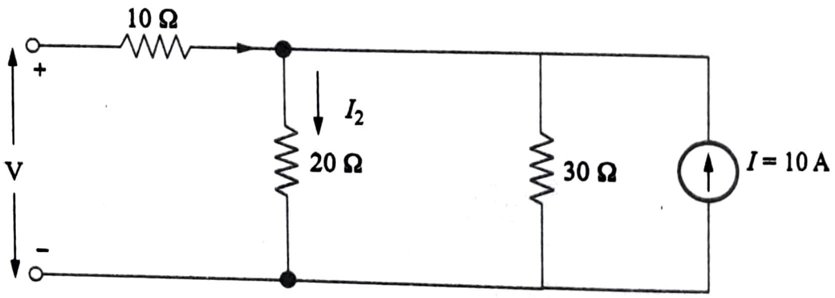

FIG. E2.60(c)

Applying the current division rule, we get

$$I_2 = I \times \frac{30}{20+30} = \frac{10 \times 30}{50} = 6\ \text{A}$$

Therefore, $V = V_{20\Omega} = 20 \times I_2 = 20 \times 6 = 120\,\text{V}$

Hence, the voltage current ratio, $\dfrac{V}{I} = \dfrac{120}{10} = 12\Omega$

Reciprocity theorem is verified from Eqn. (1) and (2) as the voltage–current rati remains same even though their positions are interchanged.

Example 2.61 Verify reciprocity theorem for the network shown in Fig. E2.61(a).

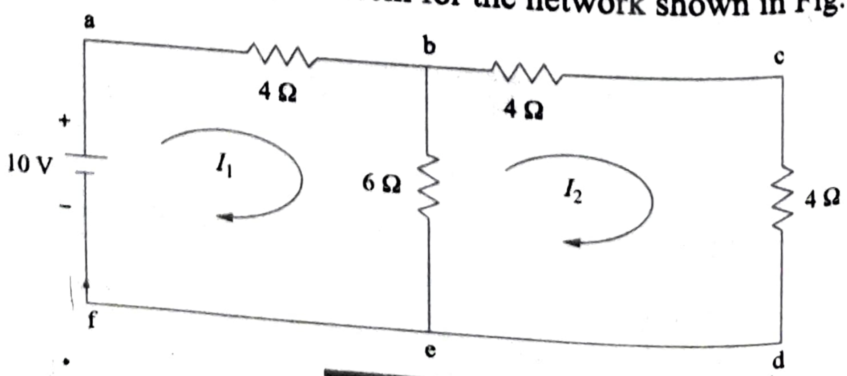

FIG. E2.61(a)

Solution

Applying KVL to the loop 'abefa', we get

$$4I_1 + 6(I_1 - I_2) = 10$$

$$10I_1 - 6I_2 = 10$$

Applying KVL to the loop 'bcdeb', we get

$$4I_2 + 4I_2 + 6(I_2 - I_1) = 0$$

$$-6I_1 + 14I_2 = 0$$

Applying Cramer's rule, we have

$$\Delta = \begin{vmatrix} 10 & -6 \\ -6 & 14 \end{vmatrix} = 140 - 36 = 104$$

$$\Delta_2 = \begin{vmatrix} 10 & 10 \\ -6 & 0 \end{vmatrix} = 60$$

Therefore, $I = I_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{60}{104} = 0.577\text{A}$

Hence, the voltage–current ratio

$$\frac{V}{I} = \frac{10}{0.577} = 17.33\Omega \qquad (1)$$

The circuit is redrawn after changing the position of the voltage source to V′ = 10 V as shown in Fig. E2.62(b).

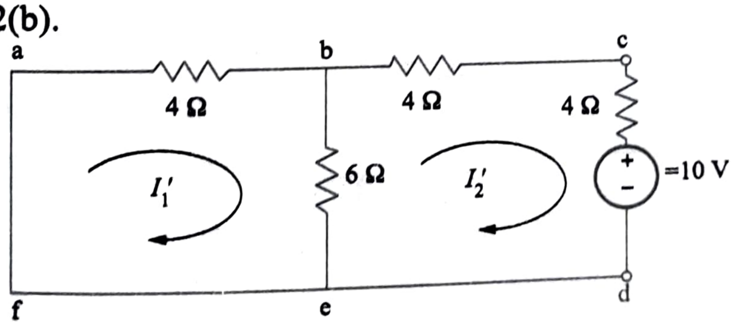

FIG. E2.62(b)

Applying KVL to the loop 'abefa', we get

$$4I_1' + 6(I_1' - I_2') = 0$$

$$10I_1' - 6I_2' = 0$$

Applying KVL to the loop 'bcdeb', we get

$$4I_2' + 4I_2' + 10 + 6(I_2' - I_1') = 0$$

$$-6I_1' + 14I_2' = -10$$

Applying Cramer's rule, we have

$$\Delta' = \begin{vmatrix} 10 & -6 \\ -6 & 14 \end{vmatrix} = 140 - 36 = 104$$

$$\Delta_1' = \begin{vmatrix} 0 & -6 \\ -10 & 14 \end{vmatrix} = -60$$

Therefore, $I' = -I_1' = -\frac{\Delta_1'}{\Delta'} = \frac{60}{104} = 0.577\text{ A}$

Hence, the voltage–current ratio,

$$\frac{V'}{I'} = \frac{10}{0.577} = 17.33\Omega \quad (2)$$

Reciprocity theorem is verified from Eqn. (1) and (2) as the voltage–current ratio remains same even though their positions are interchanged.

Example 2.63 Verify the reciprocity theorem for the circuit shown in Fig. E2.63(a).

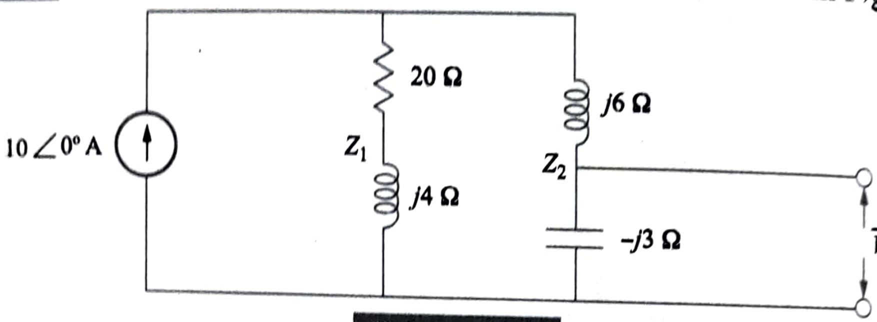

FIG. E2.63(a)

Solution

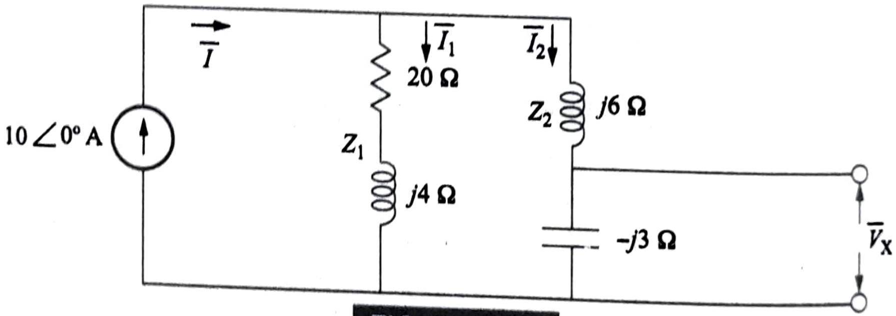

FIG. E2.63(b)

From the circuit shown in Fig. E2.63(b)

$$Z_1 = 20 + j4\Omega$$

$$Z_2 = j6 - j3 = j3\Omega$$

and the current through them are I_1 and I_2, respectively.

Applying the current division rule, we get

The current flowing through the capacitor

$$\bar{I}_2 = \bar{I}\frac{Z_1}{Z_1 + Z_2} = \frac{10\angle 0°(20 + j4)}{20 + j4 + j3} = \frac{10\angle 0°(20 + j4)}{20 + j7}$$

$$= \frac{10\angle 0°(20.396\angle 11.3°)}{21.19\angle 19.29°} = 9.62\angle -7.98°\text{A}$$

Therefore, the voltage across the capacitor

$$\bar{V}_x = \bar{I}_2(-j3)$$

$$= 9.62\angle -7.98(-j3) = 9.62\angle -7.98(3\angle -90°) = 28.86\angle -97.98°\text{V} \quad (1)$$

Removing the current source $10\angle 0°$ A and placing it parallel to $-j3\Omega$ capacitor, the circuit is drawn as shown in Fig. E2.63(c).