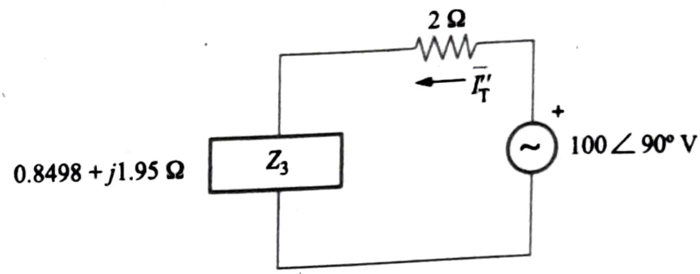

FIG. E2.53(i)

The circuit is further reduced as shown in Fig. E2.53(h).

Therefore, $$Z_3 = \frac{4.9964\angle 20.185^\circ \times 2\angle -90^\circ}{4.6896 + j1.724 - j2}$$

$$= \frac{9.9928\angle -69.815^\circ}{4.6977\angle 3.366^\circ} = 2.127\angle -66.45^\circ\Omega = 0.8498 - j1.95\Omega$$

From the circuit shown in Fig. E2.53(i)

$$\bar{I}_T'' = \frac{100\angle 90^\circ}{2 + 0.8498 - j1.95}$$

$$= \frac{100\angle 90^\circ}{3.4531\angle -34.36^\circ} = 28.959\angle 124.38^\circ\text{A}$$

Applying current division rule to the circuit shown in Fig. E2.53(h), we get

$$\bar{I}'' = \bar{I}_T'' \times \frac{-j2}{4.6896 + j1.7241 - j2} = \frac{28.959\angle 124.38^\circ \times 2\angle -90^\circ}{4.6977\angle -3.366^\circ}$$

$$= 12.329\angle 37.746^\circ\text{A} = 9.7489 + j7.5473\,\text{A}$$

According to the superposition theorem, $\bar{I} = \bar{I}' - \bar{I}''$ since $\bar{I}'$ and $\bar{I}''$ are in opposite directions, we have

$$\bar{I} = 3.144 + j5.66 - 9.7489 - j7.5473$$

$$= -(6.6049 + j1.8873)\text{A} = 6.8692\angle -164.053^\circ\text{A}.$$

2.9 MAXIMUM POWER TRANSFER THEOREM

The maximum power transfer theorem states that "*Maximum power will be transferred to the load impedance Z_L by a network, if the impedance of Z_L is the conjugate of the network impedance Z_S as viewed from the output terminals*". This theorem is also known as "Jacobi's law".

To explain the maximum power transfer theorem, consider the circuit shown in Fig. 2.12.

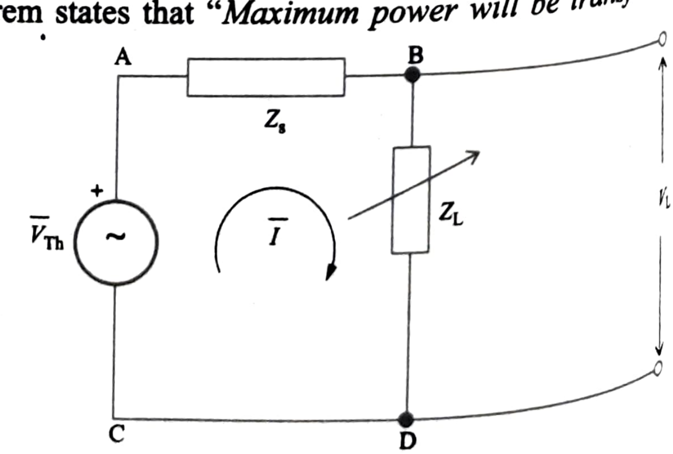

FIG. 2.12 An example circuit

This circuit has variable impedance Z_L connected to its output terminal BD

$$Z_L = R_L + jX_L$$

The impedance of the network as seen from the output terminal, with load impedance removed is

$$Z_S = Z_{Th} = R_S + jX_S$$

The current flow I in the circuit can be calculated as

$$\bar{I} = \frac{\bar{V}_{Th}}{Z_S + Z_L}$$

$$= \frac{\bar{V}_{Th}}{R_S + jX_S + R_L + jX_L}$$

The power delivered to the load can then be calculated as

$$P = |\bar{I}|^2 Z_L = \frac{|\bar{V}_{Th}|^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

The maximum power is transferred from the source to the load when the derivation of the power with respect to the load impedance is zero.

i.e., $$\frac{dP}{dZ_L} = 0$$

or $$\frac{\partial P}{\partial X_L} = 0 \quad \text{and} \quad \frac{\partial P}{\partial R_L} = 0$$

$$\frac{\partial P}{\partial X_L} = \frac{-2|\bar{V}_{Th}|^2 R_L (X_S + X_L)}{[(R_S + R_L)^2 + (X_S + X_L)^2]^2} = 0$$

Therefore, $X_S + X_L = 0$

or $$X_S = -X_L$$

Hence, maximum power is delivered to the load when the reactance of the load is equal and opposite to the reactance of the source. Substituting this condition in the equation of power, we get

$$P = \frac{|\bar{V}_{Th}|^2 R_L}{(R_S + R_L)^2}$$

Partial derivation of the above equation with respect to R_L, and equating it to zero leads to

$$\frac{\partial P}{\partial R_L} = \frac{|\bar{V}_{Th}|^2 (R_S + R_L)^2 - 2|\bar{V}_{Th}|^2 R_L (R_S + R_L)}{(R_S + R_L)^4} = 0$$

or

$$|\bar{V}_{Th}|^2 (R_s + R_L) - 2|\bar{V}_{Th}|^2 R_L = 0$$

Therefore, $R_S = R_L$

Hence, maximum power will be transferred to the load impedance Z_L by a network, if the impedance of Z_L is the conjugate of the network impedance Z_S.

i.e., when $Z_L = Z_S^*$, i.e., $R_L + jX_L = R_S - jX_S$

Therefore, the net equivalent impedance of the circuit becomes

$$R_L + jX_L + R_L - jX_L = 2R_L$$

Power delivered to the load, $P = |\bar{I}|^2 R \text{(or)} \frac{|\bar{V}|^2}{R}$

Substituting $R = 2R_L$ and $|\bar{I}| = \frac{|\bar{V}_{Th}|}{2R_L}$ in the above equation, we get

The maximum power transferred to the load $P_{max} = \frac{|\bar{V}_{Th}|^2}{4R_L}$

Limitations of the Maximum Power Transfer Theorem

1. It is defined only to help in selecting an optimal load for maximizing the power trans
2. It is not helpful if the load resistance is already specified.

Example 2.54 Find the value of R_L at which maximum power is transferred and al calculate the value of maximum power transferred in the circuit shown in Fig. E2.54(a).

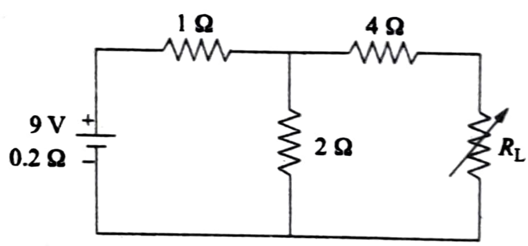

FIG. E2.54(a)

Solution

Given: Internal resistance of voltage source $= 0.2\Omega$.
The maximum power transfer occurs when $R_L = R_S$. To calculate R_S replacing the voltag source by a short circuit and removing R_L the given circuit is redrawn as shown Fig. E2.54(b).

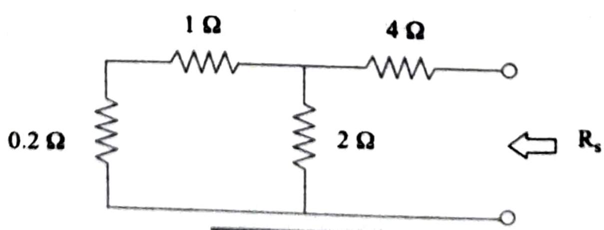

FIG. E2.54(b)

Therefore, $$R_S = \frac{1.2 \times 2}{1.2 + 2} + 4 = 4.75\Omega$$

To find the maximum power P_{max} P_{max}, find the open circuit voltage, i.e., V_{Th} after open–circuiting R_L as shown in Fig. E2.54(c).

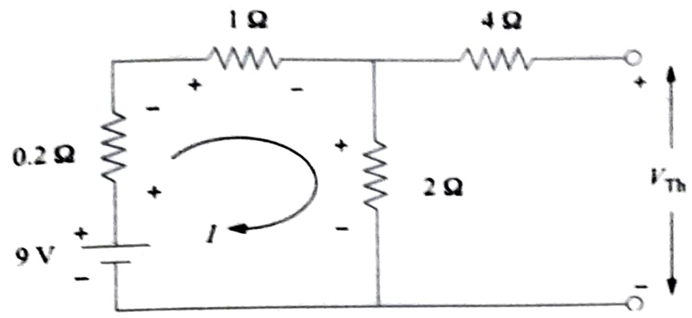

FIG. E2.54(c)

Applying KVL to the circuit shown in Fig. E2.54(c), we get

$$1I + 2I + 0.2I = 9$$

or

$$I = \frac{9}{3.2} = 2.8125 \text{ A}$$

Therefore,

$$V_{Th} = 2 \times I = 5.625 \text{ V}$$

Therefore,

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{(5.625)^2}{4 \times 4.75} = 1.665\text{W}$$

Example 2.55 Determine the load resistance that will receive maximum power from the source in the network shown in Fig. E2.55(a). Also find maximum power delivered to the load.

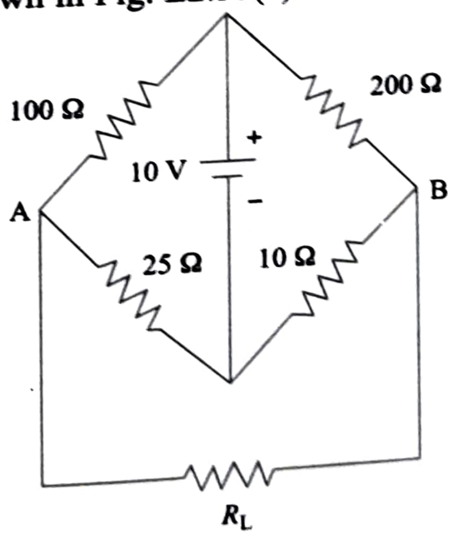

FIG. E2.55(a)

Solution

The maximum power transfer occurs when $R_L = R_S$. To calculate R_s replace the voltage source by a short circuit and remove R_L. Then the given circuit is redrawn as shown in Fig. E2.55(b).

Therefore,

$$R_S = \frac{100 \times 25}{100 + 25} + \frac{200 \times 10}{200 + 10} = 20 + 9.524 = 29.524\Omega$$

Hence,

$$R_L = R_S = 29.524\Omega$$

To find the value of maximum power P_{max}, calculate V_{Th} by open-circuiting R_L as shown in Fig. E2.55(c).

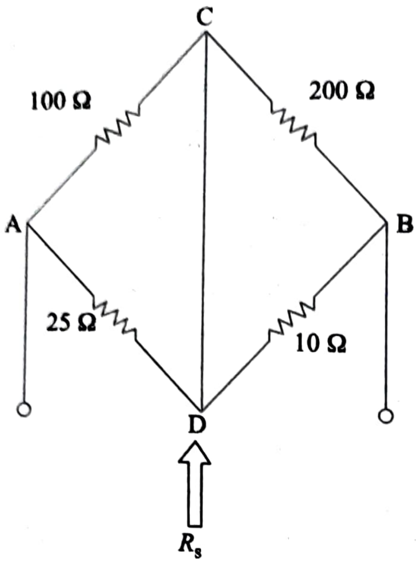

FIG. E2.55(b)

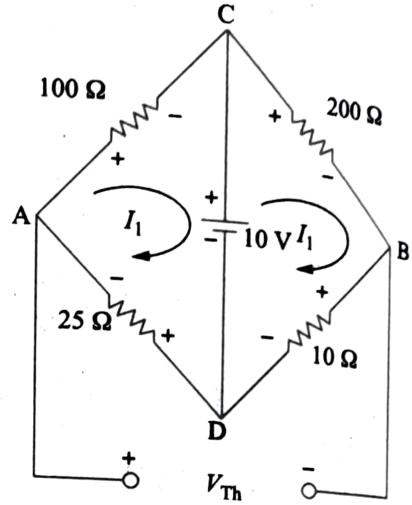

FIG. E2.55(c)

Applying KVL to the loop ACDA, we get

$$100I_1 + 25I_1 = -10$$

$$I_1 = -0.08 \text{ A}$$

Applying KVL to the loop CBDC, we get

$$200I_2 + 10I_2 = 10$$

$$I_2 = 0.0476 \text{ A}$$

Therefore, $$V_{Th} = V_{AB} = V_{AD} + V_{DB}$$

$$= 25 \times 0.08 + 10 \times -0.0476 = 1.524 \text{ V}$$

Therefore, $$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{(1.524)^2}{4 \times 29.524} = 19.679 \text{ mW}$$

Example 2.56 Find the value of maximum power transferred at the terminals 'ab' in the circuit shown in Fig. E2.56(a).

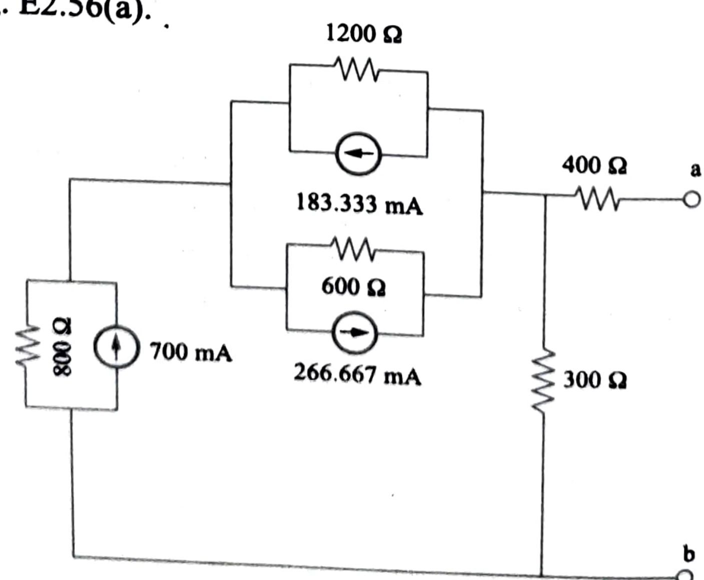

FIG. E2.56(a)

Solution

(a) To find R_N:

Open-circuiting the current sources, the circuit is drawn as shown in Fig. E2.56(b).

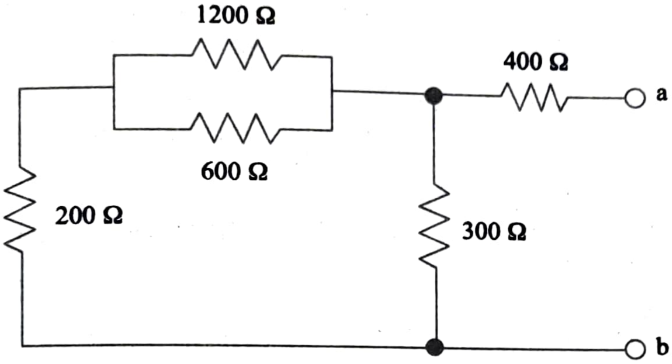

FIG. E2.56(b)

$$R_N = \left[\left(200 + \frac{1200 \times 600}{1200 + 600}\right) \| 300\right] + 400$$

$$= \frac{600 \times 300}{600 + 300} + 400$$

$$R_N = 600\ \Omega$$

For maximum power to be transferred, $R_L = 600\ \Omega$

(b) To find I_N:

The load terminals 'ab' are shorted and using source transformation, the current sources in the given circuit are converted into voltage sources as shown in Fig. E2.56(c).

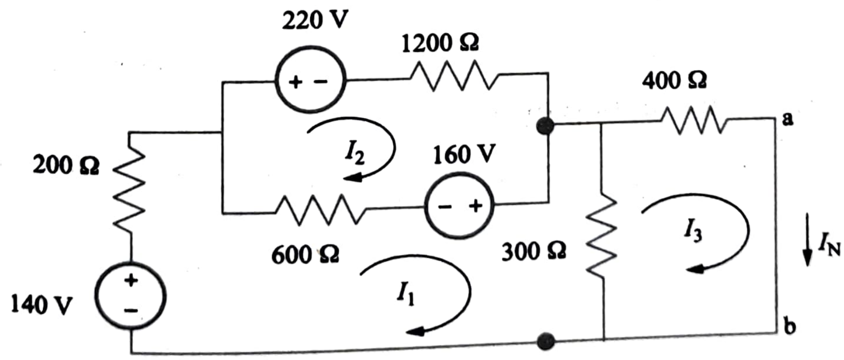

FIG. E2.56(c)

Applying KVL to the circuit, we get

$$1100I_1 - 600I_2 - 300I_3 = 300 \quad (1)$$

$$-600I_1 + 1800I_2 = -380 \quad (2)$$

$$-300I_1 + 700I_3 = 0 \quad (3)$$

Upon solving Eqn. (1), (2) and (3), we get

$$I_1 = 0.2247\,\text{A}$$

$$I_2 = -0.1362\,\text{A}$$

$$I_N = I_3 = 0.0963\,\text{A}$$

(c) To find P_{max}:

Norton's equivalent circuit is drawn as shown in Fig. E2.56(d).

According to the current division rule, we get

$$I_L = \frac{96.3\times10^{-3}\times600}{600+600} = 48.15\text{ mA}$$

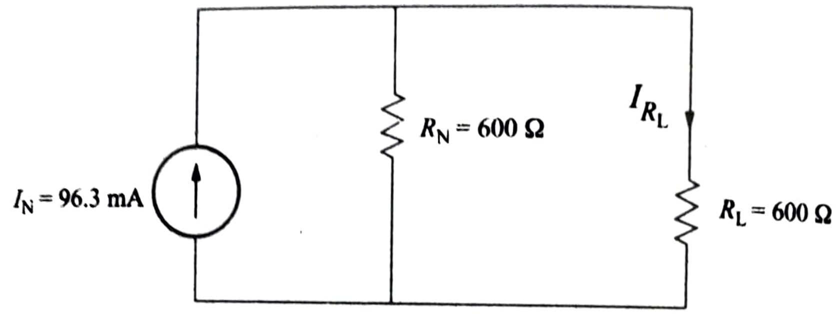

FIG. E2.56(d)

Hence, the maximum power transferred to the load

$$P_{max} = \left(I_{R_L}\right)^2 . R_L$$

$$= \left(48.15\times10^{-3}\right)^2\times600 = 1.391\text{ W}$$

Example 2.57 In the network shown in Fig. E2.57(a), find the value of Z_L so that power transfer from the source is maximum. Also, find the value of P_{max}.

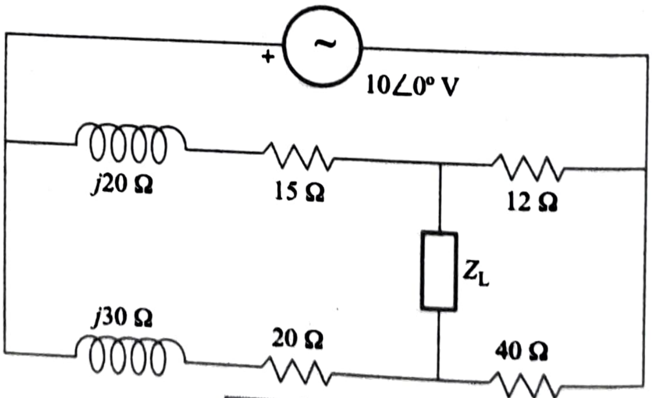

FIG. E2.57(a)

Solution

The maximum power transfer occurs when $Z_L = Z_S^*$. To calculate Z_s, the voltage source replaced by a short circuit and Z_L is removed as shown in Fig. E2.57(b).

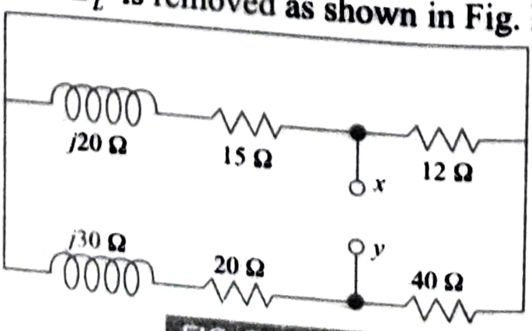

FIG. E2.57(b)