$$\bar{I}_N = 5\angle -53.13°\text{A}$$

(b) To find Z_N:

Short the voltage source as shown in Fig. E2.46(c).

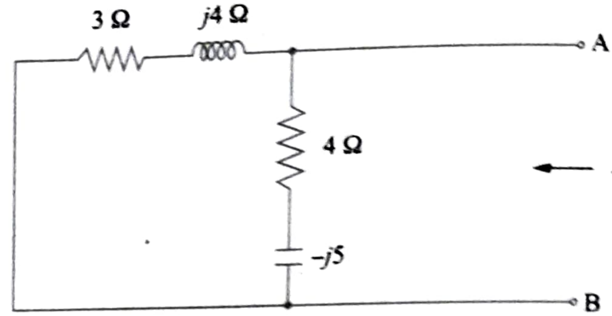


FIG. E2.46(c)

Therefore,
$$Z_N = \frac{(3+j4)(4-j5)}{3+j4+4-j5}$$
$$= \frac{5\angle 53.13° \times 6.4031\angle -51.34°}{7.071\angle -8.1301°}$$
$$= 4.5277\angle 9.9201°\Omega$$
$$= 4.459 + j0.779\Omega$$

(c) The Norton's equivalent circuit is shown in Fig. E.2.46(d).

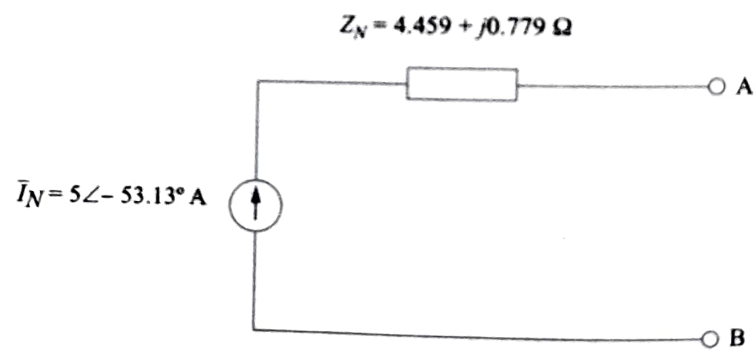


FIG. E2.46(d)

2.8 SUPERPOSITION THEOREM

Superposition theorem states that "*In a linear circuit consisting of more than one independent source, the total current in any part of the circuit equals the algebraic sum of the individual contributions of currents produced by each independent source separately*". Superposition refers to the superposition of responses arising from individual sources. It should be noted that this theorem is valid only for a linear circuit. Also, superposition theorem is not valid if the circuit consists of dependent sources.

A circuit is called linear if the system of equations describing that circuit is linear. If a linear circuit has more than one source then one independent source can be considered at a time by deactivating all other independent sources.

In the process of applying superposition, if a current source has to be deactivated then it has to be replaced with an open circuit, so that there is no current flow (or $I_s = 0$) from the deactivated source in that branch. Otherwise, if a voltage source has to be deactivated then it has to be replaced with an short circuit, so that the deactivated source does not produce any voltage ($V_s = 0$) in that branch.

Consider the circuit shown in Fig. 2.11(a) to illustrate the application of superposition theorem.

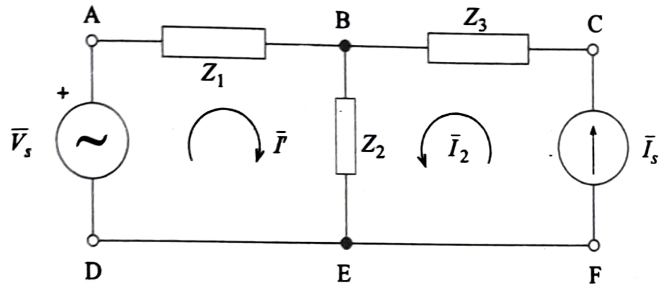


FIG. 2.11(a) A circuit with conventional current direction

This circuit has two independent sources $\overline{V}_s$ and $\overline{I}_s$. To analyze the circuit through the superposition theorem let us first consider only the voltage source $\overline{V}_s$ by deactivating the current source $\overline{I}_s$. Deactivation of the current source is done through open-circuiting its terminals C and F as shown in Fig. 2.11(b).

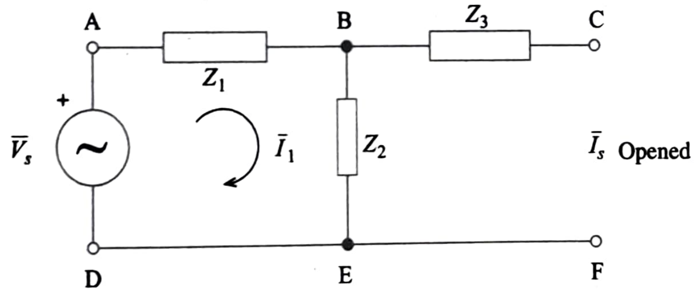


FIG. 2.11(b) Independent contribution of voltage source

The current through Z_2 due to the voltage source $\overline{V}_s$ alone can be calculated as

$$\overline{I}' = \frac{\overline{V}_S}{Z_1 + Z_2}$$

Now, to analyze the independent contribution of the current source $\overline{I}_s$ alone, short the voltage source terminals A and D as shown in Fig. 2.11(c).

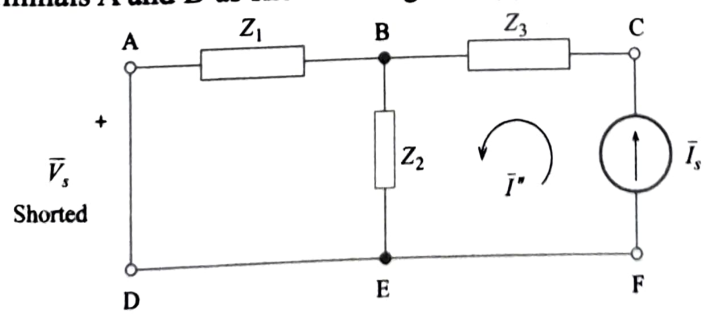


FIG. 2.11(c) Independent contribution of current source

The current flow $\overline{I}''$ through Z_2 due to current source $\overline{I}_s$ alone can be calculated as

$$\overline{I}'' = \overline{I}_S \times \frac{Z_1}{Z_1 + Z_2}$$

According to the superposition theorem, the total current passing through impedan
Z_2 due to both the sources $\overline{V}_s$ and $\overline{I}_s$ can be calculated as

$$\overline{I} = \overline{I}' + \overline{I}''$$

Limitations of Superposition Theorem

1. It is valid only for linear circuits.
2. It is not valid for unbalanced bridge circuits.
3. It is applicable to measure voltage and current, and not power.
4. It is applied only to the circuits containing two or more sources.

Example 2.47 Find the current through $R_L = 7.5\Omega$ using superposition theorem in t
network shown in Fig. E2.47(a).

Solution

(a) Let the active independent source be the current source 10 A alone. Assigning all other sources as zero, i.e., short-circuiting the voltage source 20 V, the circuit is redrawn as shown in Fig. E2.47(b) and further simplified as shown in Fig. E2.47(c) and Fig. E2.47(d). Using source transformation, the 10 A current source in Fig. E2.47(d) is converted into its equivalent voltage source shown in Fig. E2.47(e).

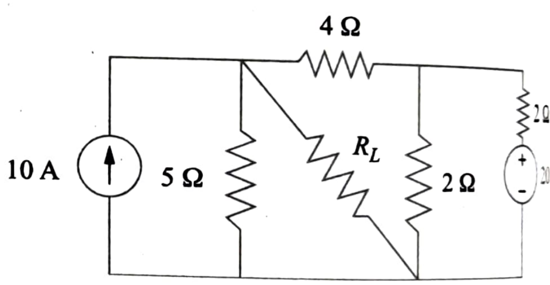


FIG. E2.47(a)

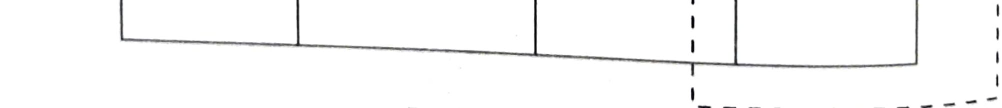
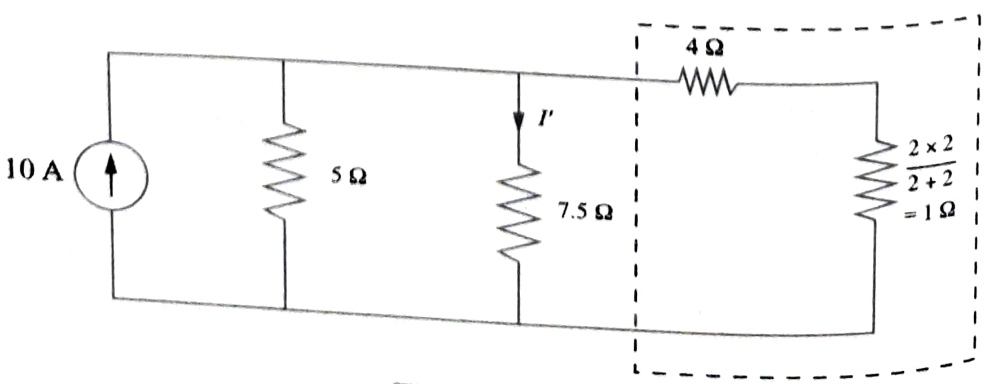


FIG. E2.47(b)



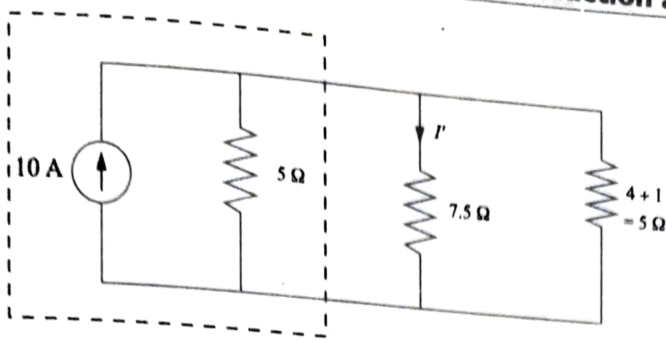


FIG. E.2.47(d)

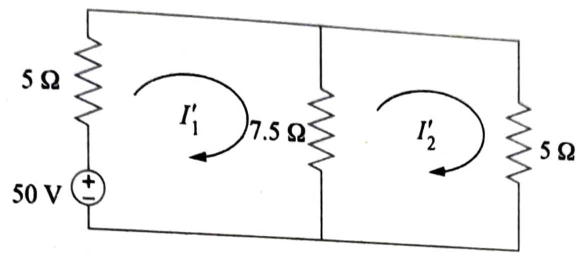


FIG. E2.47(e)

Applying KVL to this circuit, we get

$$12.5I_1' - 7.5I_2' = 50 \tag{1}$$

$$-7.5I_1' + 12.5I_2' = 0 \tag{2}$$

Upon solving Eqn.(1) and(2), we get

$$I_1' = 6.25\,\text{A} \text{ and } I_2' = 3.75\,\text{A}.$$

Therefore, $\quad I'_{7.5\,\Omega} = I_1' - I_2' = 6.25 - 3.75 = 2.5\,\text{A}$

(b) Let the active independent source be the voltage source 20 V alone. Assigning all other sources as zero, i.e., open-circuiting the current source 10 A, the circuit is redrawn with the assumed currents and direction as shown in Fig. E2.47(f).

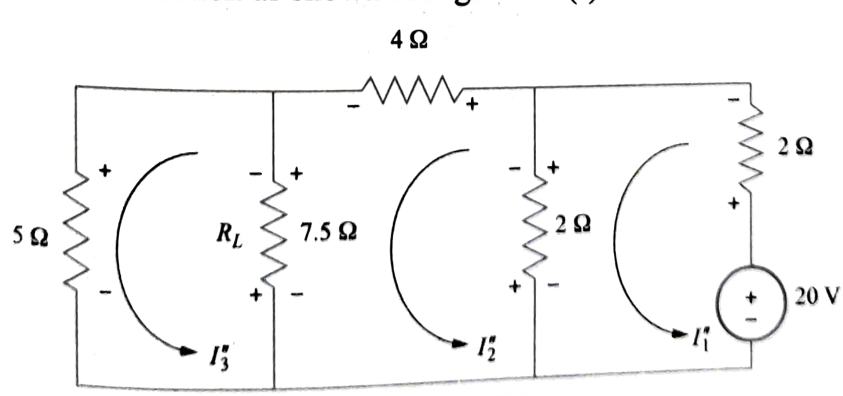


FIG. E2.47(f)

Applying KVL to the loops, we get

$$4I_1'' - 2I_2'' = 20 \tag{3}$$

$$-2I_1'' + 13.5I_2'' - 7.5I_3'' = 0 \tag{4}$$

$$-7.5I_2'' + 12.5I_3'' = 0 \tag{5}$$

Upon solving Eqn.(3), Eqn.(4) and Eqn.(5), we get

$$I_2'' = 1.25\,\text{A} \text{ and } I_3'' = 0.75\,\text{A}$$

Therefore, $$I_{7.5\Omega}'' = I_2'' - I_3'' = 1.25 - 0.75 = 0.5\,\text{A}$$

According to superposition theorem,

The total current through $R_L = 7.5\Omega$ is

$$I_{7.5\Omega} = I_{7.5\Omega}' + I_{7.5\Omega}'' = 2.5\,\text{A} + 0.5\,\text{A} = 3\,\text{A}$$

Example 2.48 Determine the current in the 10Ω resistor by the principle of superposi[tion] theorem for the circuit shown in Fig. E2.48(a).

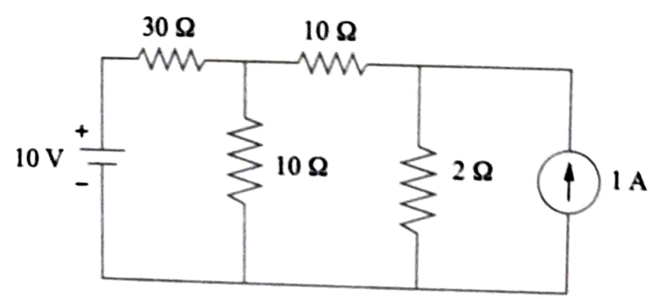


FIG. E2.48(a)

Solution

(*a*) When the 10V source alone is active, deactivating the remaining sources, i.e., ope[n] circuiting the current source the circuit is redrawn as shown in Fig. E2.48(b). Let t[he] currents be I_1' and I_2' in the direction shown in Fig. E2.48(b).

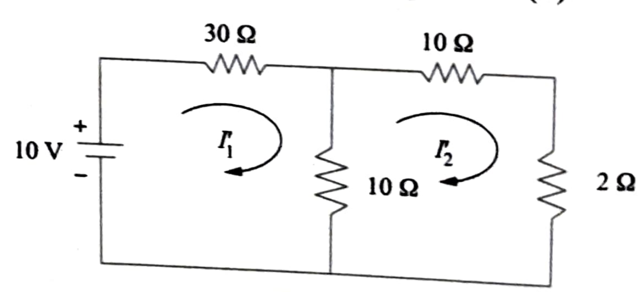


FIG. E2.48(b)

Applying KVL to the circuit, we get

For loop 1,

$$30I_1' + 10(I_1' - I_2') = 10$$

$$40I_1' - 10I_2' = 10 \quad (1)$$

For loop 2,

$$10I_2' + 2I_2' + 10(I_2' - I_1') = 0$$

$$-10I_1' + 22I_2' = 0 \quad (2)$$

Upon solving Eqn.(1) and Eqn.(2), we get

$$I_2' = 0.128 \text{ A}$$

(*b*) When the 1A source alone is active, deactivating the remaining sources, i.e., short-circuiting the voltage source the circuit is redrawn as shown in Fig. E2.48(c).

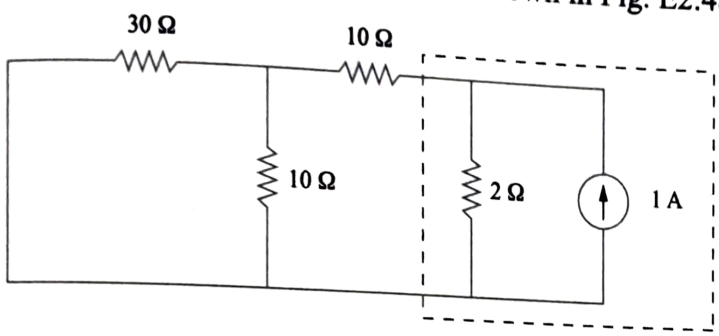


FIG. E2.48(c)

Using source transformation the current source 1 A is converted into its equivalent voltage source as shown in Fig. E2.48(c).
Let the current response in the circuit be I_1' and I_2'' as shown in Fig. E2.48(d).

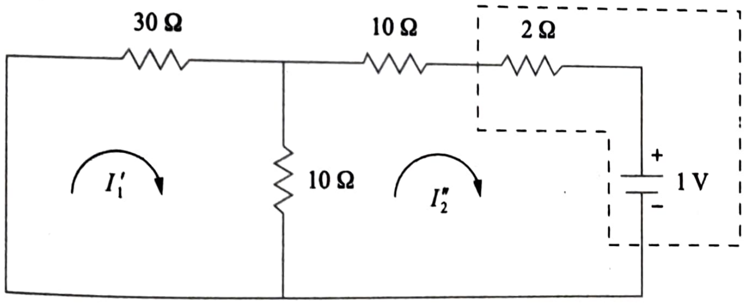


FIG. E2.48(d)

Applying KVL to the circuit, we get

$$40I_1'' - 10I_2'' = 0 \tag{1}$$

and

$$-10I_1'' + 22I_2'' = -1 \tag{2}$$

Upon solving Eqn.(1) and(2), we get

$$I_2'' = -0.0513 \text{A}$$

Since the obtained result is in negative, the actual current direction is opposite to the assumed direction.

Hence,

$$I_2'' = 0.0513 \text{A}$$

(c) According to Superposition theorem, the total current through the resistor 10 Ω,

$$I_2 = I_2' + I_2''$$

$$= 0.128 + 0.0513 = 0.179 \text{A}$$