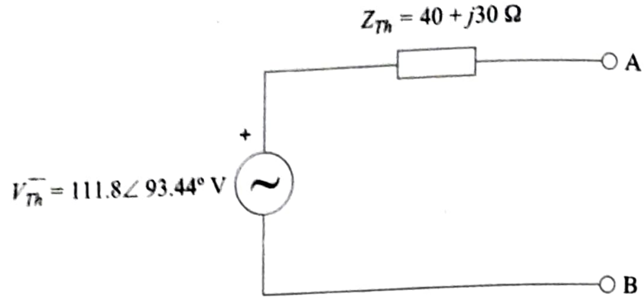


FIG. E2.39(d)

## 2.7 NORTON'S THEOREM

Norton's theorem states that "*any two-terminal bilateral linear circuit consisting of energy sources and impedances can be replaced with a Norton's equivalent circuit with a single Norton current source $\overline{I}_N$ in parallel with a Norton's impedance $Z_N$ connected to a load impedance $Z_L$*". Similar to Thevenin's theorem, Norton's theorem is also used to simplify the analysis of complex circuits.

The value of Norton's equivalent current $\overline{I}_N$ can be calculated by shorting the load impedance $Z_L$. The Norton's equivalent impedance $Z_N$ is obtained by the procedure similar to that of calculating Thevenin's equivalent impedance $Z_{Th}$.

Consider the circuit shown in Fig. 2.10(a) to illustrate the application of Norton's theorem.

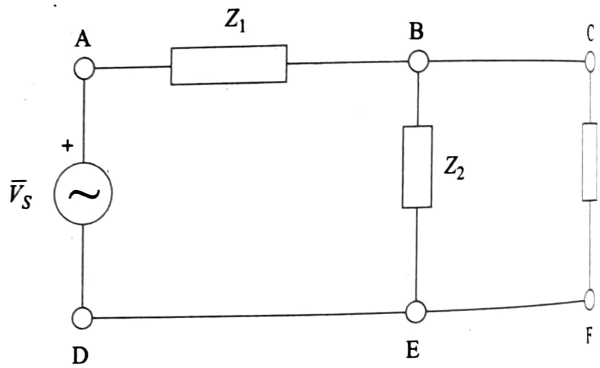


FIG. 2.10(a) A circuit with energy source and impedances

**Step 1:** To find Norton's current $\overline{I}_N$, short-circuit the load $Z_L$ as shown in Fig. 2.10(b).

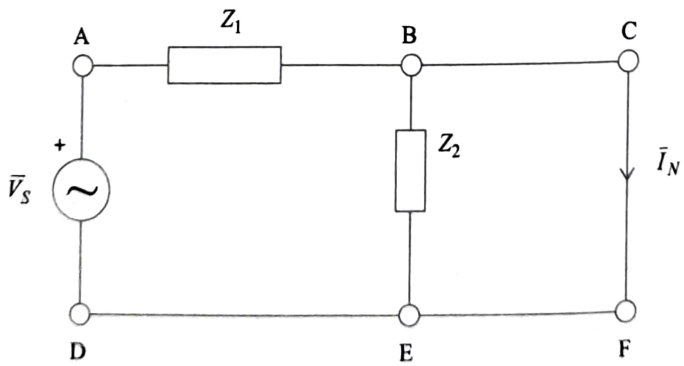


FIG. 2.10(b) Short-circuiting the load

As the entire current flows through the short-circuited terminals C and F and no currentflows through $Z_2$, we have

$$\overline{I}_N = \frac{\overline{V}_S}{Z_1}$$

**Step 2:** To find Norton's equivalent impedance ($Z_N$)

$$Z_N = Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

**Step 3:** Norton's equivalent circuit with $\bar{I}_N$, $Z_N$ and $Z_L$ can be constructed as shown in Fig. 2.10(c).

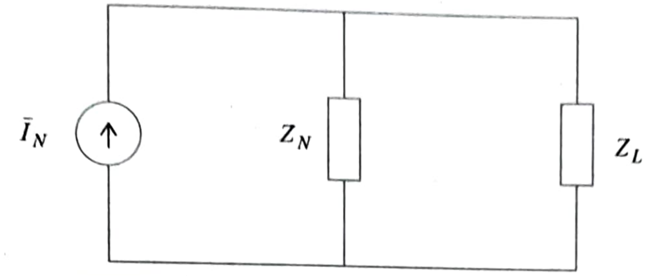


FIG. 2.10(c) Norton's equivalent circuit

It is important to note that using source transformation, Thevenin's equivalent circuit can be converted into Norton equivalent and vice versa as shown in Fig. 2.10(d).

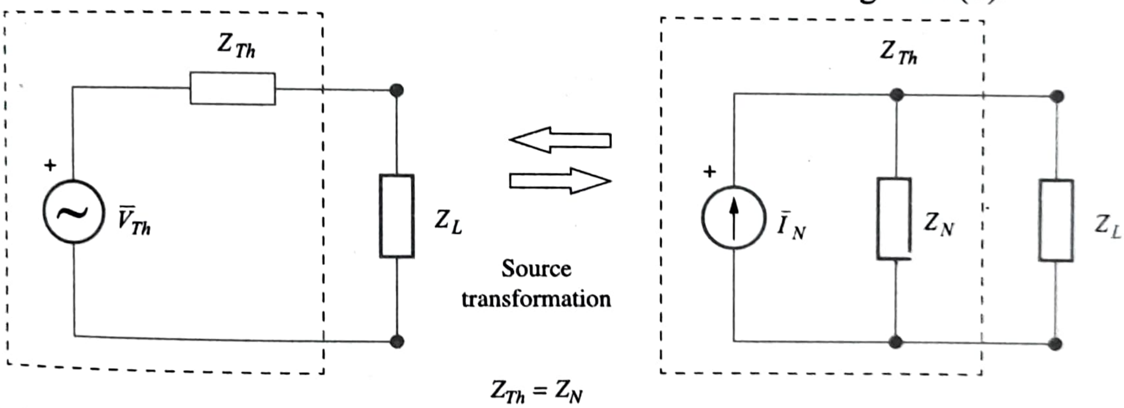


FIG. 2.10(d) Source transformation between Thevenin's equivalent circuit and Norton's equivalent circuit

If dependant sources are present in the circuit, the same procedure can be followed using Thevenin's theorem for solving problems.

**Example 2.40** Find the voltage drop across the 12 Ω resistance using Norton's theorem for the circuit shown in Fig. E2.40(a).

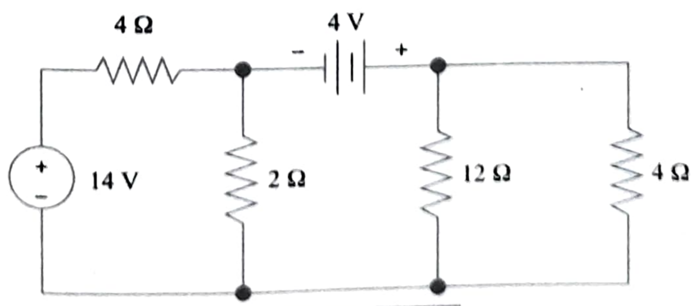


FIG. E2.40(a)

***Solution***

*(a) To find Norton's current $I_N$:*

Short the branch consisting of 12 Ω resistor as shown in Fig. E2.40(b).
Since the 4 Ω resistor is in parallel with a short-circuited terminal, it is removed from the circuit as shown in Fig. E2.40(c).

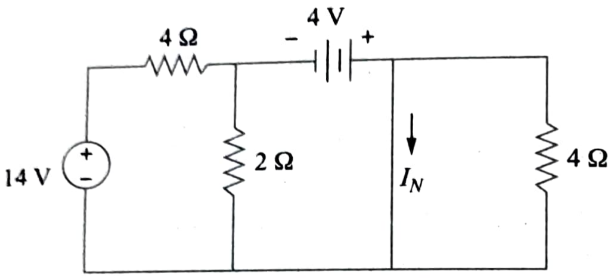


FIG. E2.40(b)

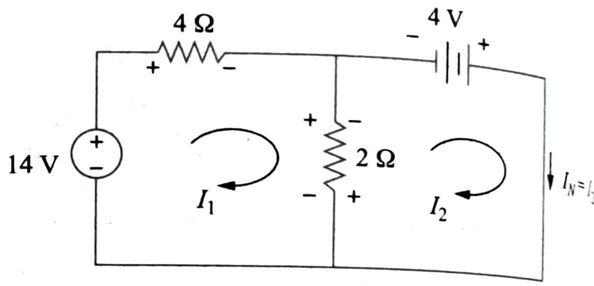


FIG. E2.40(c)

Applying KVL to the circuit shown in Fig. E2.40(c), we get

For loop 1, $4I_1 + 2(I_1 - I_2) = 14$

i.e., $$6I_1 - 2I_2 = 14 \quad (1)$$

For loop 2, $2(I_2 - I_1) = 4$

i.e., $$-2I_1 + 2I_2 = 4 \quad (2)$$

Upon solving Eqn.(1) and(2), we get

$$I_N = I_2 = 6.5\,\text{A}$$

*(b) To find $R_N$:*

Open-circuit the 12 Ω resistor, and short-circuit the voltage source as shown in Fig. E2.40(d). The circuit is further simplified as shown in Fig. E2.40(e).

Therefore, $$R_N = \frac{4 \times 1.333}{4 + 1.333} \approx 1\,\Omega$$

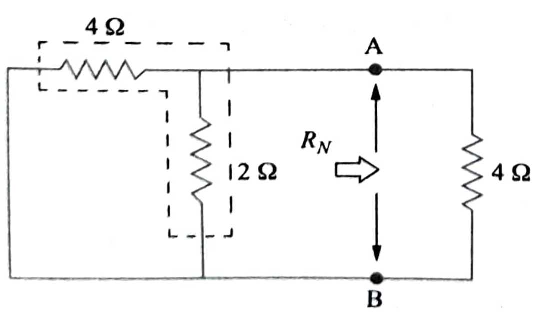


FIG. E2.40(d)

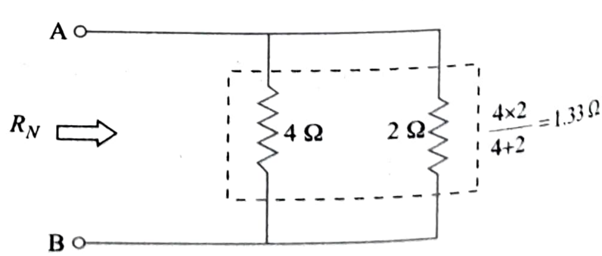


FIG. E2.40(e)

*(c) The Norton's equivalent circuit is shown in Fig. E2.40(f).*

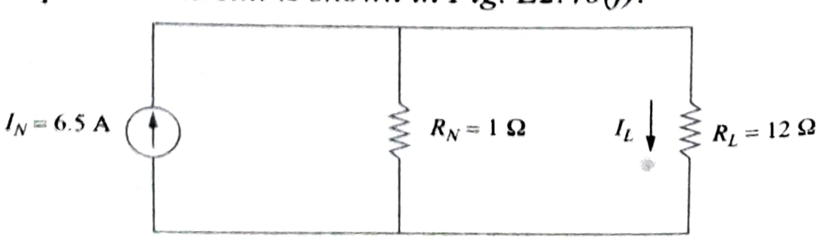


FIG. E2.40(f)

Therefore, $I_L = I_{12\,\Omega} = I_N \times \dfrac{R_N}{R_L + R_N}$

$$= 6.5 \times \frac{1}{12+1} = 0.5 \text{ A}$$

Voltage drop across the $12\,\Omega$ load resistor $V_L = I_L R_L = 0.5 \times 12 = 6$ V.

**Example 2.41** Obtain the Norton's Equivalent for the circuit shown in Fig. E2.41(a).

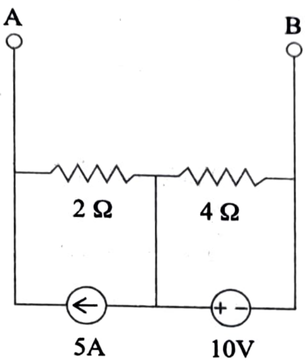


FIG. E2.41(a)

***Solution***

*(a) To find $I_N$:*

Short-circuit the branch AB as shown in Fig. E2.41(b).

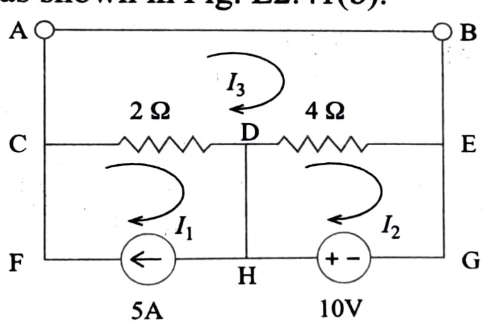


FIG. E2.41(b)

From Fig. E2.41(b), it is evident that

$$I_1 = 5 \text{ A}$$

Applying KVL to the remaining loops, we get

For the loop DEGHD,

$$4(I_2 - I_3) = 10$$

$$4I_2 - 4I_3 = 10 \quad (1)$$

For the loop ABEDCA

$$4(I_3 - I_2) + 2(I_3 - I_1) = 0$$

$$-2I_1 - 4I_2 + 6I_3 = 0$$

$$-4I_2 + 6I_3 = 2 \times 5 = 10 \quad (2)$$

Upon solving Eqn. (1) and (2), we get

$$I_2 = 12.5 \text{ A and } I_3 = 10 \text{ A}$$

Therefore, $I_N = I_3 = 10\text{ A}$

*(b) To calculate $R_N$:*

Open-circuit the current source and short-circuit the voltage source as shown in Fig. E2.4

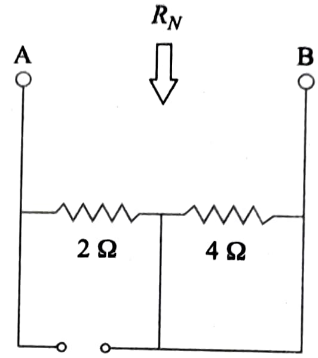


FIG. E2.41(c)

From the circuit shown in Fig. E2.41(c), it is evident that the 4 Ω resistor is in para with the short circuit. Since the equivalent resistance of this parallel combination is ze it can be removed.

Therefore, $R_N = 2\Omega$

*(c) The Norton's equivalent circuit is shown in Fig. E2.41(d).*

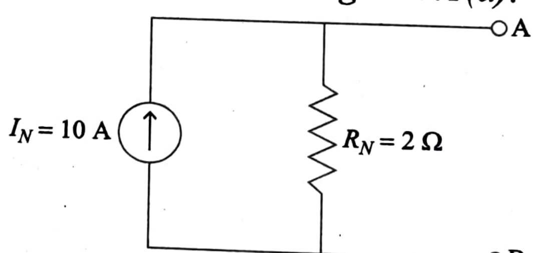


FIG. E2.41(d)

**Example 2.42** Determine the current through the 2 Ω resistor in the circuit shown in F E2.42(a), using Norton's theorem.

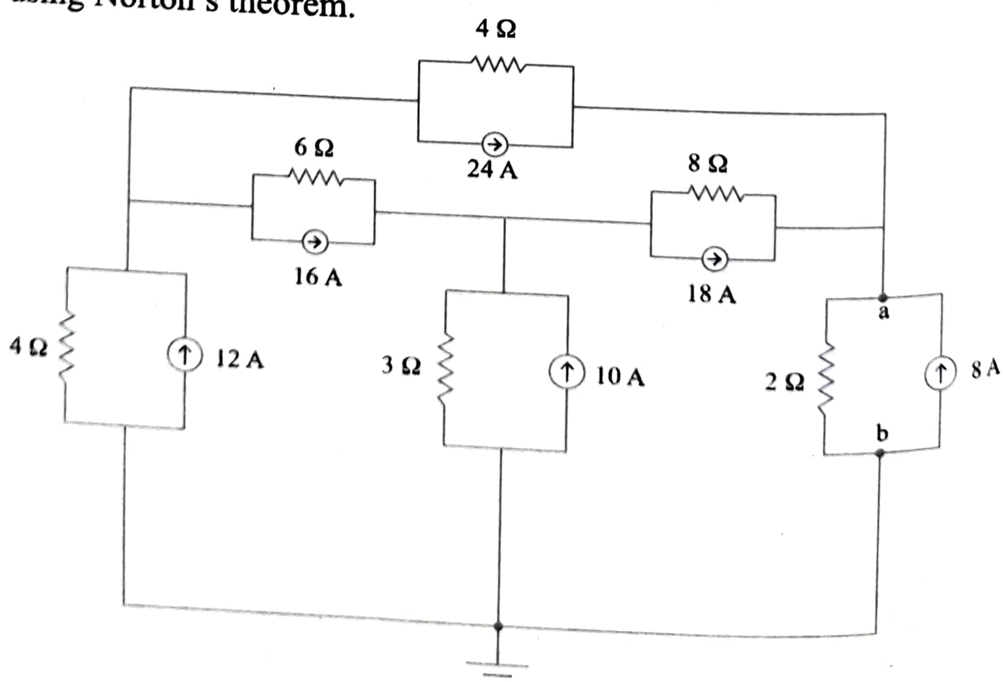


FIG. E2.42(a)

***Solution***

Using source transformation, the current sources are converted into voltage sources as shown in Fig. E2.42(b).

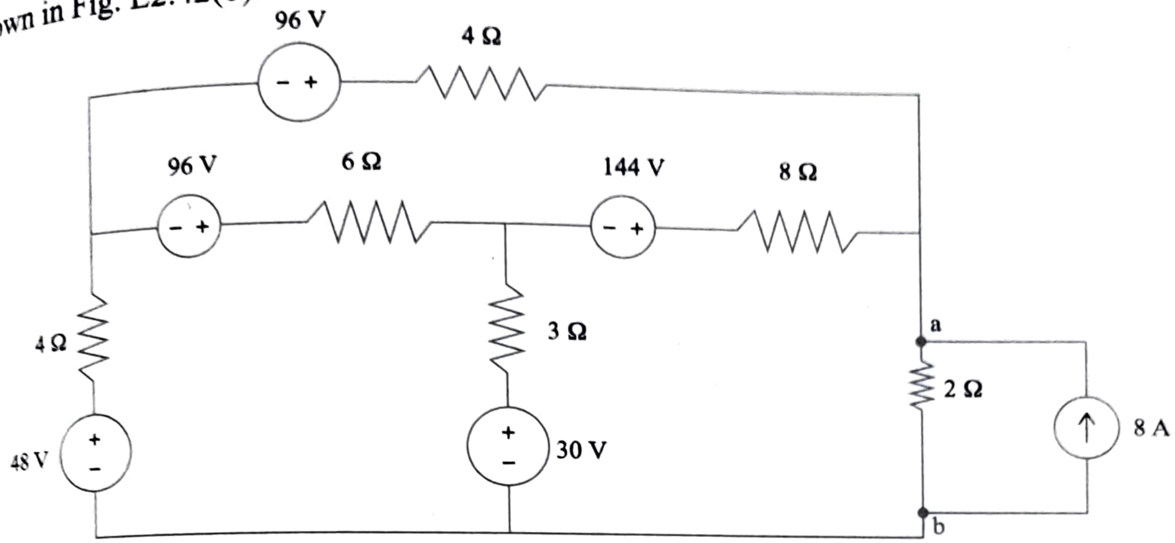


FIG. E2.42(b)

(a) *To find $I_N$:*

Short-circuit the load resistor 2 Ω as shown in Fig. E2.42(c).

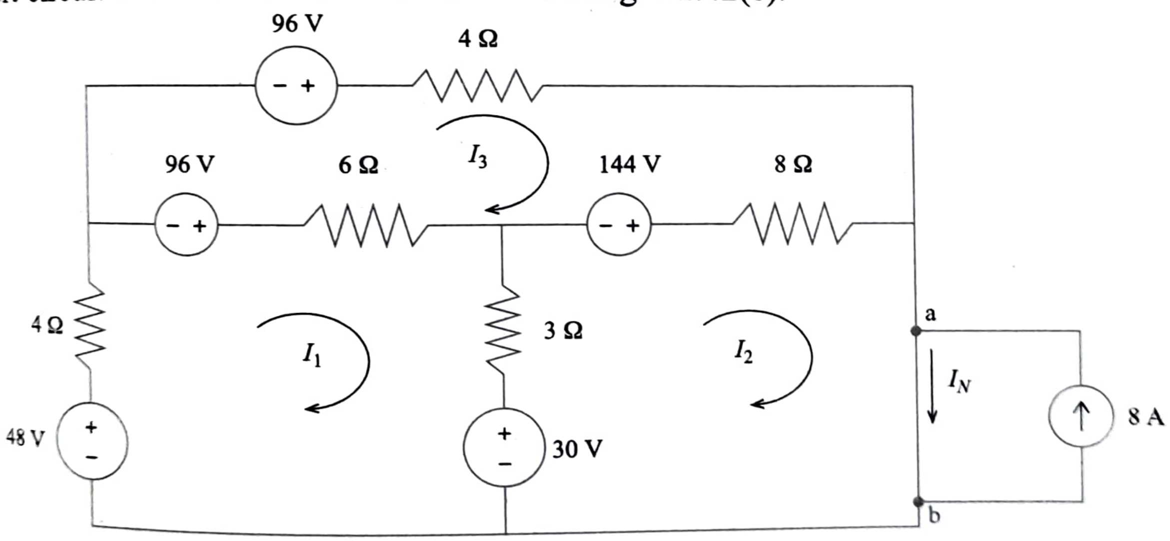


FIG. E2.42(c)

Applying KVL to this circuit, we get

For loop 1,

$$13I_1 - 3I_2 - 6I_3 = 96 - 30 + 48$$

$$13I_1 - 3I_2 - 6I_3 = 114 \qquad (1)$$

For loop 2,

$$-3I_1 + 11I_2 - 8I_3 = 144 + 30$$

$$-3I_1 + 11I_2 - 8I_3 = 174 \qquad (2)$$

For loop 3,

$$-6I_1 - 8I_2 + 18I_3 = 96 - 144 - 96$$

$$-6I_1 - 8I_2 + 18I_3 = -144 \qquad (3)$$

Writing Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{vmatrix} 13 & -3 & -6 \\ -3 & 11 & -8 \\ -6 & -8 & 18 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 114 \\ 174 \\ -144 \end{vmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 13 & -3 & -6 \\ -3 & 11 & -8 \\ -6 & -8 & 18 \end{vmatrix} = 896$$

$$\Delta_2 = \begin{vmatrix} 13 & 114 & -6 \\ -3 & 174 & -8 \\ -6 & -144 & 18 \end{vmatrix} = 28512$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{28512}{896} = 31.821\text{A}$$

From Fig. E2.42(c), It is evident that $I_N = I_2 + 8 = 31.821 + 8 = 39.821\text{A}$

*(b) To find $R_N$:*

Short-circuit the voltage sources and remove the load $R_L$ as shown in Fig. E2.42(d)

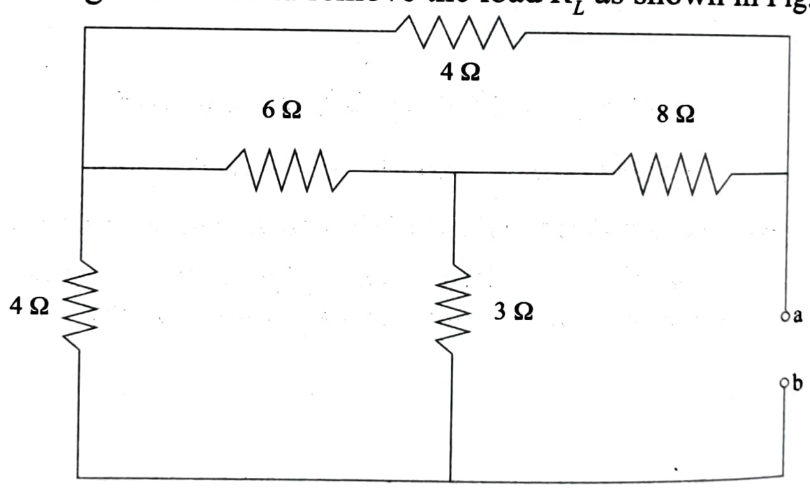


FIG. E2.42(d)

Apply delta-to-star transformation as shown in Fig. E2.42(e).

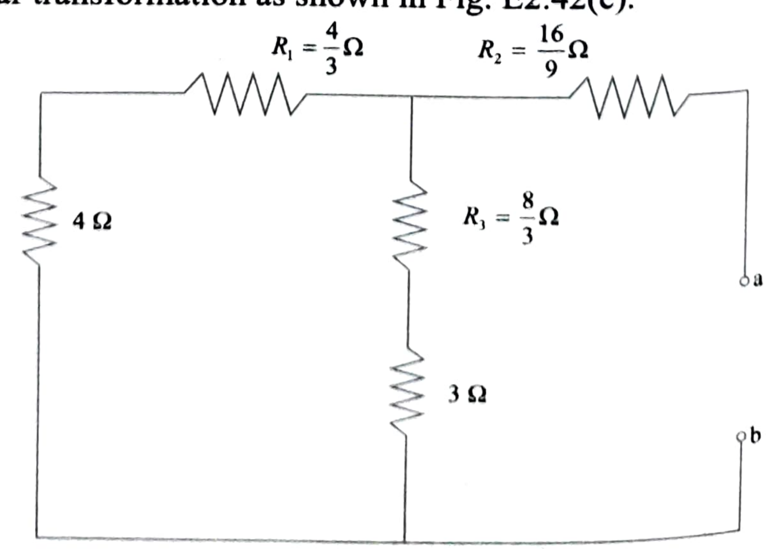


FIG. E2.42(e)

Here,
$$R_1 = \frac{6\times4}{6+4+8} = \frac{24}{18} = \frac{4}{3}\Omega$$
$$R_2 = \frac{8\times4}{6+4+8} = \frac{32}{18} = \frac{16}{9}\Omega$$
and
$$R_3 = \frac{6\times8}{6+4+8} = \frac{48}{18} = \frac{8}{3}\Omega$$
Therefore,
$$R_N = R_{ab} = \left[\left(4+\frac{4}{3}\right)\|\left(\frac{8}{3}+3\right)\right] + \frac{16}{9}$$
$$= \frac{5.333\times5.667}{5.333+5.667} + \frac{16}{9} = 4.525\Omega$$

(c) *The Norton's equivalent circuit is shown in Fig. E2.42(f).*

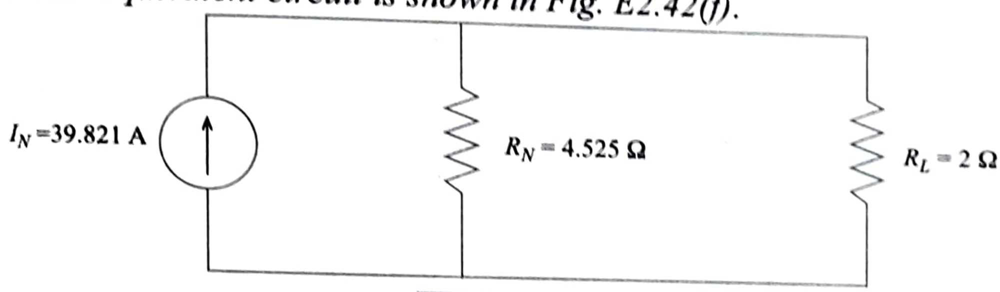


FIG. E2.42(f)

According to the current division rule,
$$I_{2\Omega} = \frac{39.821\times4.525}{4.525+2} = 27.6153 \text{ A}$$

**Example 2.43** Obtain Norton's equivalent for the circuit shown in Fig. E2.43(a) across the terminal AB.

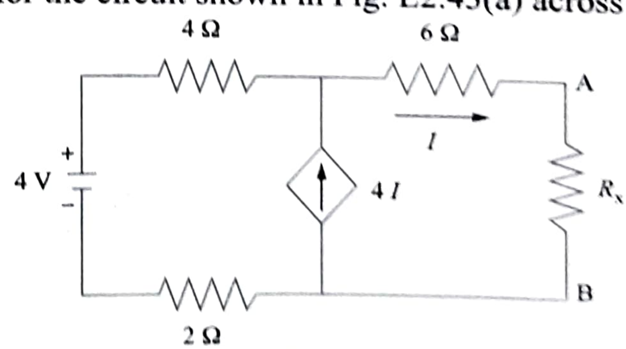


FIG. E2.43(a)

***Solution***

(a) *To find $I_N$:*

Short-circuit the load resistor $R_x$ between the terminals AB, as shown in Fig. E2.43(b). Let the current flowing through the 4Ω resistor be $I_x$ in the direction as shown in Fig. E2.43(b).

Applying KCL at node 'c', we get
$$4I = I + I_x$$
or
$$I_x = 3I$$

Applying KVL to the super mesh ECABDFE, we get
$$-4I_x + 6I - 2I_x = 4$$
$$-6(3I) + 6I = 4$$
$$-12I = 4$$

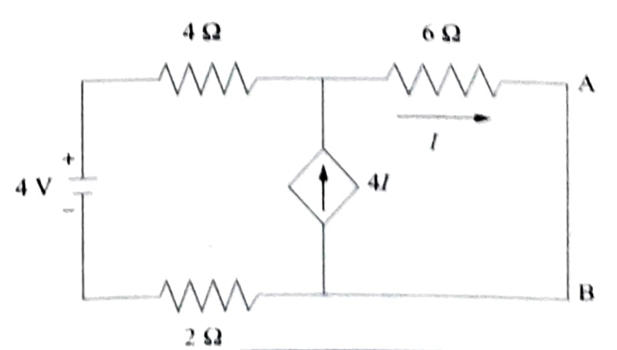


FIG. E2.43(b)