The star equivalent can be drawn as shown in Fig. E2.26(c), which is simplified using network reduction techniques as shown in Fig. E2.26(d).

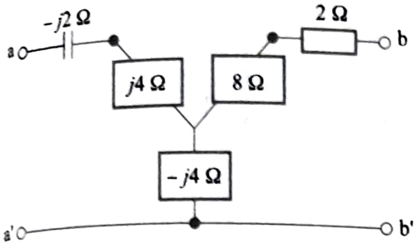

FIG. E2.26(c)

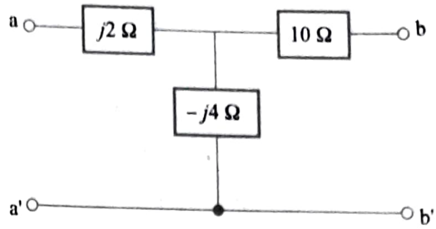

FIG. E2.26(d)

Finally, to convert the network into π, the star configuration of impedances $j2\,\Omega$, $10\,\Omega$ and $-j4\,\Omega$ is converted to equivalent delta, as shown in Fig. E2.26(e).

$$Z_B = j2 + 10 + \frac{j2\times 10}{-j4} = 5 + j2\ \Omega$$

$$Z_A = j2 - j4 + \frac{j2\times -j4}{10} = 0.8 - j2\ \Omega$$

$$Z_C = 10 - j4 + \frac{10\times -j4}{j2} = -10 - j4\ \Omega$$

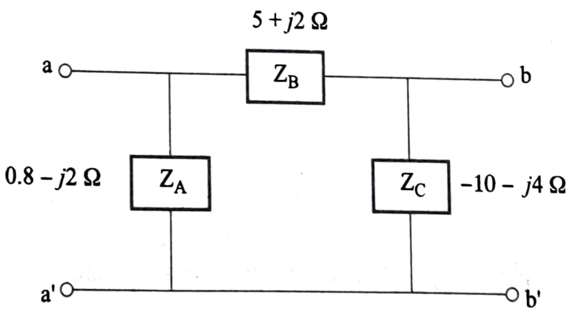

FIG. E2.26(e)

2.6 THEVENIN'S THEOREM

Thevenin's theorem states that "*for any two-terminal bilateral linear circuit consisting of energy sources and impedances can be replaced with an equivalent circuit consisting of a single Thevenin's equivalent voltage source $\bar{V}_{Th}$ in series with a Thevenin's equivalent impedance (Z_{Th})*". Thevenin's theorem is used to simplify a complex circuit comprising many sources and impedances into its Thevenin's equivalent.

The Thevenin's equivalent voltage is equal to the open circuit voltage $\bar{V}_{Th}$ calculated by removing the load impedance (Z_L). The Thevenin's equivalent impedance (Z_{Th}) is equal to the effective impedance obtained after shorting the existing voltage sources and opening the existing current sources.

Consider the example shown in Fig. 2.9(a) to illustrate the application of Thevenin's theorem.

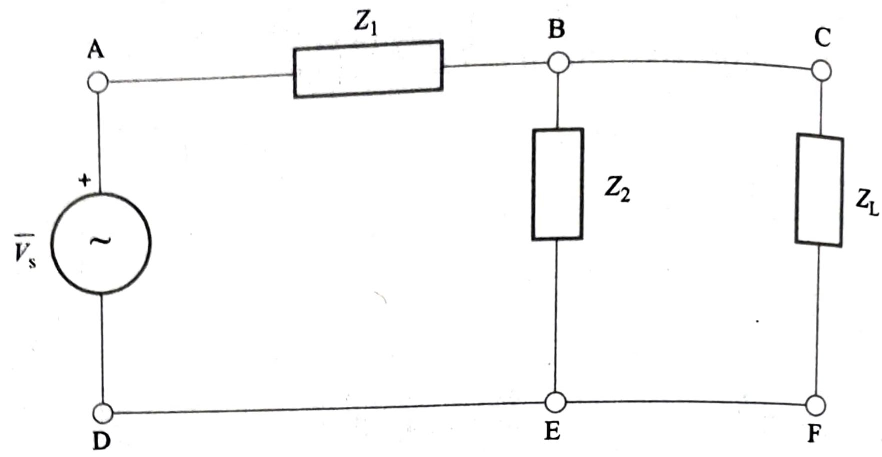

FIG. 2.9(a) A circuit with source and impedances

This circuit can be replaced by a Thevenin's equivalent circuit by replacing the vo source $\overline{V}_S$ with Thevenin's equivalent voltage $\overline{V}_{Th}$, and by replacing the impedances Z Z_2 with the Thevenin's equivalent impedance Z_{Th} as shown in Fig. 2.9(b).

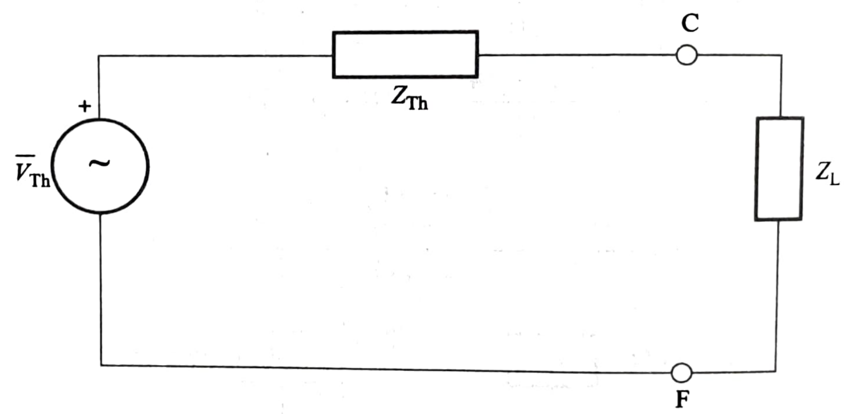

FIG. 2.9(b) Thevenin's equivalent circuit

Inspection of Fig. 2.9(a) and (b) reveals that the value of Z_L is not changed or replace the Thevenin's equivalent circuit.

The subsequent steps are followed to calculate $\overline{V}_{Th}$ and Z_{Th} for the circuit shown in Fig. 2.

Step 1 (To find $\overline{V}_{Th}$): Open the load impedance at terminal CF as shown in Fig. 2.9(c)

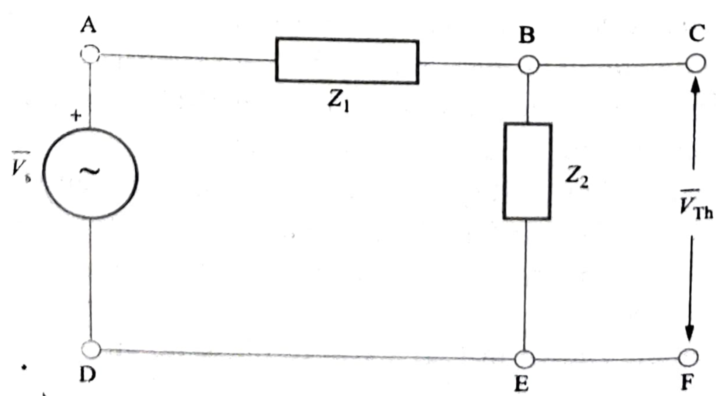

FIG. 2.9(c) Open-circuiting the load impedance

Find the open circuit voltage $\bar{V}_{Th}$. According to voltage division rule,

$$\bar{V}_{Th} = \bar{V}_s \times \frac{Z_2}{Z_1 + Z_2}$$

Step 2 (To find Z_{Th}): Short-circuit the voltage source $\bar{V}_s$ as shown in Fig. 2.9(d).

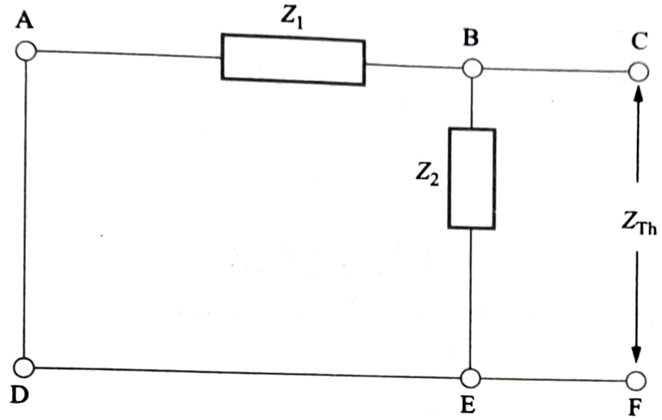

FIG. 2.9(d) Short-circuiting the voltage source

Find the Thevenin's equivalent impedance Z_{Th} as

$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Step 3: Construct the Thevenin's equivalent circuit with this $\bar{V}_{Th}$ and Z_{Th} as shown in Fig. 2.9(e)

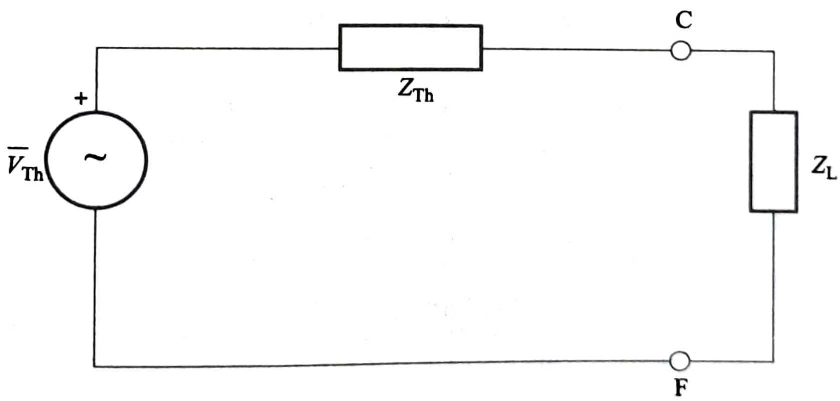

FIG. 2.9(e) Thevenin's equivalent circuit

Example 2.27 For the circuit shown in Fig. E.2.27(a), find the current through 10 Ω by using Thevenin's theorem.

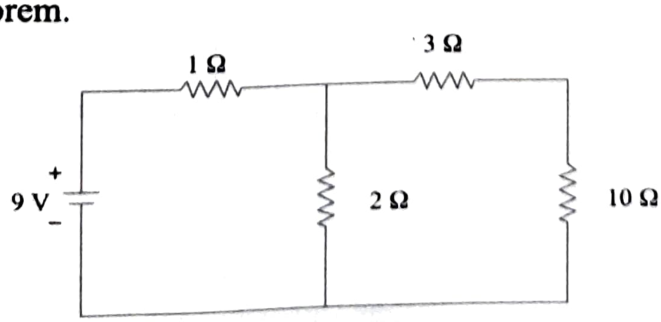

FIG. E.2.27(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

The load resistor $10\,\Omega$ is open-circuited and the circuit is redrawn as shown in Fig. E.227(

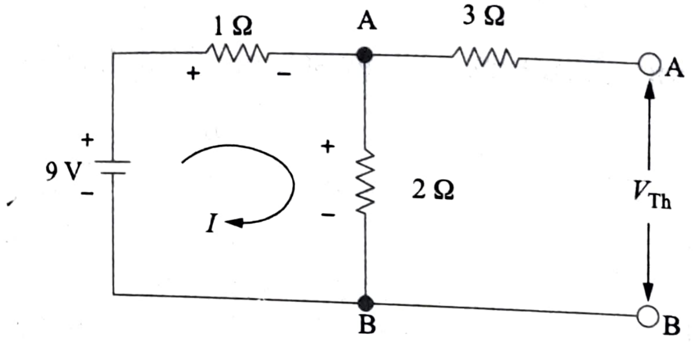

FIG. E2.27(b)

Applying KVL to the circuit shown in Fig. E.227(b), we get

$$1I + 2I = 9$$

or
$$I = \frac{9}{3} = 3\,\text{A}$$

Therefore, According to Ohm's law, $V_{Th} = V_{AB} = 3 \times 2 = 6\,\text{V}$

(b) To find Thevenin's equivalent resistance, R_{Th}:

The circuit is drawn after deactivating all the sources, i.e., short-circuiting the 9V voltag source as shown in Fig. E2.27(c).

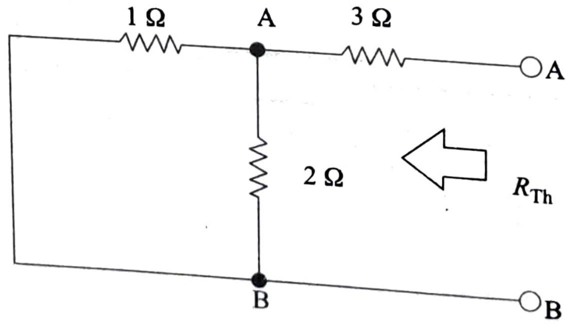

FIG. E2.27(c)

Therefore,
$$R_{Th} = 3 + \frac{2 \times 1}{2 + 1} = 3.667\,\Omega$$

(c) Thevenin's equivalent circuit is drawn as shown in Fig. E2.27(d).

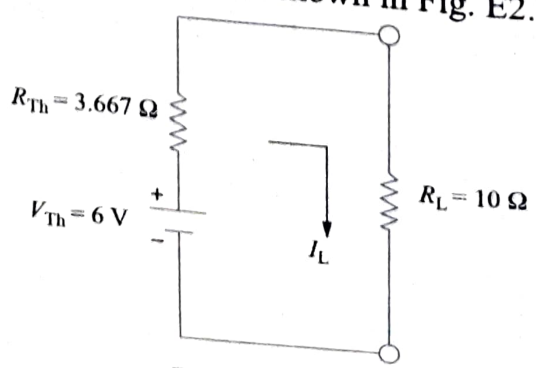

FIG. E2.27(d)

Hence, the current through 10Ω load resistor,

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} = \frac{6}{10+3.667} = 0.439\,\text{A}$$

Example 2.28 Using Thevenin's theorem find the current in branch AB having 2Ω resistor in the network shown in Fig. E2.28(a).

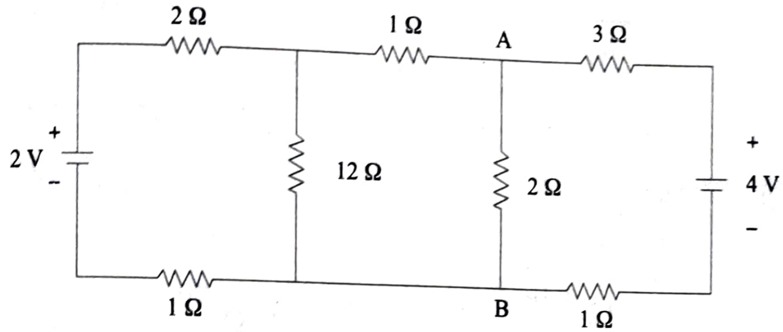

FIG. E2.28(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

Open-circuit the resistor 2Ω in the branch AB and then the circuit is redrawn as shown in Fig. E2.28(b) .

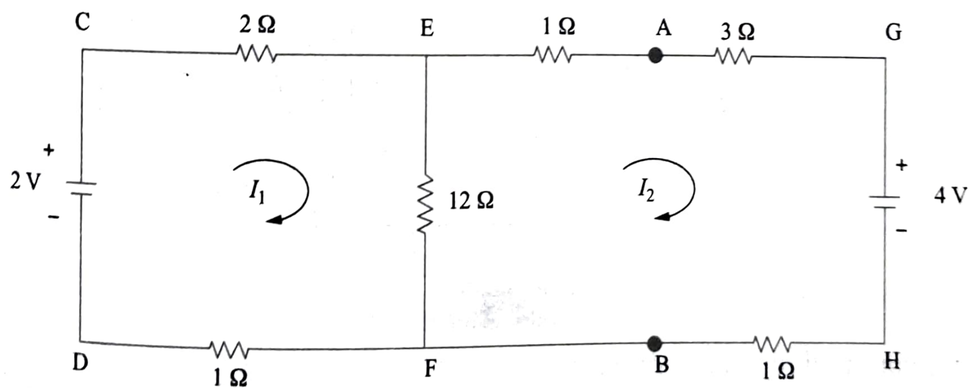

FIG. E2.28(b)

Applying KVL to the loop CEFDC, we get

$$2I_1 + 12(I_1 - I_2) + 1I_1 = 2$$

$$15I_1 - 12I_2 = 2 \qquad (1)$$

Applying KVL to the loop EGHFE, we get

$$1I_2 + 3I_2 + 4 + 1I_2 + 12(I_2 - I_1) = 0$$

$$-12I_1 + 17I_2 = -4 \qquad (2)$$

Solving Eqn.(1) and Eqn.(2), we get

$$I_1 = -0.126\,\text{A} \text{ and } I_2 = -0.324\,\text{A}$$

$$V_{Th} = V_{AG} + V_{GH} + V_{HB} = -0.324 \times 3 + 4 - 0.324 \times 1 = 2.704\ \text{V}$$

(b) To find Thevenin's equivalent resistance R_{Th}:
The circuit is redrawn after open-circuiting the resistor $2\,\Omega$ in the branch AB as shown in Fig. E2.28(c) and further simplified as shown in Fig. E2.28(d).

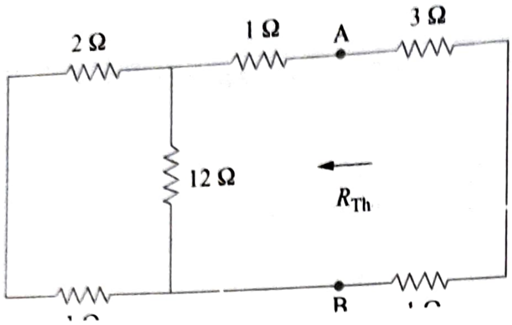

FIG. E2.28(c)

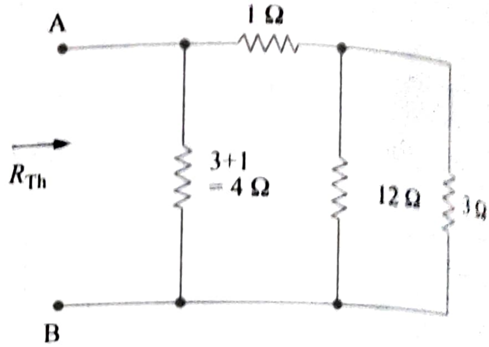

FIG. E2.28(d)

Therefore,
$$R_{Th} = \frac{\left[\frac{3\times 12}{3+12}+1\right]\times 4}{\left[\frac{3\times 12}{3+12}+1\right]+4} = \frac{3.4\times 4}{3.4+4} = 1.838\,\Omega$$

(c) Thevenin's equivalent circuit across AB is drawn as shown in Fig. E2.28(e).

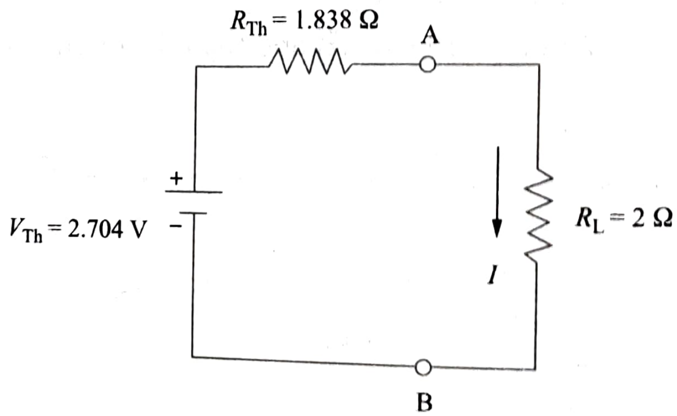

FIG. E2.28(e)

Hence, the current through $2\,\Omega$ resistor is

$$I = \frac{V_{Th}}{R_{Th}+R_L} = \frac{2.704}{1.838+2} = 0.705\,\text{A}$$

Example 2.29 Calculate the current through the $2\,\Omega$ resistor in the circuit shown in Fig. E2.29(a), using Thevenin's theorem.

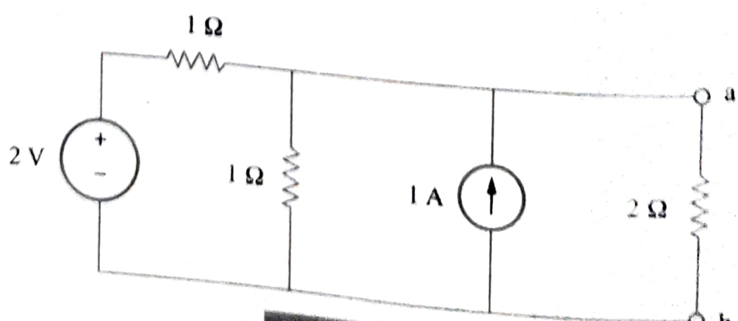

FIG. E2.29(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

The circuit is redrawn after open-circuiting the 2 Ω resistor as shown in Fig. E2.29(b). Using source transformation, the current source in Fig. E2.29(b) is converted into a voltage source as shown in Fig. E2.29(c).

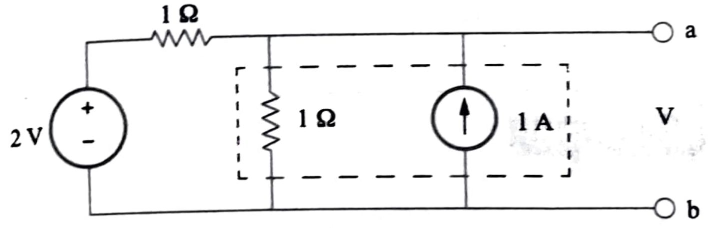

FIG. E2.29(b)

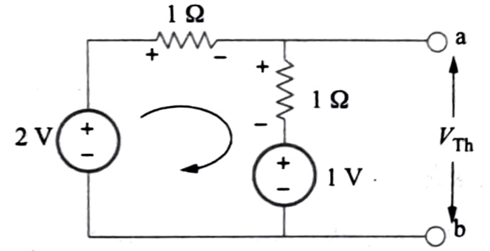

FIG. E2.29(c)

Applying KVL to the circuit shown in Fig. E2.29(c), we get

$$1I + 1I + 1 = 2 \text{, i.e., } I = 0.5\,\text{A}$$

Therefore, $V_{Th} = 1V + 1I = 1 + 0.5 = 1.5\,\text{V}$

(b) To find the Thevenin's equivalent resistance R_{Th}:

The circuit is redrawn as shown in Fig. E2.29(d) after deactivating all the sources, i.e., short-circuiting voltage source and open-circuiting current source.

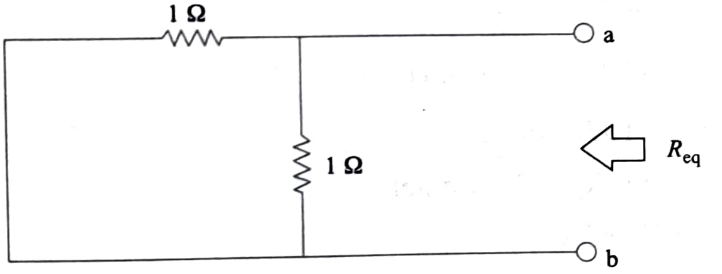

FIG. E2.29(d)

Therefore, $R_{Th} = \dfrac{1 \times 1}{1+1} = 0.5\,\Omega$

(c) Thevenin's equivalent circuit is drawn as shown in Fig. E2.29(e).

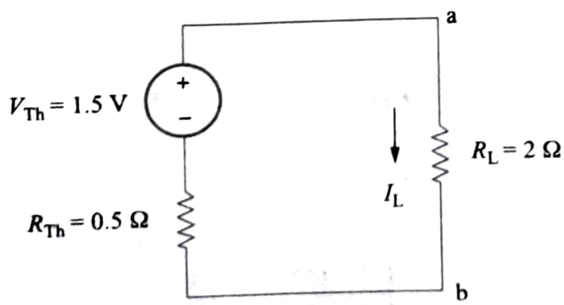

FIG. E2.29(e)

The current through 2 Ω resistor is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{1.5}{0.5 + 2} = 0.6\,\text{A}$$

Example 2.30 Find Thevenin's equivalent for the circuit shown in Fig. E2.30(a).

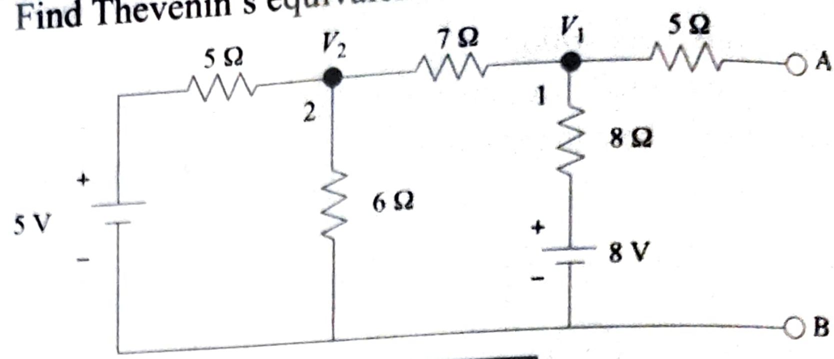

FIG. E2.30(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

Applying KCL at node 1, we get

$$\frac{V_1-8}{8}+\frac{V_1-V_2}{7}=0$$

$$7V_1-56+8V_1-8V_2=0$$

$$15V_1-8V_2=56$$

Applying KCL at node 2, we get

$$\frac{V_2}{6}+\frac{V_2-V_1}{7}+\frac{V_2-5}{5}=0$$

$$35V_2+30(V_2-V_1)+42(V_2-5)=0$$

$$-30V_1+107V_2=210$$

Upon solving Eqn. (1) and Eqn. (2), we get

$$V_{Th}=V_1=5.620\text{ V}$$

(b) To find Thevenin's equivalent resistance R_{Th}:

The circuit is redrawn after deactivating all sources, i.e., short-circuiting the voltage sources as shown in Fig. E2.30(b).

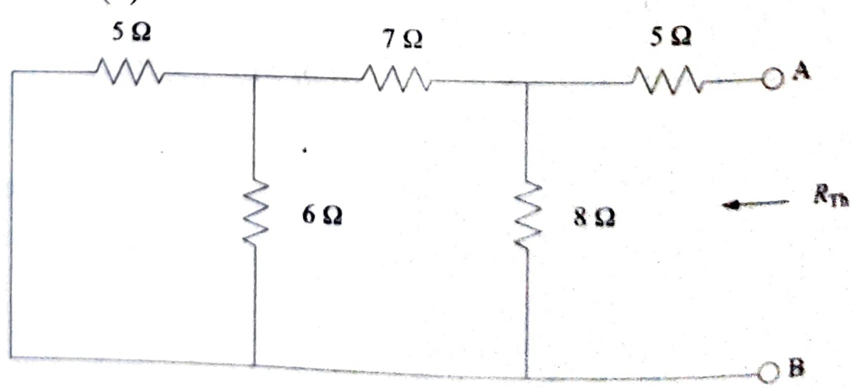

FIG. E2.30(b)

Therefore, $$R_{Th}=R_{AB}=\frac{\left(\frac{5\times 6}{5+6}+7\right)\times 8}{\left(\frac{5\times 6}{5+6}+7\right)+8}+5=9.389\ \Omega$$

(c) Thevenin's equivalent circuit is drawn as shown in Fig. E2.30(c).

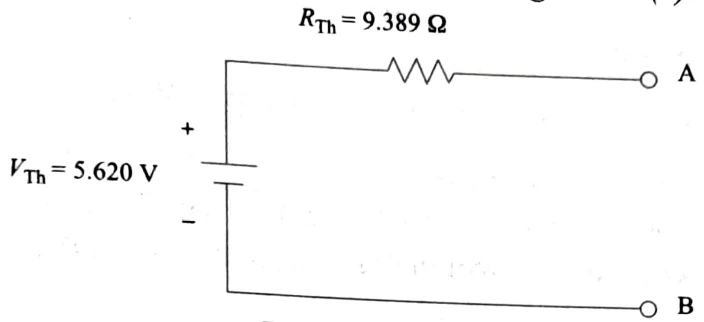

FIG. E2.30(c)

Example 2.31 Find Thevenin's equivalent circuit for the circuit shown in Fig. E2.31(a).

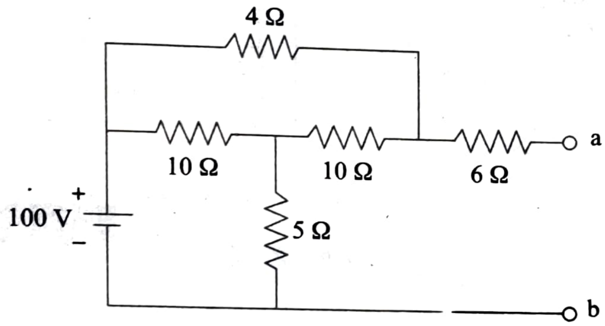

FIG. E2.31(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

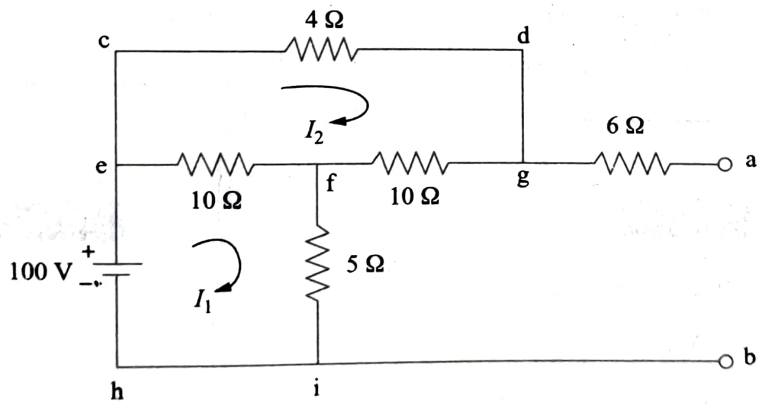

FIG. E2.31(b)

Applying KVL to the loop 'efihe', we get

$$10(I_1 - I_2) + 5I_1 = 100$$

Therefore, $\quad 15I_1 - 10I_2 = 100 \qquad (1)$

Applying KVL to the loop 'cdgec', we get

$$4I_2 + 10I_2 + 10(I_2 - I_1) = 0$$

$$-10I_1 + 24I_2 = 0 \qquad (2)$$

Upon solving Eqn. (1) and (2), we get

$$I_1 = 9.231\,\text{A}$$

and $$I_2 = 3.846\,\text{A}$$

$$V_{Th} = V_{if} + V_{fg} = 9.231 \times 5 + 3.846 \times 10 = 46.155 + 38.46 = 84.615\,\text{V}$$

(b) To find Thevenin's equivalent resistance R_{Th}:
The voltage source is short-circuited and the circuit is redrawn as shown in Fig. E2.31(c
The circuit is further reduced as shown in Fig. E2.31(d), (e) and (f).

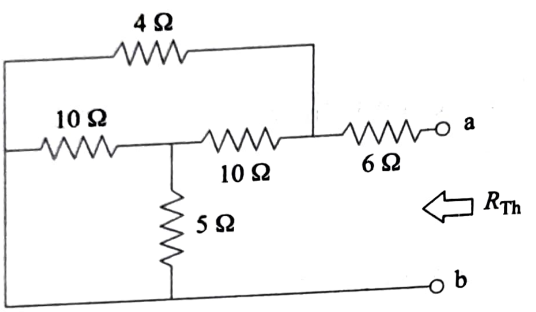

FIG. E2.31(c)

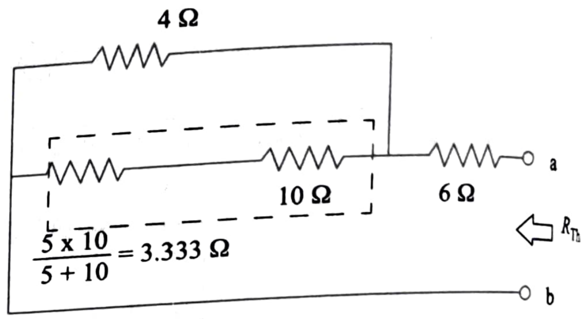

FIG. E2.31(d)

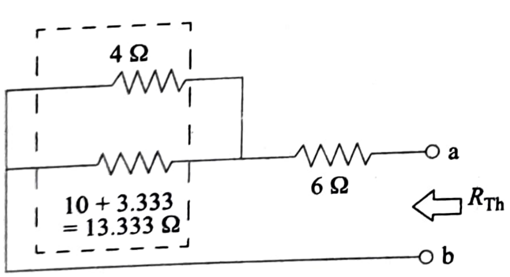

FIG. E2.31(e)

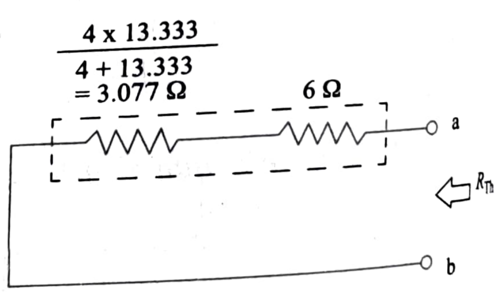

FIG. E2.31(f)

From the simplified circuit, we get

$$R_{Th} = 9.077\,\Omega$$

(c) Thevenin's equivalent circuit is shown in Fig. E2.31(g).

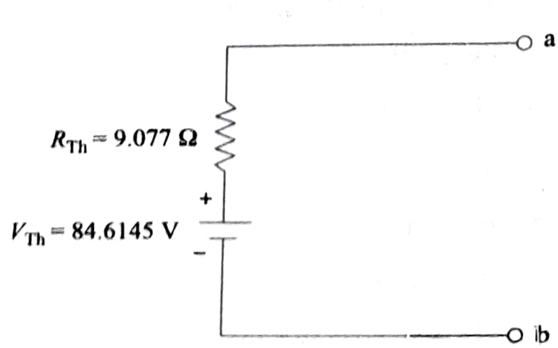

FIG. E2.31(g)

Example 2.32 For the circuit shown in Fig. E2.32(a), find the current through the $10\,\Omega$ resistor using Thevenin's theorem.

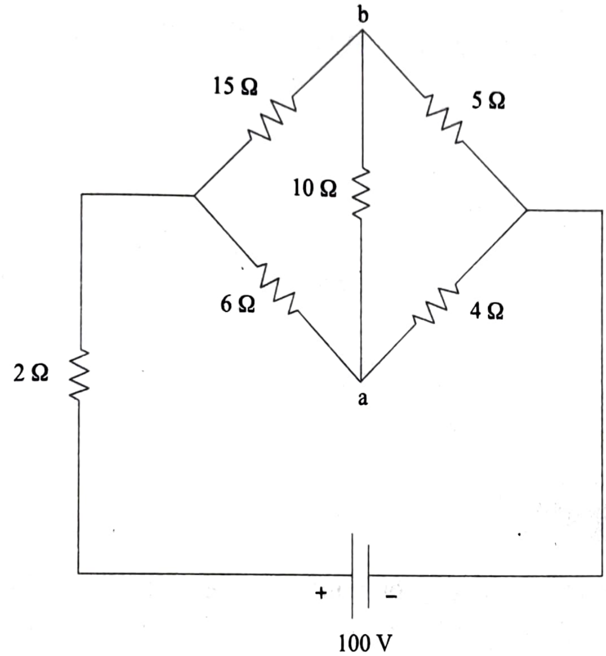

FIG. E2.32(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

The circuit is redrawn after open-circuiting the $10\,\Omega$ load resistor at terminals 'ab' as shown in Fig. E2.32(b).

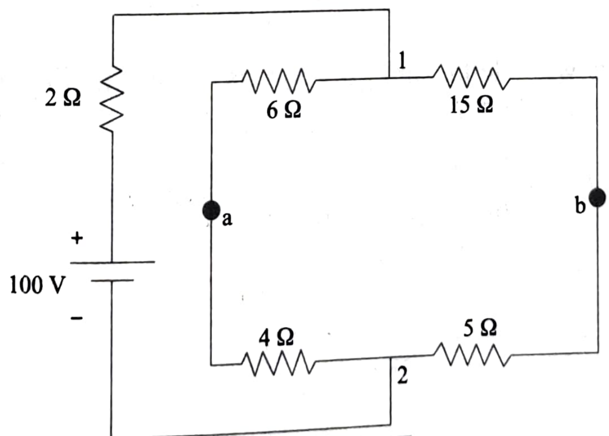

FIG. E2.32(b)

Applying KCL at node 1 for the circuit shown in Fig. E2.32(b), we get

$$\frac{V_1 - 100}{2} + \frac{V_1}{6+4} + \frac{V_1}{5+15} = 0$$

i.e.,

$$0.5(V_1 - 100) + 0.1V_1 + 0.05V_1 = 0$$

$$V_1 = 76.923\,\text{V}$$

Therefore,

$$V_{Th} = V_a - V_b = \frac{V_1}{6+4} \times 4 - \frac{V_1}{15+5} \times 5$$

$$= 30.769 - 19.230 = 11.538\,\text{V}$$

(b) To find Thevenin's equivalent resistance R_{Th}:

The circuit is redrawn after deactivating all sources, i.e., assigning all sources to zero as shown in Fig. E2.32(c). The circuit shown in Fig. E2.32(c) is simplified as shown in Fig. E2.32(d).

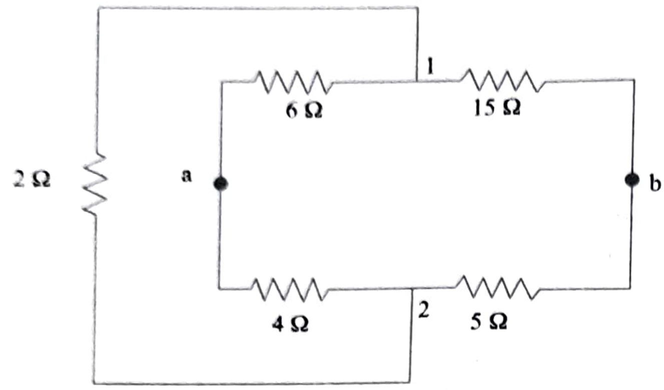

FIG. E2.32(c)

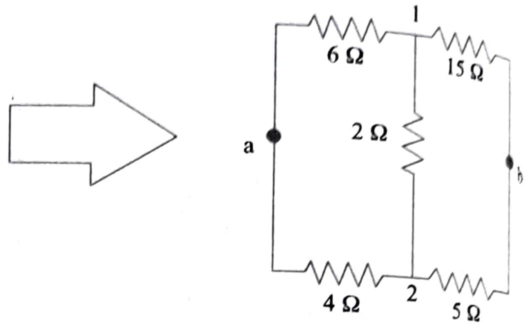

FIG. E2.32(d)

Applying star-delta conversion to the star configuration of resistors 2 Ω, 6 Ω and 15 Ω in the circuit shown in Fig. E2.32(d), we have

$$R_1 = \frac{6\times15+15\times2+2\times6}{2} = \frac{132}{2} = 66\ \Omega$$

$$R_2 = \frac{6\times15+15\times2+2\times6}{15} = \frac{132}{15} = 8.8\ \Omega$$

$$R_3 = \frac{6\times15+15\times2+2\times6}{6} = \frac{132}{6} = 22\ \Omega$$

The circuit is redrawn with delta configuration as shown in Fig. E2.32(e) and further simplified as shown in Fig. E2.32(f).

Therefore, $$R_{Th} = R_{ab} = \frac{66\times(2.75+4.074)}{66+2.75+4.074} = 6.185\ \Omega$$

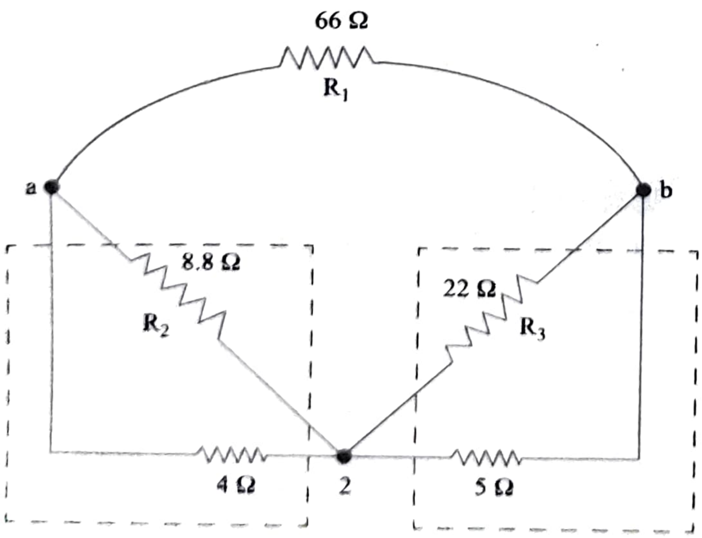

FIG. E2.32(e)

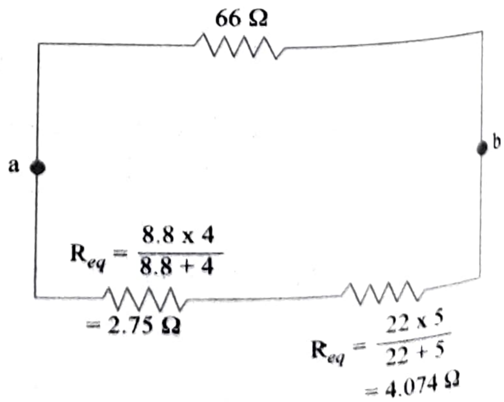

FIG. E2.32(f)

(c) Thevenin's equivalent circuit is drawn as shown in Fig. E2.32(g).

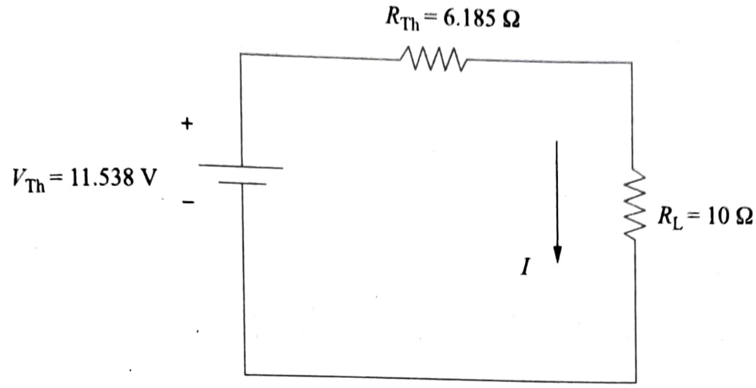

FIG. E2.32(g)

According to Ohm's law, the current through 10 Ω load resistor is

$$I = \frac{11.538}{16.185} = 0.713\,\text{A}.$$

Example 2.33 Using Thevenin's theorem, find the current through the 1 Ω resistor in the circuit shown in Fig. E2.33(a).

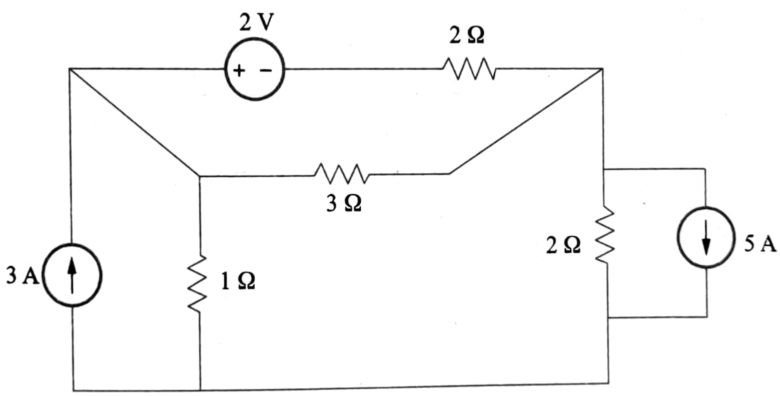

FIG. E2.33(a)

Solution

(a) To find Thevenin's equivalent voltage V_{Th}:

The circuit is redrawn after open-circuiting the 1 Ω load resistor and transforming the 5A current source to its equivalent voltage source as shown in Fig. E2.33(b).

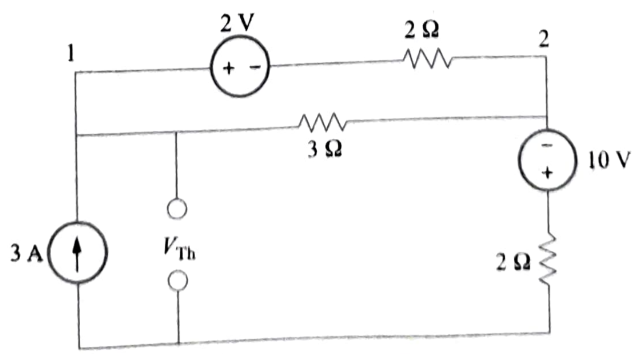

FIG. E2.33(b)

Therefore, according to Ohm's law, the current through 1Ω resistor is

$$I = \frac{0.8}{4.2} = 0.19\,\text{A}$$

Example 2.34 Determine the Thevenin's equivalent across the terminals A and B of the circuit shown in Fig. E2.34(a)

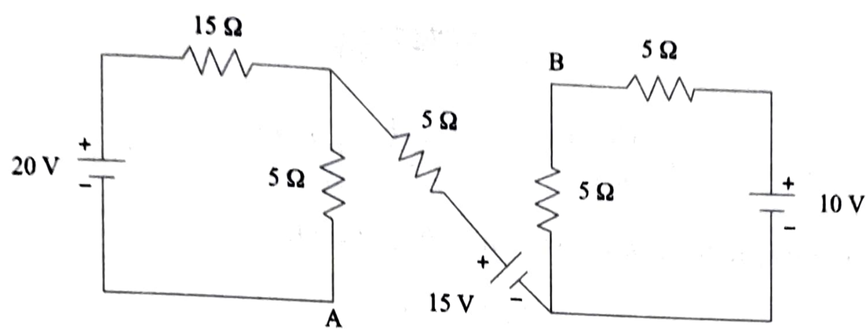

FIG. E2.34(a)

Solution

The given circuit is redrawn as shown in Fig. E2.34(b).

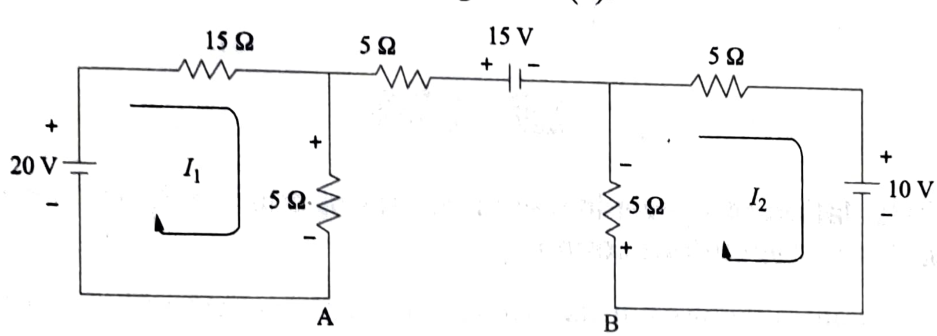

FIG. E2.34(b)

(a) To find Thevenin's equivalent voltage V_{Th}:
Applying KVL to the circuit shown in Fig. E2.34(b), we get
At loop 1,

$$(5+15)I_1 = 20$$

or
$$I_1 = \frac{20}{20} = 1\text{A}$$

At loop-2,

$$(5+5)I_2 = -10$$

or
$$I_2 = -1\text{A}$$

Therefore, $V_{Th} = V_{AB} = 15 - 5I_2 - 5I_1 = 15\text{V}$

(b) To find Thevenin's equivalent resistance, R_{Th}:
The circuit is redrawn after deactivating all sources, i.e., assigning all sources to zero as shown in Fig. E2.34(c).

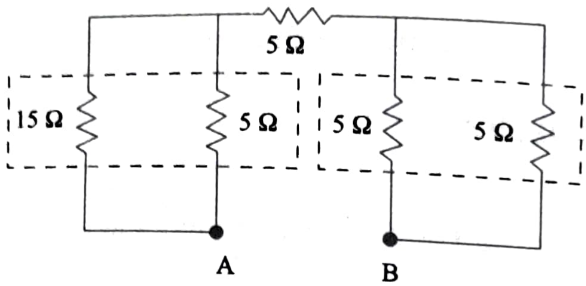

FIG. E2.34(c)

$$R_{Th} = R_{AB} = \frac{15\times 5}{15+5} + 5 + \frac{5\times 5}{5+5} = 11.25\ \Omega$$

(c) Thevenin's equivalent circuit is shown in Fig. E2.34(d).

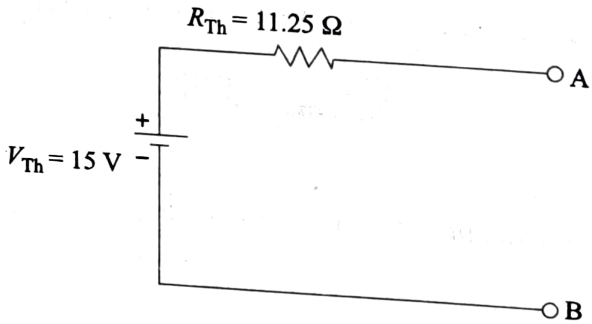

FIG. E2.34(d)

2.6.1 Calculation of Thevenin's equivalent impedance Z_{Th} for the circuit with the dependent source

Different procedure is followed in calculating Thevenin's equivalent impedance (Z_{Th}) for the circuit consisting of dependent sources. The dependent voltage and current sources are kept as they are while calculating Z_{Th}.
As an example, consider the circuit shown in Fig. 2.9(f), consisting of a dependent source NI_x.

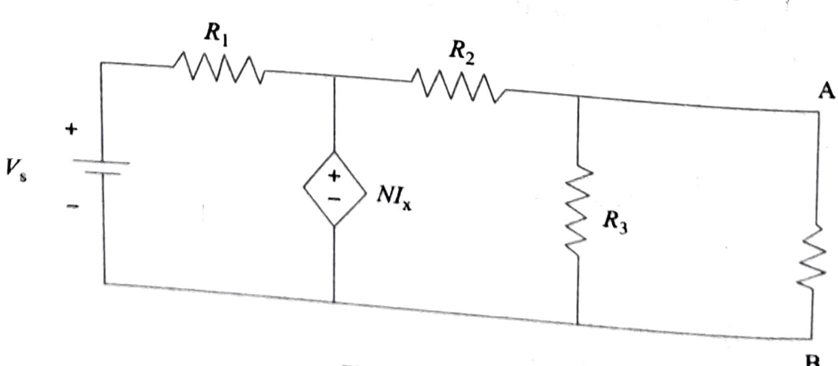

FIG. 2.9(f)

Step 1: Find the open circuit voltage V_{Th} by removing the load resistor R_L as shown in Fig. 2.9(g).