

$$= -5.97 + j29.46\Omega$$

The resultant equivalent delta can be obtained as shown in Fig. E2.20(b).

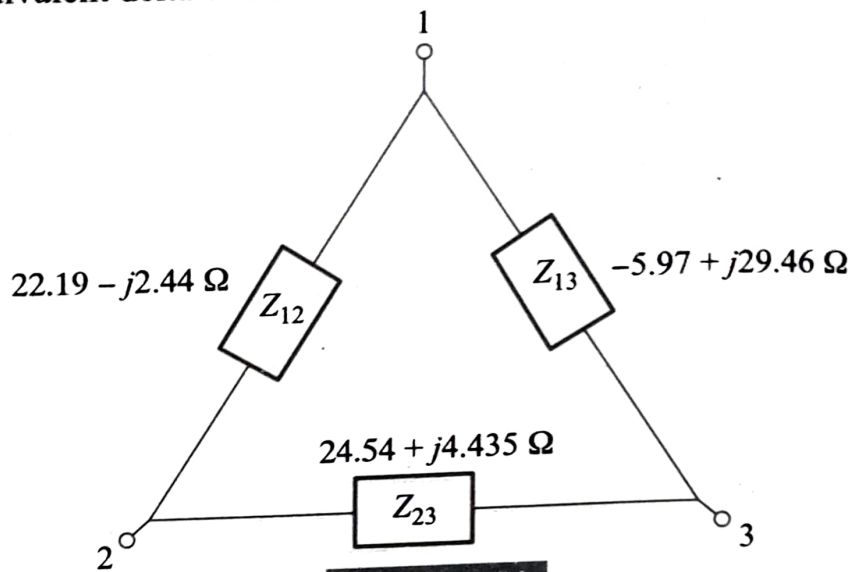


FIG. E2.20(b)

**Example 2.21** For the circuit shown in Fig. E2.21(a), using star-delta transform obtain the voltage to be applied across AB in order to drive a current of 5 A into the circuit.

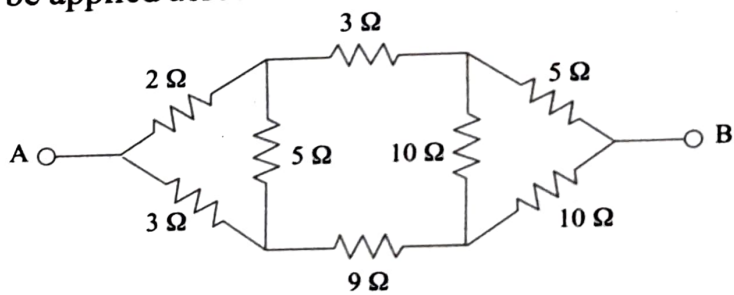


FIG. E2.21(a)

**Solution**

The equivalent star for the delta configuration of resistors  $2 \Omega$ ,  $3 \Omega$  and  $5 \Omega$  can be obtained

$$R_1 = \frac{2 \times 3}{2 + 3 + 5} = 0.6 \Omega$$

$$R_2 = \frac{2 \times 5}{2 + 3 + 5} = 1 \Omega$$

$$R_3 = \frac{3 \times 5}{2 + 3 + 5} = 1.5 \Omega$$

The star equivalent configuration is drawn as shown in Fig. E2.21(b).

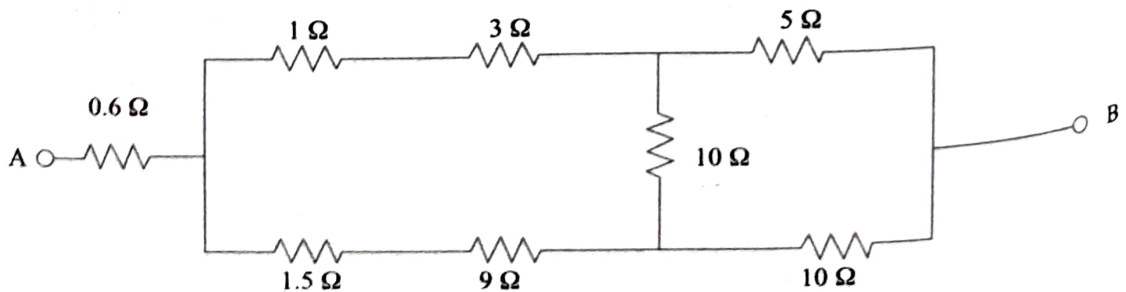


FIG. E2.21(b)

Similarly, the equivalent star as shown in Fig. E2.21(c) for the delta configuration of resistors  $5\ \Omega$ ,  $10\ \Omega$  and  $10\ \Omega$  is obtained as

$$R_1 = \frac{5 \times 10}{5 + 10 + 10} = 2\ \Omega$$

$$R_2 = \frac{5 \times 10}{5 + 10 + 10} = 2\ \Omega$$

$$R_3 = \frac{10 \times 10}{5 + 10 + 10} = 4\ \Omega$$

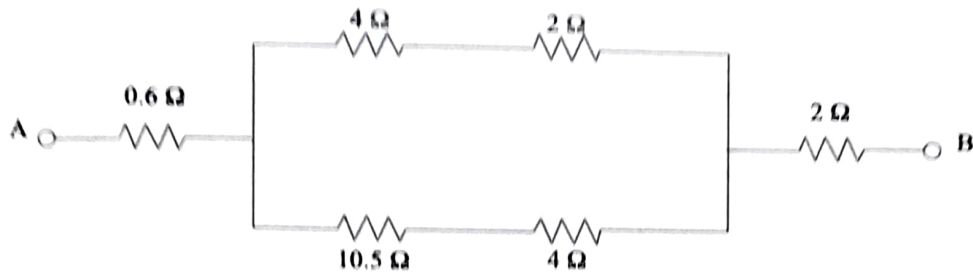


FIG. E2.21(c)

The total resistance between the terminal AB is

$$R_{AB} = 0.6 + \frac{6 \times 14.5}{6 + 14.5} + 2 = 6.843\ \Omega$$

Given, the current through the terminal AB,  $I_{AB} = 5\ \text{A}$ .

Therefore, according to Ohm's law, the voltage across AB is

$$V_{AB} = I_{AB} R_{AB} = 5 \times 6.843 = 34.215\ \text{V}.$$

**Example 2.22** Calculate the resistance  $R_{ab}$  when all the resistance values are equal to  $1\ \Omega$  for the circuit in Fig. E2.22.

**Solution**

Given: The value of all resistances in the circuit is  $1\ \Omega$ . The equivalent star for the delta configuration of resistors connected to the nodes 'a-e-b' is obtained as

$$R_1 = R_2 = R_3 = \frac{1 \times 1}{1 + 1 + 1} = \frac{1}{3}\ \Omega$$

Similarly, star conversion of the delta configuration of resistors connected to the nodes 'c-d-e' yields

$$R_1 = R_2 = R_3 = \frac{1 \times 1}{1 + 1 + 1} = \frac{1}{3}\ \Omega$$

The star equivalent configuration is drawn as shown in Fig. E2.22(b) and its simplified circuit is shown in Fig. E2.22(c).

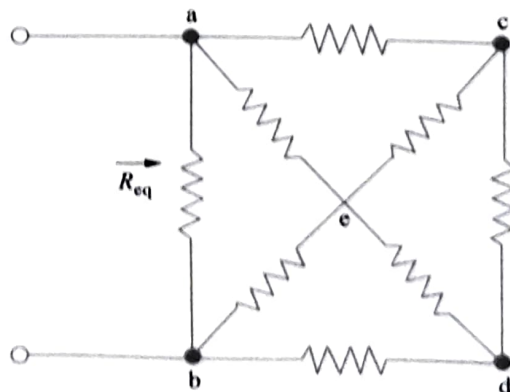


FIG. E2.22

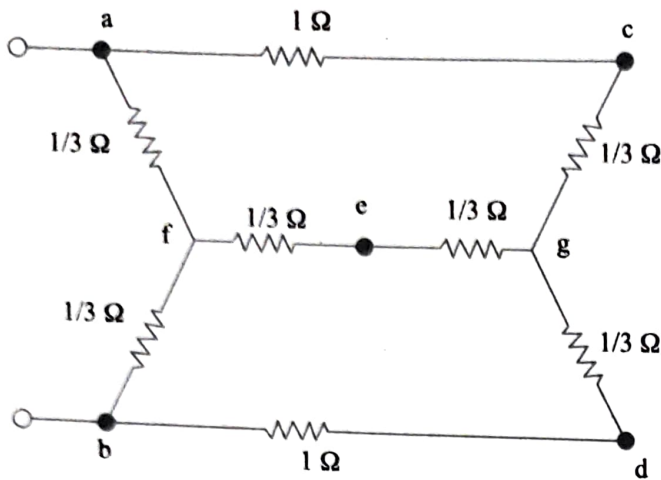


FIG. E2.22(b)

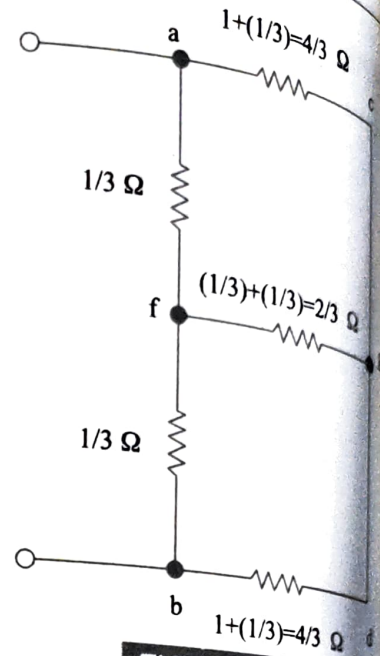
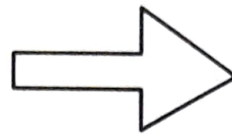


FIG. E2.22(c)

The equivalent star as shown in Fig. E2.22(d) for the delta configuration of resistors connected to the nodes 'b-g-f' is obtained as

$$R_1 = \frac{\frac{2}{3} \times \frac{1}{3}}{\frac{2}{3} + \frac{1}{3} + \frac{4}{3}} = 0.095 \Omega$$

$$R_2 = \frac{\frac{1}{3} \times \frac{4}{3}}{\frac{2}{3} + \frac{1}{3} + \frac{4}{3}} = 0.190 \Omega$$

$$R_3 = \frac{\frac{2}{3} \times \frac{4}{3}}{\frac{2}{3} + \frac{1}{3} + \frac{4}{3}} = 0.381 \Omega$$

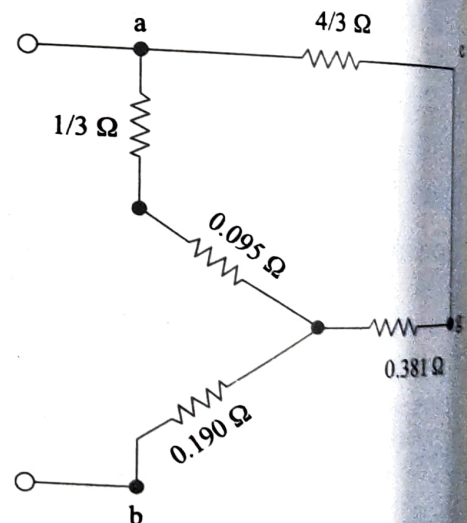


FIG. E2.22(d)

The circuit is further simplified using network reduction techniques as shown in Fig. E2.22(e).

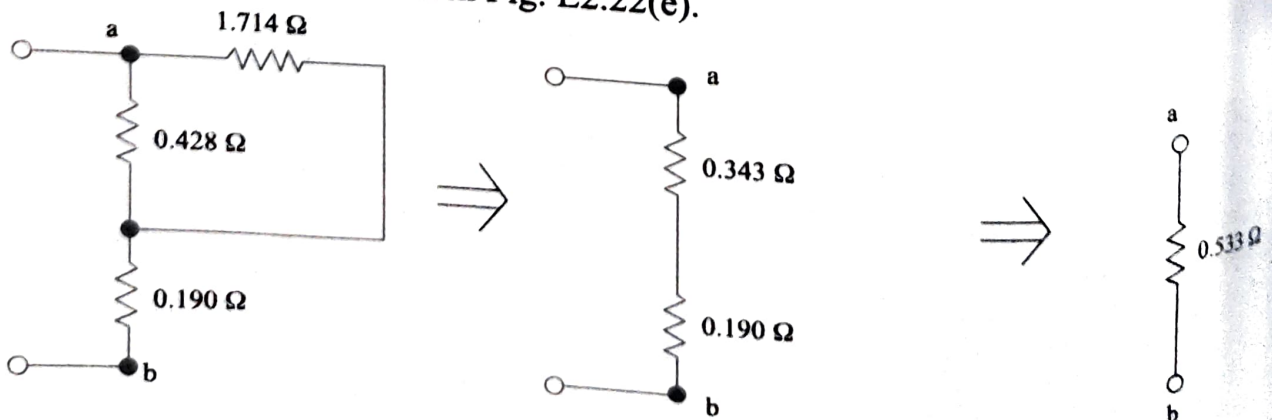


FIG. E2.22(e)

Hence, the resistance between terminal ab,  $R_{ab} = 0.533 \Omega$ .

**Example 2.23** For the circuit shown in Fig. E2.23(a), using star-delta conversion, find the current flowing through the  $10\Omega$  resistor.

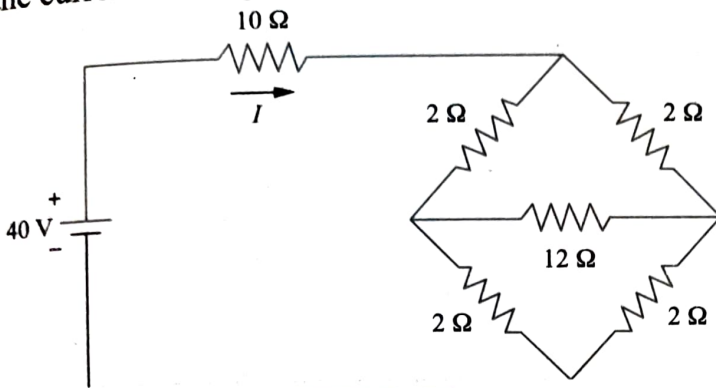


FIG. E2.23(a)

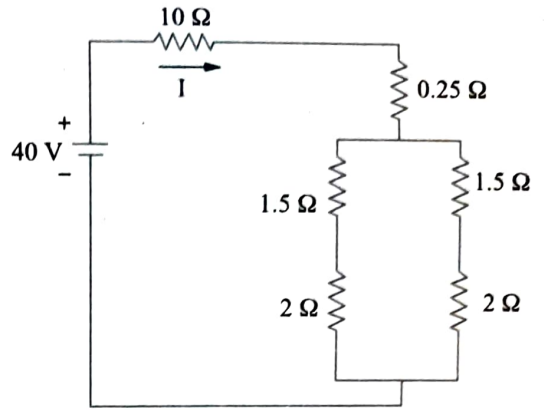


FIG. E2.23(b)

**Solution**

The equivalent star as shown in Fig. E2.23(b) for the delta configuration of resistors  $2\Omega$ ,  $2\Omega$  and  $12\Omega$  can be obtained as

$$R_1 = \frac{2 \times 2}{2 + 2 + 12} = 0.25 \Omega$$

$$R_2 = \frac{2 \times 12}{2 + 2 + 12} = 1.5 \Omega$$

$$R_3 = \frac{2 \times 12}{2 + 2 + 12} = 1.5 \Omega$$

The total resistance of the equivalent circuit shown in Fig. E2.23(b) is calculated as

$$R_T = 10 + 0.25 + \frac{3.5 \times 3.5}{3.5 + 3.5} = 12 \Omega$$

Hence, according to Ohm's law, the current through  $10\Omega$  resistor is

$$I_{10\Omega} = \frac{V}{R_T} = \frac{40}{12} = 3.333 \text{ A}$$

**Example 2.24** Find the value of  $R$  and current through it in the circuit shown in Fig. E2.24(a) when the branch AD carries zero current.

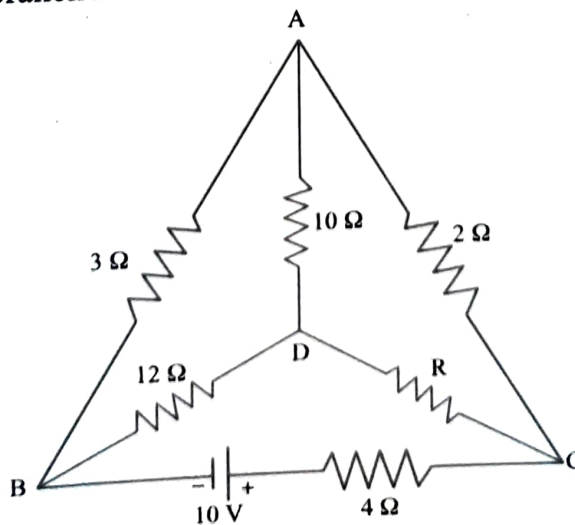


FIG. E2.24(a)

**Solution**

Since the current in the branch AD is zero, it can be replaced by an open circuit as shown in Fig. E2.24(b). The circuit is further reduced as shown in Fig. E2.24(b), (c) and (d).

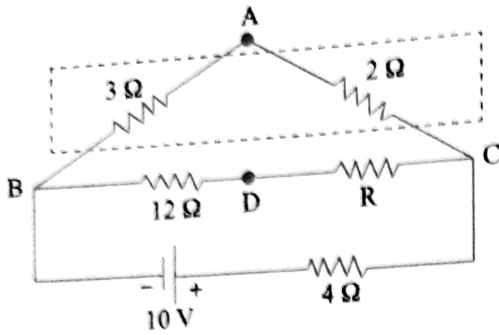


FIG. E2.24(b)

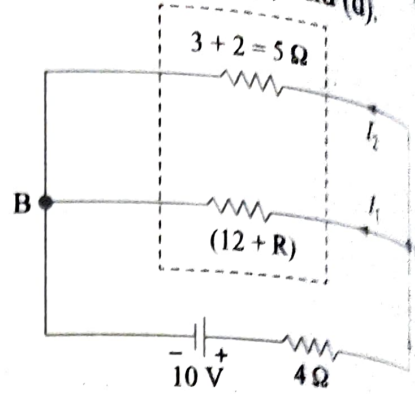


FIG. E2.24(c)

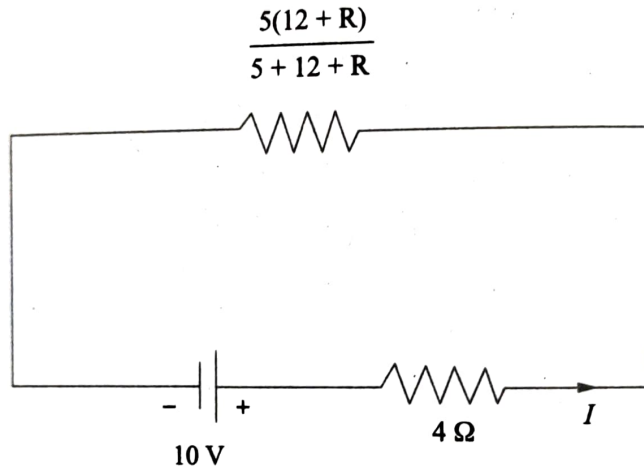


FIG. E2.24(d)

Therefore, the current, 
$$I = \frac{10}{4 + \frac{5 \times (12 + R)}{5 + 12 + R}}$$

$$= \frac{10(17 + R)}{68 + 4R + 60 + 5R} = \frac{10(17 + R)}{9R + 128}$$

Applying the current division rule to the circuit shown in Fig. E2.24(c), we get

$$I_1 = I \times \frac{5}{(5 + 12 + R)} = \frac{10(17 + R)}{(9R + 128)} \times \frac{5}{(17 + R)} = \frac{50}{(9R + 128)}$$

and

$$I_2 = I \times \frac{(12 + R)}{(5 + 12 + R)} = \frac{10(17 + R)}{(9R + 128)} \times \frac{(12 + R)}{(17 + R)} = \frac{10(12 + R)}{(9R + 128)}$$

Since the branch current AD is zero, the nodes A and D in the circuit shown in Fig. E2.24(a) must have equal potential.

i.e., 
$$V_{AC} = V_{CD}, \text{ or } I_2 \times 2 = I_1 \times R$$

or, 
$$\frac{2 \times 10 \times (12 + R)}{9R + 128} = \frac{50R}{9R + 128}$$

Therefore, 
$$240 + 20R = 50R$$
  

$$R = 8 \Omega.$$

**Example 2.25** Find the equivalent resistance between B and C, for the circuit shown in Fig. E2.25(a).

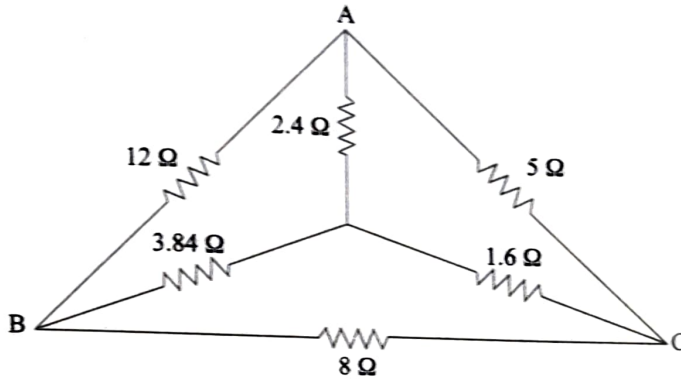


FIG. E2.25(a)

**Solution**

The star configuration of resistors 24 Ω, 1.6 Ω and 3.84 Ω is converted to delta configuration as shown in Fig. E2.25(b), where

$$R_{AB} = \frac{3.84 \times 1.6 + 2.4 \times 1.6 + 3.84 \times 2.4}{1.6} = 3.84 + 2.4 + \frac{3.84 \times 2.4}{1.6} = 12 \Omega$$

$$R_{BC} = \frac{3.84 \times 1.6 + 2.4 \times 1.6 + 3.84 \times 2.4}{2.4} = \frac{3.84 \times 1.6}{2.4} + 1.6 + 3.84 = 8 \Omega$$

$$R_{CA} = \frac{3.84 \times 1.6 + 2.4 \times 1.6 + 3.84 \times 2.4}{3.84} = 1.6 + \frac{2.4 \times 1.6}{3.84} + 2.4 = 5 \Omega$$

The circuit shown in Fig. E2.25(b) is reduced further as shown in Fig. E2.25(c) and (d).

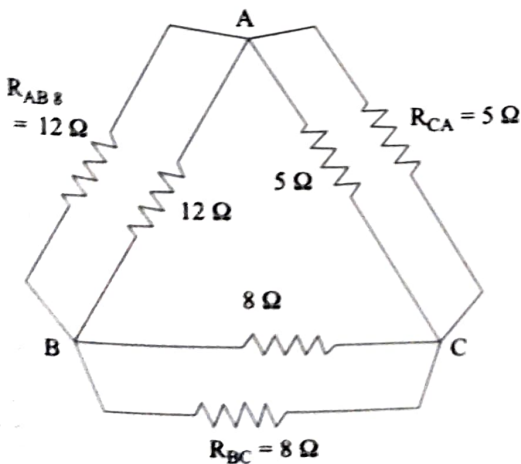


FIG. E2.25(b)

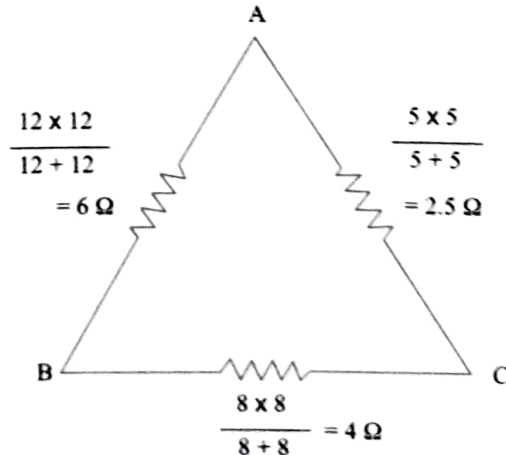


FIG. E2.25(c)